Beyond the Plane-Wave Impulse Approximation in $(e,e'p)$ Reactions

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The orthonormal-correlated-states method is used to study the $^4\mathrm{He}(e,e'p)^3\mathrm{H}$ reaction. The effects of short-range correlations, orthogonality corrections, $p+{}^3\mathrm{H}$ interaction in the final states, and two-body charge and current operators brings the theoretical predictions into fair agreement with the NIKHEF and Saclay data.

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In the past decade single- and double-coincidence electron-scattering experiments performed on few-body nuclei[1-7] have clearly proven the inadequacy, at intermediate energies, of a theoretical description of the quasielastic nucleon-knockout process based on the plane-wave impulse approximation (PWIA). For example, PWIA calculations of the ^{3}H, ^{3}He, and ^{4}He longitudinal and transverse response functions measured in (e,e') reactions[1-3] predict more strength than is experimentally observed in the quasielastic peak region,[8,9] particularly for the longitudinal response at low momentum transfer, not surprisingly so since the Coulomb sum rule is violated in the PWIA.[10]

Progress beyond the PWIA has been made in the description of three-nucleon scattering states at the relatively high excitation energies of interest in studying the electromagnetic response either by solving the Faddeev equations in the continuum with simple interaction models,[11] or by approximately including some of the final-state interactions affecting the knocked-out nucleon.[12]

The objective of the present work is to extend the orthonormal-correlated-states (OCS) approach,[10,13] which has recently been shown to provide a reasonable description of the ^{3}He and ^{3}H inclusive data,[10,14] to the four-nucleon system, and to report on a microscopic calculation of the $^4\mathrm{He}(e,e'p)^3\mathrm{H}$ cross section in the specific kinematic setups of the NIKHEF[6] and Saclay[7] experiments.

In the one-photon-exchange approximation the $^Ai(e,e'p)^{(A-1)}f$ cross section is expressed as[15]

$$\frac{d^6\sigma}{dE_e'\,d\Omega_e'\,dE\,d\Omega}=\sigma_M pE(V_LR_L+V_TR_T+V_{LT}R_{LT}\cos\phi+V_{TT}R_{TT}\cos2\phi)\,,$$

where σ_M is the Mott cross section, E_e',E and Ω_e',Ω are the energies and solid angles of the final electron and proton, and ϕ is the angle between the electron-scattering plane and that specified by the electron three-momentum transfer $\mathbf{k}$ and the proton momentum $\mathbf{p}$. The coefficients V_a are defined in terms of the electron variables, while the structure functions R_a involve matrix elements of the charge and current operators between the initial ^Ai and final $^{(A-1)}f+p$ nuclear states and depend on the momenta k and p, the angle θ between them, and the electron energy transfer ω. In the reaction under consideration the required matrix elements are given by

$$^{(-)}\langle p+{}^3\mathrm{H};\mathbf{p}\mathbf{p}_3,\sigma\sigma_3|O_{L,T}(\mathbf{k})|{}^4\mathrm{He}\rangle\,,$$

where $|^4\mathrm{He}\rangle$ is the ^{4}He ground state, $O_{L,T}(\mathbf{k})$ are the charge (L) and current (T) operators, $|p+{}^3\mathrm{H};\mathbf{p}\mathbf{p}_3,\sigma\sigma_3\rangle^{(-)}$ is the $p+{}^3$H ingoing scattering state, $\mathbf{p}_3=\mathbf{k}-\mathbf{p}$ is the ^{3}H momentum (it is related to the missing momentum $\mathbf{p}_m$ by $\mathbf{p}_m=-\mathbf{p}_3$), and σ and σ_3 are the p and ^{3}He spin projections, respectively.

Correlated-basis functions (CBF) for the $1+3$ states are written as

$$\psi^{(-)}_{\mathbf{q}\sigma\tau,\sigma_3\tau_3}(1+3)=\tfrac{1}{2}\sum_p(-)^pS[F_{ij}F_{ik}F_{il}]\eta^{(-)}_{\mathbf{q}\sigma\tau}(i)\phi_{\sigma_3\tau_3}(jkl)\,,$$

