Threshold Enhancement and the Flavor-Changing Electromagnetic Vertex

B. Dutta Roy,[a] B. A. Irwin, B. Margolis, J. Robinson, and H. D. Trottier
Physics Department, McGill University, Montreal, Quebec, Canada H3A 2T8

C. Hamzaoui[b]
Department of Physics, Concordia University, Montreal, Quebec, Canada H3G 1M8
(Received 4 June 1990)

We analyze the evolution of the flavor-changing electromagnetic vertex as a function of the external initial quark mass m_e, in the three-generation standard model. We find a substantial threshold enhancement in the vertex, to one-loop order, when m_e is of the order of the mass of the W boson. We comment on QCD corrections.

PACS numbers: 13.40.Hq, 12.15.Ji, 12.15.Mm

Flavor-changing radiative decays, such as $b \to s\gamma$, have been the subject of intensive investigation in the recent literature.[1-6] The conventional approach to such neutral-current vertices has been to exploit the large mass of the top quark in processes where it participates as an *internal* particle, in order to "evade" the suppression of the neutral current due to the Glashow-Iliopoulos-Maiani (GIM) mechanism.[7]

On the other hand, little is apparently known about the behavior of the flavor-changing electromagnetic vertex when the mass m_e of an *external* particle becomes large, say, comparable to M_W.[8] The question then naturally arises as to how the GIM mechanism for the vertex might be modified for $m_e \sim M_W$. With the current limit on the top-quark mass from the Collider Detector at Fermilab passing 90 GeV, this question could well be relevant to phenomenology.

In fact, we have found that the vertex receives a significant enhancement as m_e becomes $\sim M_W$, at least to one-loop order. The origin of this enhancement is completely different from the case where the mass of an *internal* quark becomes large, and has to do with the onset of physical thresholds in the internal loop integration.

The vertex function V^μ for $q_e \to q_{e'}\gamma$ takes the form[3,4]

$$V^\mu = (k^2\gamma^\mu - k^\mu \not{k})(F_1^L L + F_1^R R) + i\sigma^{\mu\nu}k_\nu(F_2^L m_{e'} L + F_2^R m_e R)\,, \tag{1}$$

where $R,L \equiv \frac{1}{2}(1 \pm \gamma^5)$. For real-photon emission ($k^2 = 0$), only the spin-flip term contributes, and in the case where the mass of the initial quark m_e is much larger than the mass of the produced quark $m_{e'}$, the width is

$$\Gamma(q_e \to q_{e'}\gamma) = \frac{\alpha}{128\pi^4} m_e^5 G_F^2 \left| \sum_l \tilde{F}_2^R(m_e,m_l) V_{el} V_{le'}^\dagger \right|^2 , \tag{2}$$

where V_{el} is the Kobayashi-Maskawa matrix,[9] and where we introduce the dimensionless form factor $\tilde{F}_2^R$,

$$F_2^R \equiv \frac{ieg^2}{32\pi^2 M_W^2} \tilde{F}_2^R\,. \tag{3}$$

A typical diagram contributing to $q_e \to q_{e'}\gamma$ to one-loop order is illustrated in Fig. 1. The one-loop form factors were evaluated in the "low-energy" limit of small external quark masses, $m_e \ll M_W$, in Ref. 4, for very light and very heavy internal quarks:

$$\tilde{F}_2^R(m_e, m_l \to 0) = \tilde{F}_2^R(m_e,0) + (\tfrac{1}{4}\hat{e}_W + \tfrac{1}{2}\hat{e}_l)\left(\frac{m_l}{M_W}\right)^2 , \tag{4}$$

$$\tilde{F}_2^R(m_e, m_l \to \infty) = \tilde{F}_2^R(m_e,0) + (\tfrac{1}{2}\hat{e}_W + \tfrac{1}{4}\hat{e}_l)\,. \tag{5}$$

$\hat{e}_W$ and $\hat{e}_l$ denote the charges of the W and the quarks in the loop, in units of the electron charge. The m_l-independent term $\tilde{F}_2^R(m_e, m_l = 0)$ in the low-energy form factor does not contribute to the decay rate, which is the main feature of the GIM suppression mechanism [due to Kobayashi-Maskawa (KM) matrix unitarity].

