# Collapse of Integer Hall Gaps in a Double-Quantum-Well System

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For coupled double-quantum-well systems in which tunneling is important, the symmetric to antisymmetric energy gap leads to a quantum Hall effect. In this Letter we show that interaction effects in strong magnetic fields can destroy this gap, and present a theory which predicts the occurrence or nonoccurrence of a quantum Hall effect.

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The quantum Hall effects [integer (IQHE) and fractional (FQHE)] are remarkable phenomena arising from the peculiar dynamics of 2D electrons in strong perpendicular magnetic fields.[1] The IQHE is related to gaps in the single-particle density of states comparable to the cyclotron energy $\hbar\omega_c$. For the FQHE, the gaps in the excitation spectrum are a direct result of the Coulomb interaction between electrons.

Recently[2] interest has focused on the behavior of double-quantum-well structures (DQWS). These structures allow the controlled introduction of additional degrees of freedom in the third ($z$) direction. For a pair of identical quantum wells with interwell tunneling, the lowest subband states of each well mix to form symmetric and antisymmetric states. The symmetric-antisymmetric energy gap, $\Delta_{SAS}$, is controllable and is typically much smaller than $\hbar\omega_c$, though comparable to Coulomb correlation energies ($\sim 0.1e^2/\varepsilon_0 l$) in strong magnetic fields ($\geq 5$ T). Herein, $l=(\hbar c/eB)^{1/2}$ is the magnetic length, $\nu=2\pi l^2 n_{2D}$ is the number of filled energy levels, and $n_{2D}$ is the total electron areal density. In the absence of Coulombic effects and in the strong-magnetic-field limit ($\hbar\omega_c, g^*\mu_B B > \Delta_{SAS}$), the excitation gap at $\nu=$odd is $\Delta_{SAS}$ and results in a QHE, i.e., an activated longitudinal resistivity ($\rho_{xx}$) and a quantized Hall plateau in $\rho_{xy}$. The experiments in Ref. 2 demonstrate that strong magnetic fields can destroy the QHE at $\nu=$odd in a DQWS. In this Letter we show that Coulomb interactions account for this magnetic-field-driven destruction of the QHE, and present a theory which reliably predicts the occurrence or nonoccurrence of a QHE.

In the absence of Coulombic effects at $\nu=1$, the electrons of a DQWS occupy the lowest (symmetric) state and the pair of wells act as a single well. Coulombic effects could drastically modify the properties of this DQWS by mixing antisymmetric states into the many-body wave function. This would allow electrons within the same well to be more strongly correlated than electrons in different wells. If the Coulomb energy reduction per antisymmetric state were to exceed $\Delta_{SAS}$, the symmetric to antisymmetric excitation gap would vanish, thereby destroying the QHE. If $\Delta_{SAS}$ were held fixed while Coulomb interactions were increased, at the point where the energy gap vanishes we would expect a *transition* to a new ground state with qualitatively different correlations (very weak interwell and strong intrawell correlations). The missing QHE plateaus at strong magnetic field[2] result when Coulombic effects, which scale as $e^2/\varepsilon_0 l \propto \sqrt{B}$, drive the DQWS through this transition to the new ground state.

Our calculation of the phase boundary of the QHE is based on a single-mode-approximation[3] (SMA) calculation of the symmetric to antisymmetric excitation energy of a DQWS. We consider a strong perpendicular magnetic field $B$ in order to assume maximal spin polarization and neglect Landau-level mixing. For the case of odd integral filling factors and $\Delta_{SAS}=0$, our results for the excitation energy are identical to those obtained by Fertig[4] using a diagrammatic approach. We find that the nondispersive excitation $\Delta_{SAS}$ becomes dispersive with a minimum energy near $kl=1$. For sufficiently small $\Delta_{SAS}/(e^2/\varepsilon_0 l)$ and sufficiently large $d/l$, the mode goes soft. We will identify the mode softening with the disappearance of the QHE gaps in Ref. 2.