where the symmetrized product $S[F_{ij}F_{ik}F_{il}]$ describes short-range correlations between the spectator nucleon i in spin-isospin state $\sigma\tau$ and the bound cluster of nucleons jkl in spin-isospin state $\sigma_3\tau_3$, and $\mathbf{q}$ is the relative momentum between i and jkl. The sum over permutations p ensures the antisymmetry of the CBF wave function. The pair-correlation operator F_{ij} contains central, spin, isospin, and tensor correlations, and satisfies the healing requirement $F_{ij}(r_{ij}\geq d)=1$ ($d=1.9$ fm). It is determined from the NN interaction with the method discussed in Ref. 10. The ^{4}He and three-nucleon bound states are represented by variational wave functions obtained for the Argonne[16] two-nucleon and Urbana-VII three-nucleon[17] interaction models. The quality of these wave functions has previously been assessed by quantitatively successful predictions for the binding energies,[17] elastic electromagnetic form factors,[18,19] and asymptotic D/S ratios in the $d+n$, $d+p$, and $d+d$ breakup channels of ^{3}H, ^{3}He, and ^{4}He, respectively. The spectator wave function $\eta^{(-)}_{\mathbf{q}\sigma\tau}$ is taken to be the scattering solution of a Schrödinger equation containing a complex energy-dependent optical-potential of the form

$$V^o(r;T_q)=V(r;T_q)+V^\tau(r;T_q)\boldsymbol{\tau}\cdot\boldsymbol{\tau}_3\,,$$

where T_q is the relative energy between the clusters i and

jkl. The term $V(r;T_q)$ has central, spin-orbit, and l-dependent components, and was determined by van Oers *et al.*[20] from an analysis of $p+{}^3$He elastic-scattering data in the energy range $T_q=117$–450 MeV. I parametrized the charge-exchange term $V^\tau(r;T_q)$ with a complex Woods-Saxon potential whose real and imaginary parts have depths linearly dependent on T_q, $v^a_{R,I}+v^b_{R,I}T_q$ (MeV), with $v^a_R,v^a_I=-30.4,-3.57$ MeV, and $v^b_R,v^b_I=0.175,0.0134$, radii $R_R,R_I=1.2,1.8$ fm, and diffusenesses $a_R,a_I=0.15,0.20$ fm, and obtained the values for these parameters by fitting the available $p+{}^3$H elastic and charge-exchange data at $T_q=43$ (Ref. 21) and 117 MeV.[22] It should be noted that the $p+{}^3$H relative energy in the NIKHEF and Saclay kinematics covers the range 26–168 MeV. In the results presented below I have included all partial waves in $\eta^{(-)}_{\mathbf{q}\sigma\tau}$ with full account of interaction effects in those with $l\leq 5$, and have explicitly verified for the NIKHEF kinematical setup denoted I (for which the $p+{}^3$H relative energy is 75 MeV) that the numerical importance of final-state interactions in higher partial waves is small.

The CBF states have correct energies; however, they are not orthogonal. The OCS basis is obtained by orthonormalizing the CBF states by a combination of Schmidt and Löwden transformations designed to preserve their energies.[10,13]

In the lowest order of correlated-basis theory, the $p+{}^3$H OCS states are used to calculate the amplitudes. The electroexcitation operator $O_{L,T}(\mathbf{k})$ contains one- and two-body currents. The one-body current operators have the standard impulse-approximation expressions (with inclusion, however, of the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator). The two-body charge and current operators consist of a "model-independent" part, which is constructed from the NN interaction with the methods described in Refs. 18 and 19, and a "model-dependent" part of far less numerical importance associated with π- and ρ-exchange excitation of an intermediate Δ_{33} resonance, and the $\rho\pi\gamma$ and $\omega\pi\gamma$ mechanisms.[18] The Höhler parametrization[23] is used for the electromagnetic form factors of the nucleon. Since the corrections due to the Löwdin transformation are expected to be small,[10] only the Schmidt orthogonalization of the $p+{}^3$H states to the ground state is retained in the calculation of the amplitudes, which is then carried out without further approximations with the Monte Carlo integration techniques developed in Refs. 10 and 18.