We now consider the evolution of the one-loop vertex, with light internal quarks, as a function of the *external* quark mass m_e. In the standard model with three generations of quarks, the only relevant application is to the decay $t \to q\gamma$, $q = (c,u)$. For our present purposes, we will confine our attention to this decay mode, although our results have application to various extensions to the standard model.[10]

For the top-quark decay, we use the unitarity of the

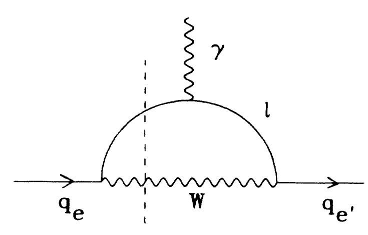

FIG. 1. A typical one-loop diagram contributing to $q_e \to q_{e'}\gamma$. The dashed line represents the unitarity cut corresponding to the physical threshold $q_e \to \text{real}\{Wl\} \to q_{e'}\gamma$, where l is the internal quark.

three-generation KM matrix to reduce Eq. (2) for the width to

$$\Gamma(t \to q\gamma) = \frac{\alpha}{128\pi^4} m_t^5 G_F^2 |V_{tb} V_{bq}|^2 |\Delta \tilde{F}_2^R(m_t)|^2, \qquad (6)$$

where we define a "net" form factor $\Delta \tilde{F}_2^R$,

$$\Delta \tilde{F}_2^R(m_t) \equiv \tilde{F}_2^R(m_t, m_b) - \tilde{F}_2^R(m_t, m_d). \qquad (7)$$

The contribution to the amplitude from the difference in the form factors of the internal s and d quarks can be neglected, given that $m_b \gg m_s$, and the fact that $V_{ts} V_{sq}^\dagger \approx V_{tb} V_{bq}^\dagger$.

The expression for $\tilde{F}_2^R(m_t, m_l)$ to one-loop order can be extracted from Ref. 4. After some simplification, we obtain (neglecting $m_q = m_{c,u}$ compared to m_t)

$$\tilde{F}_2^R(m_t, m_l) = \int_0^1 d\alpha_1 \, d\alpha_2 (\hat{e}_W \alpha_1 + \hat{e}_l \bar{\alpha}_1) \frac{1}{Y} \times [2\alpha_1(1 - \bar{\alpha}_1 \alpha_2) + \hat{m}_l^2 \bar{\alpha}_1 (1 - \alpha_1 \alpha_2)], \qquad (8)$$

where $\bar{\alpha}_1 \equiv 1 - \alpha_1$,

$$Y = \alpha_1 + \hat{m}_l^2 \bar{\alpha}_1 - \hat{m}_t^2 \alpha_1 \bar{\alpha}_1 \alpha_2 - i\epsilon, \qquad (9)$$

and $\hat{m}_t \equiv m_t / M_W$, $\hat{m}_l \equiv m_l / M_W$.

The $i\epsilon$ prescription [Eq. (9)] is crucial for handling the onset in the vertex of the physical thresholds $t \to \text{real}\{Wl\} \to q\gamma$ (where l denotes the various quark flavors in the loop), illustrated by the cut in Fig. 1. The presence of several physical thresholds in the vertex makes it possible to evade the GIM subtraction between pairs of form factors. To demonstrate this effect explicitly, we have evaluated the absorptive part of the form factor $\tilde{F}_2^R$ [Eq. (8)] analytically,

$$\mathrm{Im}\tilde{F}_2^R(m_t, m_l) = \frac{\pi}{m_t^4} \Theta(m_t - M_W - m_l) \left[\hat{e}_W M_W^2 (4E_l m_t - 3m_l^2) \ln \left(\frac{E_W + P}{E_W - P} \right) + \hat{e}_l m_l^2 (2E_W m_t - 3M_W^2) \ln \left(\frac{E_l + P}{E_l - P} \right) + 4(\hat{e}_W - \hat{e}_l)(E_W m_l^2 - 2E_l M_W^2) P \right], \qquad (10)$$