The Hamiltonian describing the low-energy excitations of the system when a given orbital Landau level of a given spin is partially filled is

$$H=-\frac{1}{2}\Delta_{SAS}\sum_i \sigma_{zi}+\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}[V_0(\mathbf{q})\bar{\rho}(-\mathbf{q})\bar{\rho}(\mathbf{q})+V_x(\mathbf{q})\bar{\sigma}_x(-\mathbf{q})\bar{\sigma}_x(\mathbf{q})]\equiv T+U_0+U_x\,. \quad (1)$$

In Eq. (1) we have assumed that there is negligible overlap between envelope functions centered in opposite wells and the symmetric-antisymmetric degree of freedom is described in an "isospin" language. Eigenstates of $\sigma_z$ with eigenvalues $\pm 1$ are the symmetric and antisymmetric combinations of the single-layer envelope functions and the eigenstates of

 

$\sigma_x$ with eigenvalues $\pm 1$ are the single-layer envelope functions, $\phi(z)$ and $\phi(z-d)$. In Eq. (1), $\Delta_{SAS}/2$ is the hopping matrix element between wells, $V_0(\mathbf{q})=[V_A(\mathbf{q})+V_E(\mathbf{q})]/2$ and $V_x(\mathbf{q})=[V_A(\mathbf{q})-V_E(\mathbf{q})]/2$ with

$$V_A(\mathbf{q})=\frac{2\pi e^2}{\varepsilon_0 q}\int_{-\infty}^{\infty} dz\,dz'\,\phi^2(z)\phi^2(z')e^{-q|z-z'|}, \quad (2)$$

$$V_E(\mathbf{q})=\frac{2\pi e^2}{\varepsilon_0 q}\int_{-\infty}^{\infty} dz\,dz'\,\phi^2(z)\phi^2(z'-d)e^{-q|z-z'|}. \quad (3)$$

Also,

$$\bar{\rho}(\mathbf{q})\equiv\sum_i B_i(\mathbf{q}) \quad (4)$$

is the projection of the total density operator onto the lowest Landau level[3] for particles with in-plane coordinates $\mathbf{r}_i$. Similarly,

$$\bar{\sigma}_x(\mathbf{q})=\sum_i \sigma_{xi}B_i(\mathbf{q}) \quad (5)$$

is the projection of the operator for the density difference between the two layers onto the lowest Landau level. In the narrow-layer limit $V_A(\mathbf{q})=2\pi e^2/\varepsilon_0 q$ and $V_E(\mathbf{q})=V_A(\mathbf{q})e^{-qd}$. Because of overall charge neutrality, we may take $V_0(q=0)=0$. On the other hand,

$$V_x(q=0)=\pi e^2 d/\varepsilon_0 \quad (6)$$

is nonzero and the $\mathbf{q}=0$ term in $U_x$ describes the capacitative energy associated with charge transfer between the two wells. For the Landau-level index $n\neq 0$, $V_0(\mathbf{q})$ and $V_x(\mathbf{q})$ must be multiplied by $[L_n(l^2q^2/2)]^2$, where $L_n(x)$ is a Laguerre polynomial.

Previous theoretical studies of two-layer systems have focused on the limit where $\Delta_{SAS}=0$.[4-6] In this limit, the $x$ component of total isospin ($S_x^{tot}$), corresponding to the difference in the number of electrons in the two layers, is a good quantum number. In such a system it has been demonstrated[5,6] that electron-electron interactions can produce incompressible states with excitation gaps and hence be responsible for a quantum Hall effect. When the hopping term is added to the Hamiltonian ($\Delta_{SAS}\neq 0$), $S_x^{tot}$ is no longer a good quantum number and, for sufficiently strong $\Delta_{SAS}$, these incompressible states will be destroyed. Herein we focus on the limit where $\Delta_{SAS}$ is large and must be incorporated into the theory from the beginning.