In Figs. 1 and 2 the predicted values for the cross section are compared with those measured at NIKHEF[6] in two different kinematic setups denoted as I and II, for both of which the $p+{}^3$H relative energy is 75 MeV. The cross-section values obtained by including the effects of short-range correlations, orthogonality corrections, final-state interactions (FSI), and two-body charge and current operators are displayed by the solid curves, and

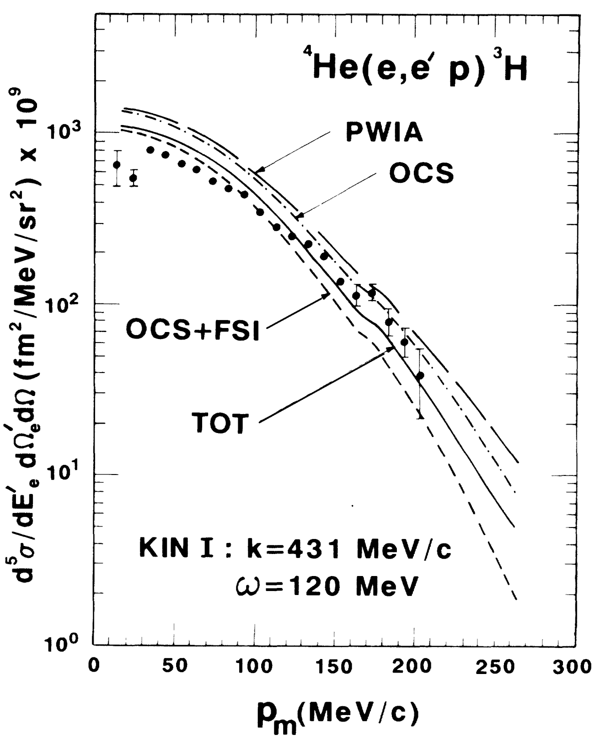

FIG. 1. Fivefold differential cross sections obtained for kinematics I in the PWIA and in the approximations OCS, OCS+FSI, and TOT.

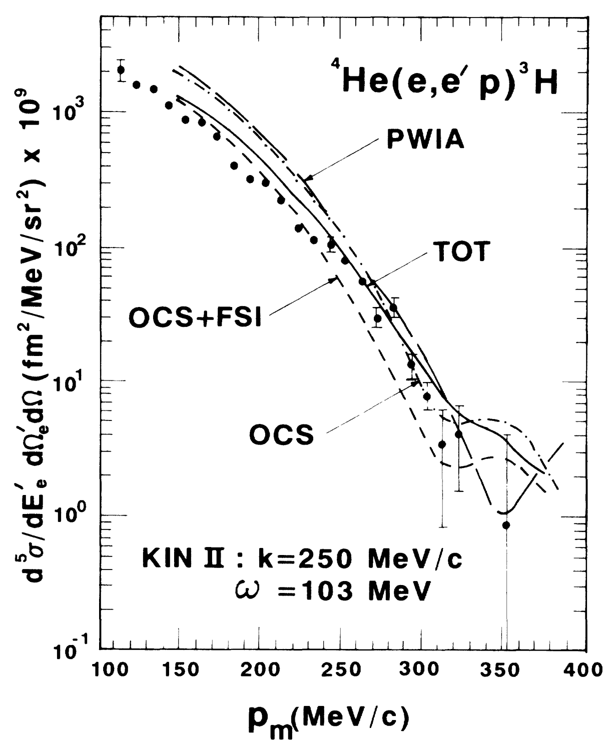

FIG. 2. Same as in Fig. 1, but for kinematics II.

are in fair agreement with the data. The results calculated in the PWIA and in the approximations denoted as OCS and OCS+FSI, in which the orthogonality corrections and short-range correlations induced by the F_{ij}'s are retained, but the spectator wave function is either a plane wave or a scattering solution corresponding to the optical potential V^o, are also shown. The contributions due to two-body charge operators are found to be negligible (for example, at $p_m = 150$ MeV/c in kinematics I they reduce to R_L by only 2%); however, those associated with two-body current operators are important in both kinematics, particularly for $p_m \geq 200$ MeV/c, where they increase the OCS+FSI predictions by more than 50%. It should be noted that these kinematics correspond to different regions of the inclusive ω spectrum: I is slightly off the top of the quasielastic peak, whereas II is in the so-called "dip" region between the quasielastic and delta production peaks. The present results reinforce the findings of Ref. 24, in which an analysis of the ^{4}He(e,e') transverse data[3] based on the non-energy- and energy-weighted sum rules showed that two-body mechanisms could significantly enhance the inclusive transverse response, even in the quasielastic regime. Final-state interactions play a dominant role by substantially reducing the OCS cross-section values. However, neglecting the isospin-exchange term in V^o has a small effect in the p_m range where kinematics I and II overlap (for example, at $p_m = 170$ MeV/c it leads to an increase of 3% in kinematics I and to a reduction of 13% in kinematics II of the full cross section), which indicates that corrections due to two-step processes $(e,e',n)(n,p)$ are not important.[6]