where E_W, E_l, and P are the c.m. energies and momentum of the W-quark pair in the decay $t \to \text{real}\{Wl\}$, $E_{W,l} = (m_t^2 \pm M_W^2 \mp m_l^2)/2m_t$, $P = \lambda^{1/2}(m_t^2, M_W^2, m_l^2)/2m_t$. We can obtain the real part of $\tilde{F}_2^R$ from the dispersion relation

$$\mathrm{Re}\tilde{F}_2^R(m_t, m_l) = \frac{1}{\pi} \mathrm{P} \int_{(M_W + m_l)^2}^{\infty} d\tilde{m}^2 \frac{\mathrm{Im}\tilde{F}_2^R(\tilde{m}, m_l)}{\tilde{m}^2 - m_t^2}. \qquad (11)$$

Our result for the net form factor $\Delta \tilde{F}_2^R(m_t)$ [Eq. (7)] is plotted in Fig. 2, showing a dramatic enhancement in the neighborhood of the thresholds for $t \to \text{real}\{W + (d,b)\}$, compared to the low-energy limit.

This enhancement is due entirely to the evasion of the GIM subtraction between the individual form factors of the internal d and b quarks, when m_t lies near the upper threshold. This is illustrated in Fig. 3, where we plot the absorptive parts of the d- and b-quark amplitudes. As m_t passes $M_W + m_d$, the d-quark amplitude acquires an absorptive part, which apparently increases quadratically

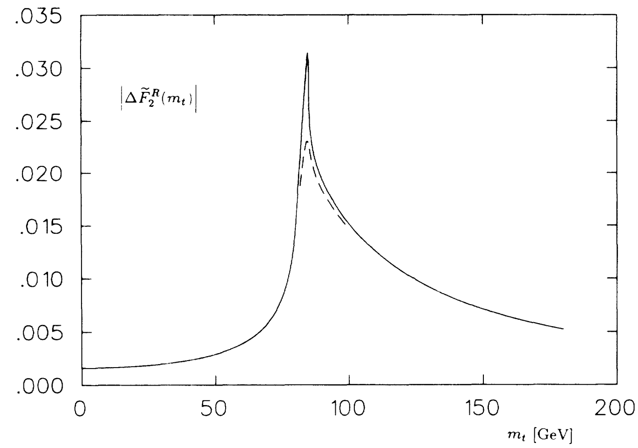

FIG. 2. The net form factor $\Delta \tilde{F}_2^R(m_t)$ [Eq. (7)] for $t \to q\gamma$, as a function of m_t. $M_W = 80$ GeV, $m_b = 5$ GeV, and $m_d \approx 0$ are fixed. The dashed line shows the effect of including the width $\Gamma_W \approx 2$ GeV of the W propagators in the vertex function.

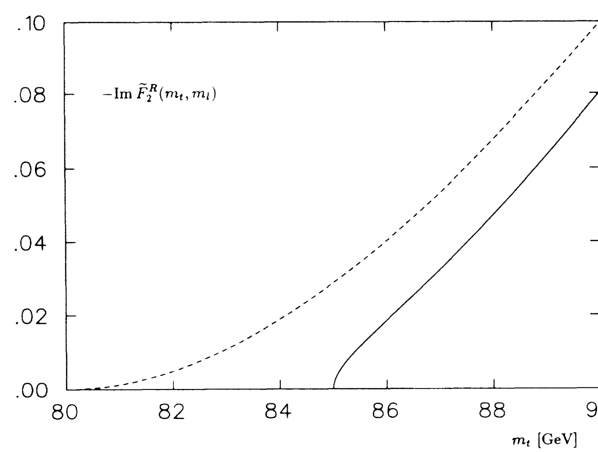

FIG. 3. Absorptive parts of the d-quark (dashed line) and b-quark (solid line) form factors, as functions of m_t near threshold. M_W, m_b, and m_d are as in Fig. 2.