Our calculation of the collective-mode energy associated with the symmetric to antisymmetric excitation, a type of magnon mode in our spin description, is based on the single-mode approximation[3] in which the isomagnon wave function is given by

$$|\Psi_-(\mathbf{k})\rangle=\frac{e^{l^2k^2/4}}{\sqrt{N}}\bar{s}_-(\mathbf{k})|\Psi_0\rangle. \quad (7)$$

Here $|\Psi_0\rangle$ is the fully polarized ground state of the system[7] and $\bar{s}_\pm(\mathbf{k})=[\bar{\sigma}_x(\mathbf{k})\pm i\bar{\sigma}_y(\mathbf{k})]/2$. Recalling that[3]

$$B_i(\mathbf{k}_1)B_i(\mathbf{k}_2)=B_i(\mathbf{k}_1+\mathbf{k}_2)\exp(-l^2k_1^*k_2/2),$$

it is easy to verify that $|\Psi_-(\mathbf{k})\rangle$ is normalized. The full Hamiltonian can be separated into a term which conserves $S_z^{tot}$ ($H_0$), and terms ($H_\pm$) which change $S_z^{tot}$ by 2, i.e.,

$$H_0=T+U_0+\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}V_x(\mathbf{q})[\bar{s}_+(-\mathbf{q})\bar{s}_-(\mathbf{q})+\bar{s}_-(-\mathbf{q})\bar{s}_+(\mathbf{q})], \quad (8)$$

$$H_\pm=\frac{1}{2}\int\frac{d^2q}{(2\pi)^2}V_x(\mathbf{q})\bar{s}_\pm(-\mathbf{q})\bar{s}_\pm(\mathbf{q}). \quad (9)$$

Using the spin-commutation relations, a tedious but straightforward calculation yields

$$\langle\Psi_-(\mathbf{k})|H_0|\Psi_-(\mathbf{k})\rangle=E_0+\varepsilon(\mathbf{k}), \quad (10)$$

where $E_0=\langle\Psi_0|H_0|\Psi_0\rangle$ and

$$\varepsilon(\mathbf{k})=\Delta_{SAS}+\frac{\nu V_x(\mathbf{k})e^{-l^2k^2/2}}{2\pi l^2}+\int\frac{d^2q\,e^{-l^2q^2/2}}{(2\pi)^2}[V_x(\mathbf{q})\tilde{h}(\mathbf{k}+\mathbf{q})-V_0(\mathbf{q})\tilde{h}(\mathbf{q})]+\int\frac{d^2q\,e^{-l^2q^2/2}}{(2\pi)^2}V_0(\mathbf{q})\tilde{h}(\mathbf{q})e^{il^2\mathbf{q}\cdot(\hat{\mathbf{z}}\times\mathbf{k})}. \quad (11)$$

In Eq. (11), $\tilde{h}(\mathbf{q})=h(\mathbf{q})\exp(l^2q^2/2)$ and $h(\mathbf{q})$ is the Fourier transform of the pair-correlation function. For $V_x=0$ these results reduce to those obtained previously.[8]

The various terms in Eq. (11) have clear physical interpretations. The first term is the bare symmetric to antisymmetric excitation energy while the second is a Hartree correction associated with the spin-dependent part of the interaction. The third term is the difference between the self-energies of an antisymmetric and a symmetric electron in the presence of a correlated state of symmetric electrons. For nearby layers, $V_x(\mathbf{q})\ll V_0(\mathbf{q})$, the exchange energy of the excited symmetric electron will be small, and the self-energy difference will be large and positive [$\tilde{h}(\mathbf{q})$ is negative]. On the other hand, for well-separated layers, the self-energy difference is small. Physically this occurs because the symmetric to antisymmetric excitation does not alter intralayer correlations and only intralayer correlations are energetically important at large layer separations. The fourth term in Eq. (11) represents the excitonic attraction between the antisymmetric electron and symmetric hole, separated by $l^2\hat{\mathbf{z}}\times\mathbf{k}$.[9] This excitonic term involves intrawell interactions and so it remains strong even for large layer separations. Thus for large layer separations and $kl\gtrsim 1$, $\varepsilon(\mathbf{k})$ can become negative. The Coulombic gain in intralayer interactions can exceed costs in hopping energy and interlayer correlation energy, leading to a collapse in the excitation gap and hence in the QHE based on a fully polarized state.