In parallel kinematics (**p** parallel to **k**) the LT and TT structure functions vanish,[15] thus allowing us to make a Rosenbluth separation of the exclusive cross section into its longitudinal and transverse parts. The ratios $\rho_L = R_L/(G_E^p)^2$ and $\rho_T = (2m^2/k^2) R_T/(G_M^p)^2$, where G_E^p and G_M^p are the proton electric and magnetic form factors taken as functions of the four-momentum transfer, are given in Table I for the specific kinematics investigated at Saclay.[7] In the PWIA ρ_L and ρ_T are both proportional to the spectral function,[25] which for the $p + {}^3$H channel only depends upon p_m. The transverse data are in reasonable agreement with theory, while the longitudinal ones are significantly smaller than the theoretical predictions. (The present results appear to be in qualitative accord with those obtained by Laget,[12] although the lack of a detailed account of his calculation in Ref. 12 prevents a more thorough comparison.) This quenching is reminiscent of that observed in the inclusive and exclusive longitudinal data on heavier nuclei, such as ^{40}Ca,[26] which led to the suggestion of a medium-induced modification of the nucleon electromagnetic structure.[27] However, no missing strength is found in the ^{4}He longitudinal response function measured at Bates in (e,e') experiments.[3] Indeed, an analysis of the Coulomb sum rule in this nucleus as well as in ^{2}H, ^{3}H, and ^{3}He was found to be consistent with the free-proton form factor,[28] a conclusion which is further corroborated by the y-scaling behavior exhibited by the available electron-scattering data at very high k (GeV/c) and low ω.[29] On the other hand, an important factor of uncertainty is associated with the sensitivity of the calculated cross sections to the optical potential V^o, which may not be accurate in the energy range relevant to the NIKHEF and Saclay kinematics. For example, artificially increasing by 30% the depths of radii of the imaginary central components of V^o drastically reduces the discrepancy between the theoretical and empirical values, as is shown in Table I, although not in the largest k's (≥ 650 MeV/c) considered. In fact, more accurate data in this k range are needed in order to ascertain whether or not the presently observed lack of longitudinal and transverse strength persists (or increases) with increasing momentum transfer, a feature, which if confirmed, could be in dramatic contrast with theoretical expectations.

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TABLE I. The experimental ratios (Expt.) ρ_L and ρ_T and those obtained in the PWIA and in the approximations OCS, OCS+FSI, and TOT. The total results (TOT) obtained by increasing the depths and radii of the imaginary central components of V^o by 30% are listed in parentheses. At $k = 829$ MeV/c only a backward electron angle measurement was made (Ref. 7).

k	p_m			ρ_L [(GeV/c)$^{-3}$sr^{-1}]				ρ_T [(GeV/c)$^{-3}$sr^{-1}]		
(MeV/c)	(MeV/c)	PWIA	OCS	OCS+FSI	TOT	Expt.	OCS	OCS+FSI	TOT	Expt.
421	+30	231	217	172	170 (130)	110 ± 7	226	175	190 (151)	167 ± 8
299	−90	108	98.2	65.2	65.0 (49.3)	44.0 ± 2.5	103	71.1	77.9 (61.3)	53.8 ± 4.6
380	+90	108	79.7	78.8	77.8 (55.3)	56.5 ± 3.2	98.1	79.8	90.1 (67.1)	84.2 ± 4.2
544	+90	108	104	88.6	86.4 (67.5)	57.7 ± 8.7	108	88.5	99.7 (80.9)	93.8 ± 3.5
572	+90	108	103	88.4	86.0 (68.3)	70.1 ± 10.6	106	88.3	99.6 (81.9)	90.4 ± 4.7
650	+90	108	105	91.1	88.3 (73.8)	43.2 ± 13.3	106	90.7	102 (87.7)	87.4 ± 4.7
829	+90	108	105	94.5	91.0 (83.5)		107	97.7	109 (100)	65.0 ± 2.0
682	+190	11.5	9.91	9.85	8.60 (6.52)	2.8 ± 3.8	10.3	8.95	11.9 (9.35)	18.4 ± 1.1

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[1]C. Marchand *et al.*, Phys. Lett. **153B**, 29 (1985).