with $m_t-(M_W+m_d)$ [see Eq. (13)]. The evasion of GIM occurs in the sense that the absorptive part of the d-quark amplitude is *unsubtracted* until m_t passes the b-quark threshold, where it makes a contribution $\propto (m_b-m_d)^2$ to the amplitude. While this term is still quadratic in the loop-quark masses, the *coefficient* of this quadratic piece turns out to be much larger than the coefficient of the low-energy quadratic form factor [Eq. (4)]. From Eq. (10), we have the $m_l=0$ limit (cf. $m_d\approx 0$)

$$\mathrm{Im}\tilde{F}_2^R(m_t,m_l=0)=\frac{4\pi}{m_t^4}\Theta(m_t-M_W)\left[-2(\hat{e}_W-\hat{e}_l)M_W^2P^2+\hat{e}_W m_t M_W^2 P\ln\left(1+\frac{2Pm_t}{M_W^2}\right)\right]. \tag{12}$$

If m_t is not too far above the massless-quark threshold, $m_t-M_W\ll M_W$, we find

$$\mathrm{Im}\tilde{F}_2^R(m_t,m_l=0)\simeq 8\pi\hat{e}_l\left(\frac{m_t-M_W}{m_t}\right)^2. \tag{13}$$

We therefore obtain the following expression for the "enhancement" of the $t\to q\gamma$ form factor, for m_t at the internal b-quark threshold, compared to its low-energy limit [cf. Eq. (4)]:

$$\frac{|\Delta\tilde{F}_2^R(m_t=M_W+m_b)|}{\Delta\tilde{F}_2^R(m_t\ll M_W)}\approx\frac{32\pi\hat{e}_l}{2\hat{e}_l+\hat{e}_W}\sim 20\,, \tag{14}$$

neglecting $\mathrm{Re}\tilde{F}_2^R$, which only contributes $\approx 7\%$ to the net form factor at $m_t=M_W+m_b$. [Including the width $\Gamma_W\approx 2$ GeV of the W propagators in the vertex function flattens the peak in $\Delta\tilde{F}_2^R(m_t)$ somewhat, reducing the threshold enhancement from ≈ 20 to ≈ 14.5 near $m_t=M_W+m_b$ (see Fig. 2); the enhancement of the rate for $t\to q\gamma$ therefore amounts to a factor of ≈ 200 at the b-quark threshold.]

We stress again that this evasion of the low-energy GIM mechanism is completely different from the case of a heavy quark in the loop. In the low-energy limit, $\tilde{F}_2^R(m_e,m_l=0)$ is completely subtracted out in the net amplitude, due to the GIM mechanism. Near threshold, however, this part of the form factor [Eq. (13)] produces the large enhancement observed in Fig. 2.

A more conventional softening of the GIM mechanism also occurs for m_t well above threshold, where the m_l-independent part of the form factor is mostly subtracted out,

$$\mathrm{Im}\tilde{F}_2^R(m_t,m_l\to 0)=\mathrm{Im}\tilde{F}_2^R(m_t,m_l=0)-\frac{\pi}{m_t^4}\hat{e}_l(m_t^2-2M_W^2)\ln(m_l^2)m_l^2 \tag{15}$$

[neglecting terms of $O(m_l^2)$]. This accounts for the asymmetry of the curve in Fig. 2 about the peak, since the low-energy GIM mechanism is softened by a logarithm at large m_t.

Despite the significant threshold enhancement, the rate for $t\to q\gamma$ is still very small, at least to one-loop order. For example, $\Gamma_{1\text{-loop}}(t\to q\gamma)\approx 0.2|V_{bq}|^2$ eV, for $m_t\approx 100$ GeV, despite a threshold enhancement of ≈ 90 in the rate.

