Nucleation of Magnetization Reversal via Creation of Pairs of Bloch Walls

J. S. Broz, H. B. Braun, O. Brodbeck, and W. Baltensperger
Theoretische Physik, Eidgenossische Technische Hochschule Hönggerberg, CH 8093 Zürich, Switzerland

J. S. Helman
Centro Brasileiro de Pesquisas Físicas (CBPF), Rua Dr. Xavier Sigaud, 150, Rio de Janeiro, 22290, Brazil
(Received 24 April 1990)

For a ferromagnet with exchange, uniaxial anisotropy, demagnetizing field, and an external magnetic field directed oppositely to the magnetization, the Landau-Lifshitz equations have time-independent, planar, local solutions of the Bloch and Néel type. These solutions are unstable. Analogous to a water droplet in supersaturated vapor for which a radius of unstable equilibrium exists, the solution for a given magnetic field is the configuration of smallest energy through which a magnetization reversal can nucleate.

PACS numbers: 75.60.Ch, 75.60.Ej, 75.60.Gm

The dynamics of ferromagnets is phenomenologically described by the Landau-Lifshitz-Gilbert equations[1]

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma[\mathbf{M}\times\mathbf{H}_{\rm eff}] + \frac{\alpha}{M}\left[\mathbf{M}\times\frac{\partial \mathbf{M}}{\partial t}\right], \qquad (1)$$

where the magnetization $\mathbf{M}(\mathbf{x},t)$ (of constant modulus M) depends on space and time, $\gamma > 0$ is the gyromagnetic ratio, and α the damping constant. The effective field

$$\mathbf{H}_{\rm eff} = -\frac{\delta W_{\rm tot}}{\delta \mathbf{M}}, \quad W_{\rm tot} = \int d^3x\, w, \qquad (2)$$

is obtained as the variational derivative of the total energy, which is a functional of the magnetization and its gradients. The energy density then reduces to

$$\begin{aligned} w = \frac{1}{8\pi}\left\{\left(\frac{\partial\theta}{\partial x}\right)^2 + \sin^2\theta\left(\frac{\partial\phi}{\partial x}\right)^2\right\} &\quad \text{exchange term} \\ +(Q/8\pi)\sin^2\theta &\quad \text{axial anisotropy term} \\ +(1/8\pi)\sin^2\theta\cos^2\phi &\quad \text{demagnetization term} \\ -(2H_z/8\pi)\cos\theta &\quad \text{Zeeman term}. \end{aligned} \qquad (3)$$

θ is the angle between the magnetization and the magnetic field H_z, and ϕ is the azimuthal angle measured from the x direction. In (3) the dimensionless units of

$$\begin{aligned} &\text{length}, \ \Delta Q^{1/2} = 1, \\ &\text{frequency}, \ 4\pi\gamma M = 1, \\ &\text{magnetic field}, \ 4\pi M = 1, \end{aligned} \qquad (4)$$

are used, where $Q = K_u/2\pi M^2$ is the quality factor, $\Delta = \sqrt{A/K_u}$ is the width of a static Bloch wall, A is the exchange coupling, and K_u is the uniaxial anisotropy constant.[1]

The Landau-Lifshitz-Gilbert equations then read

$$\begin{aligned} \dot\theta &= \phi''\sin\theta + 2\theta'\phi'\cos\theta + \sin\theta\sin\phi\cos\phi - \alpha\dot\phi\sin\theta, \\ \dot\phi\sin\theta &= -\theta'' + \{(\phi')^2 + Q + \cos^2\phi\}\sin\theta\cos\theta \\ &\quad + H_z\sin\theta + \alpha\dot\theta. \end{aligned} \qquad (5)$$

Several solutions to these nonlinear equations are known, in particular the static and the moving Bloch wall (Walker's solution),[2] the static 2π wall,[3,4] and local solutions traveling along the anisotropy axis.[5]

We consider the boundary conditions that the magnetization at infinity is opposed to the magnetic field: $\theta(\pm\infty) = \pi$. Then the following exact static solution exists:[6]

$$\tan(\theta/2) = \beta\cosh(x/\delta), \qquad (6)$$

where in the Bloch configuration ($\phi = \pm\pi/2$),

$$\delta^2 = 1/(Q - H_z), \quad \beta^2 = H_z/(Q - H_z), \qquad (7)$$

while for the Néel configuration [$\phi = 0 (\mathrm{mod}\,\pi)$], Q has to be replaced by $Q+1$ in (7) and in all subsequent expressions.