[2]K. Dow *et al.*, Phys. Rev. Lett. **61**, 1706 (1988).

[3]K. F. von Reden *et al.*, Phys. Rev. C **41**, 1084 (199).

[4]E. Jans *et al.*, Phys. Rev. Lett. **49**, 974 (1982).

[5]C. Marchand *et al.*, Phys. Rev. Lett. **60**, 1703 (1988).

[6]J. F. J. van den Brand *et al.*, Phys. Rev. Lett. **60**, 2006 (1988); J. F. J. van den Brand, Ph.D. thesis, NIKHEF, 1988.

[7]A. Magnon *et al.*, Phys. Lett. B **222**, 352 (1989); A. Magnon (private communication).

[8]H. Meier-Hajduk *et al.*, Nucl. Phys. **A395**, 332 (1983).

[9]C. Ciofi degli Atti, E. Pace, G. Salme, Phys. Lett. **141B**, 14 (1984).

[10]R. Schiavilla and V. R. Pandharipande, Phys. Rev. C **36**, 2221 (1987).

[11]E. van Meijgaard and J. A. Tjon, Phys. Rev. Lett. **57**, 3011 (1986); **61**, 1461 (1988); Phys. Lett. B **228**, 307 (1989).

[12]J. M. Laget, Phys. Rev. **151B**, 325 (1985); B **199**, 493 (1987).

[13]S. Fantoni and V. R. Pandharipande, Phys. Rev. C **37**, 1697 (1988).

[14]R. Schiavilla, Phys. Lett. B **218**, 1 (1989).

[15]S. Frullani and J. Mougey, Adv. Nucl. Phys. **14**, 1 (1984).

[16]R. B. Wiringa, R. A. Smith, and T. A. Ainsworth, Phys. Rev. C **29**, 1207 (1984).

[17]R. Schiavilla, V. R. Pandhairpande, and R. B. Wiringa, Nucl. Phys. **A449**, 219 (1986).

[18]R. Schiavilla, V. R. Pandharipande, and D. O. Riska, Phys. Rev. C **40**, 2294 (1989); R. Schiavilla and D. O. Riska, University of Helsinki Report No. HU-TFT-90-15, 1990 (to be published).

[19]R. Schiavilla, V. R. Pandharipande, and D. O. Riska, Phys. Rev. C **41**, 309 (1990).

[20]W. T. H. van Oers *et al.*, Phys. Rev. C **25**, 390 (1982); it should be pointed out, however, that because of the paucity of the $p+{}^3$H data, $p+{}^3$He elastic-scattering data in the energy range $T_q=64$–750 MeV were also analyzed to determine $V(r;T_q)$.

[21]R. Darves-Blanc *et al.*, Nuovo Cimento Lett. **4**, 16 (1972).

[22]H. Langevin-Joliot *et al.*, Nucl. Phys. **A158**, 309 (1970).

[23]G. Höhler *et al.*, Nucl. Phys. **B114**, 505 (1976).

[24]R. Schiavilla, Nucl. Phys. **A499**, 301 (1989).

[25]In the PWIA the charge and current operators are given by $\rho^{\rm PWIA}=\exp(i\mathbf{k}\cdot\mathbf{r}_1)G_E^p(k_\mu^2)$ and $\mathbf{j}^{\rm PWIA}=(i/2m)\exp(i\mathbf{k}\cdot\mathbf{r}_1)\boldsymbol{\sigma}_1\times\mathbf{k}G_M^p(k_\mu^2)$, respectively. In parallel kinematics the proton convection current does not contribute, since only the transverse components of **j** are required to calculate R_T.

[26]Z. E. Meziani *et al.*, Phys. Rev. Lett. **52**, 2130 (1984); D. Reffay-Pikeroen *et al.*, Phys. Rev. Lett. **60**, 776 (1988).

[27]J. V. Noble, Phys. Rev. Lett. **46**, 412 (1981); L. S. Celenza, A. Rosenthal, and C. M. Shakin, Phys. Rev. C **32**, 248 (1985).

[28]R. Schiavilla, V. R. Pandharipande, and A. Fabrocini, Phys. Rev. C **40**, 1484 (1989).

[29]I. Sick, in *Electron Scattering in Nuclear and Particle Science,* edited by C. N. Papanicolas, L. S. Cardman, and R. A. Eisenstein, AIP Conference Proceedings No. 161 (American Institute of Physics, New York, 1987), p. 58.