On the other hand, it is well known that there is a large QCD correction to the one-loop form factor for light internal quarks, at least in the low-energy limit where the QCD-corrected vertex has been evaluated using an effective Lagrangian approach.[5,6,11] However, the QCD-corrected form factor for heavy external quarks is unknown, and therefore the theoretical situation with respect to $t\to q\gamma$ is uncertain; based on the known low-energy form factor, one might expect a large QCD correction in this case, given that the internal quarks (b,s,d) are light. To make an order-of-magnitude estimate, we use the low-energy QCD-corrected form factor. We then find $\Gamma_{\mathrm{QCD}}(t\to c\gamma)\approx 1$ eV, at $m_t\approx 100$ GeV (using $\alpha_s\approx 0.1$ and $m_d\approx 10$ MeV).[12] This would imply a branching ratio of $\approx 10^{-8}$ [given $\Gamma_{\mathrm{tot}}\approx\Gamma(t\to bW)\approx 90$ MeV].

The possibility of a threshold enhancement to the low-energy QCD-corrected vertex then becomes extremely important, since a large enhancement, such as occurs in the one-loop vertex, could make $t\to q\gamma$ accessible at the Superconducting Super Collider. A simple qualitative argument, however, suggests that a significant threshold enhancement is unlikely. This has to do with the fact that the absorptive part of the amplitude for the internal d quark (which is responsible for the enhancement of the one-loop amplitude) increases *at least* linearly with m_t above threshold [cf. Eqs. (12) and (13)] due to the phase space for the on-shell internal particles. The contribution from the absorptive part of the QCD-corrected vertex, near the b-quark threshold, will therefore be suppressed at least by a power of m_b compared to the low-energy (logarithmic) form factor.

However, a full calculation of the QCD-corrected flavor-changing electromagnetic vertex for large external quark masses is clearly needed in order to fully explore the possibility of significant threshold effects.

(a)On leave from Saha Institute of Nuclear Physics, 92, Acharya Prafulla Chandra Road, Calcutta, 700 009, India.

(b)Also at Department of Physics, Rockefeller University, New York, NY 10021.

1T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981).

2B. A. Campbell and P. J. O'Donnell, Phys. Rev. D **25**, 1989 (1982); P. J. O'Donnell, Phys. Lett. B **175**, 369 (1986).

3N. G. Deshpande and G. Eilam, Phys. Rev. D **26**, 2463

(1982).

[4]N. G. Deshpande and M. Nazerimonfared, Nucl. Phys. **B213**, 390 (1983).

[5]S. Bertolini, F. Borzumati, and A. Masiero, Phys. Rev. Lett. **59**, 180 (1987); N. G. Deshpande, P. Lo, J. Trampetic, G. Eilam, and P. Singer, Phys. Rev. Lett. **59**, 183 (1987).

[6]B. Grinstein *et al.*, Phys. Lett. B **202**, 138 (1988); R. Grigjanis *et al.*, *ibid.* **213**, 355 (1988); **224**, 209 (1989).

[7]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).

[8]The softening of the GIM mechanism in the context of a heavy quark on an *external* line, in the case of the neutral-current quark-quark-Higgs-boson coupling, can be inferred from results presented in P. Krawczyk, Z. Phys. C **44**, 509 (1989); G. Eilam, B. Haeri, and A. Soni, Phys. Rev. D **41**, 875 (1990); W.-S. Hou and R. G. Stuart, CERN Report No. TH.5686/90, 1990 (to be published).

[9]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[10]B. Dutta Roy, B. A. Irwin, B. Margolis, J. Robinson, H. D. Trottier, and C. Hamzaoui (to be published).

[11]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Rev. D **18**, 2583 (1978).

[12]For the purpose of making an order-of-magnitude estimate, we use the QCD-corrected form factor given in Ref. 5. More recent analyses of the form factor (leading to quantitative results for $b \rightarrow s\gamma$ of the same order of magnitude as those given in Ref. 5) are given in Ref. 6.