This system may be approximately described in terms of the isomagnon collective degrees of freedom. The effective isomagnon Hamiltonian is

$$H_s = E_0 + \sum_{\mathbf{k}} [b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} \varepsilon(\mathbf{k}) + \tfrac{1}{2}\lambda(\mathbf{k})(b_{\mathbf{k}} b_{-\mathbf{k}} + b_{\mathbf{k}}^{\dagger} b_{-\mathbf{k}}^{\dagger})], \tag{12}$$

where $H_0$ is described by the free-boson piece $\sum_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} \varepsilon_{\mathbf{k}}$ and the terms proportional to $\lambda(\mathbf{k})$ approximate $H_+ + H_-$. We choose $\lambda(\mathbf{k})$ to reproduce the exact result for the matrix element of $H_-$ between zero- and two-magnon states, which leads to

$$\lambda(\mathbf{k}) = \frac{\nu}{2\pi l^2} V_x(\mathbf{k}) e^{-l^2 k^2/2} + \int \frac{d^2q}{(2\pi)^2} e^{-l^2 q^2/2} e^{il^2 \mathbf{q}\cdot(\hat{\mathbf{z}}\times\mathbf{k})} V_x(\mathbf{q}) \tilde{h}(\mathbf{k}-\mathbf{q}). \tag{13}$$

The second term can be understood as an exchange contribution to the isomagnon interaction energy. $H_s$ can be diagonalized by a Bogoliubov[10] transformation with the result that the spin-excitation energies are given by

$$E(\mathbf{k}) = [\varepsilon^2(\mathbf{k}) - \lambda^2(\mathbf{k})]^{1/2}. \tag{14}$$

In order to make a comparison with the experimental results of Ref. 2, we consider the case of a fully filled energy level ($\nu = 1$) and for the moment neglect finite-thickness corrections. We associate the vanishing of the excitation gap with the loss of a QHE to obtain the phase boundary of the incompressible state (solid curves in Figs. 1 and 2). In this simplified model, the boundary depends only on the dimensionless variables $d/l$ and $\Delta_{SAS}/(e^2/\epsilon_0 l)$. The data for the three different samples of Ref. 2 are placed in the figures by choosing $d$ as the center-to-center distance between the wells and $\Delta_{SAS}$ to be the experimentally measured gap at zero field. Solid circles, labeled by filling factor, indicate the observed IQHE states, while open circles indicate the missing IQHE states reported in Ref. 2. Note that $\nu = 1,3$ correspond to the lowest Landau level (Fig. 1) and $\nu = 5,7$ correspond to the second Landau level (Fig. 2). (Data for $\nu$ = even are not shown since the bare energy gap is not $\Delta_{SAS}$, but $g^*\mu_B B$ for $\nu = 2,6$ and $\hbar\omega_c$ for $\nu = 4,8$. These gaps are never destroyed by the Coulombic effects analyzed here.) We see that our theory with no adjustable parameters describes the principle experimental trend that the QHE is more easily destroyed by strong magnetic fields for samples with wider barriers. If the finite width of the wells were taken into account, the intrawell interaction, which favors collapse, will be weakened and the effective layer separation will be reduced. These effects can be modeled by approximating $\phi^2(z)$ equal to $1/d_W$ in a well of width $d_W$. The dashed lines in the figures give the phase boundary for this model with $d_W/d = 0.75$. (The experimental values of $d_W/d$ are 0.83, 0.78, and 0.73 for the $d_B = 28$, 40, and 51 Å samples.) As the figures show, this simple correction for finite well widths removes most of the discrepancy between theoretical and experimental phase boundaries.