It is convenient to introduce the reduced field $h = H_z/Q$, which in laboratory units has the value $H_z/(2K_u/M)$ for the Bloch case and $H_z/[(2K_u/M) + 4\pi M]$ in the Néel case. The solution (6),(7) is plotted in Fig. 1 for several values of h. The local structure represents deviations of the magnetization from the fully opposed direction, where the departure is sideways ($\phi = \pm\pi/2$) in the Bloch case. For very small fields, $h \ll 1$, a region of magnetization parallel to the outside field appears, which is delimited by a mirror-symmetric pair of Bloch walls.

The integrated magnetization of the struction (6), i.e.,

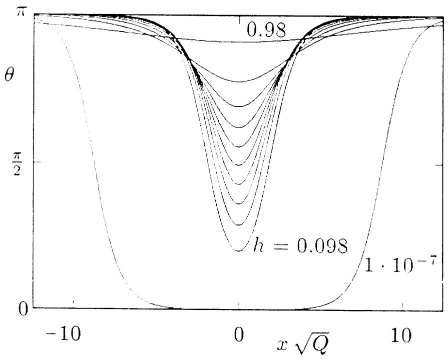

FIG. 1. Static magnetization configurations for equidistant values of the reduced external field $h = H_z/Q$.

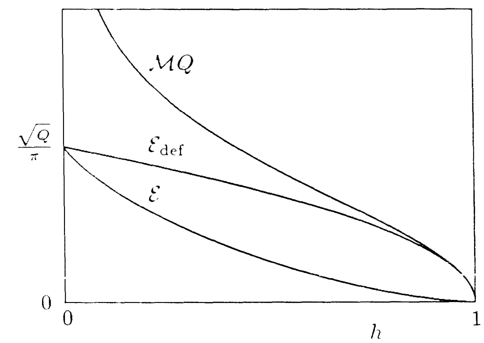

FIG. 2. Energy $\mathcal{E}$, deformation energy $\mathcal{E}_{\mathrm{def}}$, and integrated magnetization $\mathcal{M}$ as dimensionless functions of the reduced external field $h = H_z/Q$.

the difference from the fully opposed alignment, becomes

$$\mathcal{M} = \frac{1}{4\pi}\int_{-\infty}^{\infty} dx\{\cos(\theta)+1\} = \frac{\sqrt{Q}}{\pi}\frac{1}{2Q}\ln\frac{1+\sqrt{1-h}}{1-\sqrt{1-h}} \tag{8}$$

so that

$$h = \mathrm{sech}^2(\pi\sqrt{Q}\mathcal{M}). \tag{9}$$

The energy of the structure is

$$\mathcal{E} = \int_{-\infty}^{\infty} dx[w - (H_z/4\pi)] = \frac{\sqrt{Q}}{\pi}\left\{\sqrt{1-h} - \frac{h}{2}\ln\frac{1+\sqrt{1-h}}{1-\sqrt{1-h}}\right\} = \mathcal{E}_{\mathrm{def}} - H_z\mathcal{M}, \tag{10}$$

where

$$\mathcal{E}_{\mathrm{def}} = (\sqrt{Q}/\pi)\sqrt{1-h} = (\sqrt{Q}/\pi)\tanh(\pi\sqrt{Q}\mathcal{M}). \tag{11}$$

Note that $\mathcal{E}$ is positive for $0 < h < 1$. For $h \to 1$ the deformation energy $\mathcal{E}_{\mathrm{def}}$ is canceled by the first term of an expansion of the Zeeman term $-H_z\mathcal{M}$, so that

$$\mathcal{E} = (2\sqrt{Q}/3\pi)(1-h)^{3/2} + \cdots. \tag{12}$$

For $h \to 0$, $\mathcal{E} \to \sqrt{Q}/\pi$, which is the energy of two Bloch walls; the divergence of $\mathcal{M} \simeq (1/2\pi\sqrt{Q})\ln(4/h)$ is due to the reversed magnetization between the two Bloch walls. The energy of the structure $\mathcal{E}$, the deformation energy $\mathcal{E}_{\mathrm{def}}$, and the integrated magnetization $\mathcal{M}$ are plotted as dimensionless functions of the reduced external field h in Fig. 2.

Since $\mathcal{E}$ contains the Zeeman energy $-H_z\mathcal{M}$, it plays the role of an enthalpy $\mathcal{E}(H_z)$ of the system in the outside field H_z,[7] while the Legendre transformed $\mathcal{E}_{\mathrm{def}}(\mathcal{M})$ is the internal energy. Thus

$$\frac{\partial\mathcal{E}(H_z)}{\partial H_z} = -\mathcal{M}, \quad \frac{\partial\mathcal{E}_{\mathrm{def}}(\mathcal{M})}{\partial\mathcal{M}} = H_z. \tag{13}$$

We shall see immediately below, however, that the field H_z cannot be used to keep the system in the corresponding state.

We now examine the stability of the solution (6),(7). As variational functions to the solution which belongs to the field H_z we use the solutions for neighboring values $\tilde{H}_z$. Their energy in the outside field H_z is

$$\mathcal{E}_{H_z}(\tilde{h}) = (\sqrt{Q}/\pi)(1-\tilde{h})^{1/2} - H_z\mathcal{M}(\tilde{h}), \tag{14}$$

where $\tilde{h} = \tilde{H}_z/Q$. Then the first and second derivatives of (14) give for $0 < h < 1$,

$$\left.\frac{\partial\mathcal{E}_{H_z}(\tilde{h})}{\partial\tilde{h}}\right|_{\tilde{h}=h} = 0, \quad \left.\frac{\partial^2\mathcal{E}_{H_z}(\tilde{h})}{\partial\tilde{h}^2}\right|_{\tilde{h}=h} < 0. \tag{15}$$

This proves the instability of the solution (6),(7). Since there are also configurations of higher energy near this solution (for example, $\phi \neq \pm\pi/2$ in the Bloch case), the energy has a saddle point in function space at this solution.

For nucleation processes the model example is the water droplet in supersaturated vapor. It is known[8] that there is a droplet radius at which the system is in thermodynamic equilibrium. This, however, is unstable: a smaller drop shrinks, while a larger drop grows. An analogous situation holds for the solution (6). It is a nucleation configuration for the magnetization reversal and $\mathcal{E}(h)$ is the nucleation energy. A neighboring configuration belonging to $\tilde{h} < h$ will expand, while for $\tilde{h} > h$ it will shrink. Of course, in neither case will the time development follow the series of functions of the form (6); in particular the first Landau-Lifshitz-Gilbert equation (5) implies that $\dot{\theta} = 0$ for constant $\phi = 0(\mathrm{mod}\,\pi/2)$. When the magnetic field is near the value $h = 1$, the motion of the two separated Bloch walls will be beyond the range of the Walker solution[1] unless $Q < \alpha/2$, which is not typical. In the case that $h > 1$, the energy $\mathcal{E}_{H_z}(\tilde{h})$ given by (14) with $0 < \tilde{h} < 1$ is negative, so that a spontaneous creation of these structures is energetically possible. Note that the Néel case is not relevant to the above discussion, since with increasing magnetic field, $h=1$ is reached first in the Bloch case.

Several mechanisms of magnetization reversal have been discussed.[9,10] At low fields Bloch-wall motion dominates. Nucleation requires fields approaching or even exceeding the field $h=1$. Since nucleation occurs throughout the volume, it can be a fast mechanism of reversal. A limitation of the present theory is that it describes planar structures; more complex spatial configurations cannot be discarded. Experimentally, nucleation may be observed in fast optical experiments. Also, the spectrum of Barkhausen noise could distinguish this from other reversal mechanisms.

H.B.B. thanks the Swiss National Science Foundation for a grant and W.B. acknowledges the financial support of the Fundação de Amparo a Pesquisa do Rio de Janeiro (FAPERJ) and of I.B.M. do Brasil during his visits to the CBPF.

[1]F. H. de Leeuw, R. van den Doel, and U. Enz, Rep. Prog. Phys. **43**, 689 (1980).

[2]J. F. Dillon, in *Magnetism,* edited by G. R. Rado and H. Suhl (Academic, New York, 1963), Vol. III, pp. 415–464.

[3]R. M. Hornreich and H. Thomas, Phys. Rev. B **17**, 1406 (1978).

[4]K. A. Long and A. R. Bishop, J. Phys. A **12**, 1325 (1979).

[5]A. M. Kosevich, Phys. Met. Metall. **53**, 1 (1982).

[6]A particular time-dependent solution and periodic static solutions will be discussed elsewhere. Solution (6) is the symmetric case of a solution obtained by A. Aharoni, Phys. Rev. **119**, 127 (1960), Eq. (18).

[7]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media,* Course of Theoretical Physics, Vol. 8 (Pergamon, New York, 1975).

[8]L. D. Landau and E. M. Lifshitz, *Statistical Physics,* Course of Theoretical Physics, Vol. 5, Pt. 1 (Pergamon, New York, 1980).

[9]E. M. Gyorgy, in *Magnetism,* edited by G. R. Rado and H. Suhl (Academic, New York, 1963), Vol. III, p. 525.

[10]Jiang-Gang Zhu and H. Neal Bertram, J. Appl. Phys. **66**, 1291 (1989).