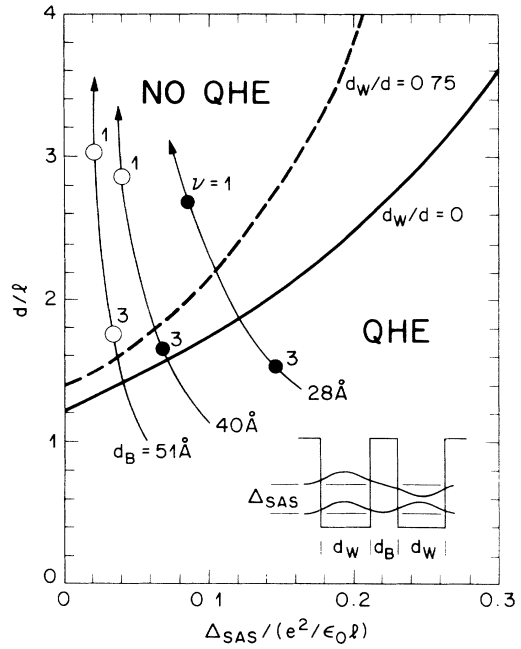


FIG. 1. Phase diagram for the quantum Hall effect in a double quantum well (shown in the inset). $\Delta_{SAS}$ is the symmetric-antisymmetric energy gap, $d = d_W + d_B$ is the interwell spacing, and $l = (\hbar c/eB)^{1/2}$ is the magnetic length. The heavy solid line is the calculated phase boundary of the $n = 0$ Landau level for collapse of the energy gap above the incompressible state. The dashed line includes our approximate finite-well-thickness correction. The experimental data for $\nu = 1$ and 3 from the three samples of Ref. 2 are represented by open and solid circles which denote the missing and observed QHE states, respectively. The arrows indicate increasing magnetic field for a given sample.

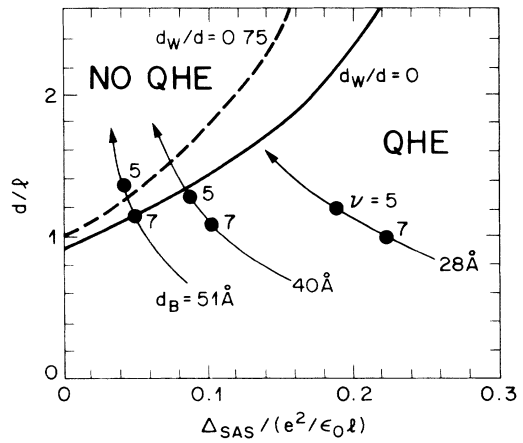


FIG. 2. Same as Fig. 1, for the $n = 1$ Landau level. The data for $\nu = 5$ and 7 are shown.

We believe that the agreement shown in Figs. 1 and 2 demonstrates that our theory captures the important physics of the observed collapse of the QHE in the DQWS at high magnetic fields. We are unable to quantitatively explain the observed magnetic-field dependence of the activation energies at $\nu$=odd reported in Ref. 2, but believe this dependence is due to increased mixing of orbital Landau levels by the Coulomb interaction at weaker magnetic fields. In our calculations, only electrons in the highest occupied energy level take part in correlations. In this case, the electron charge is spread out in the plane over a distance set by the magnetic length $l$. In the presence of increased Landau-level mixing at lower magnetic fields, it is possible for the electron charge to become more localized in the plane. As a result, intralayer correlations will lead to larger reductions of the Coulomb energy and greater suppression of the symmetric-antisymmetric energy gap.

We have not addressed the case of fractional filling factors here. However, we believe that the phase diagram for the FQHE could be extremely rich. For example, along the $d/l=0$ line of the phase diagram, the interaction is spin independent and the physics is equivalent to that of a two-spin system where $\Delta_{SAS}$ plays the role of a Zeeman energy. It has been observed that phase transitions occur at some fractional filling factors between states with different isospin polarization.[11,12] On the other hand, the $\Delta_{SAS}=0$ limit in the phase diagram corresponds to the situation in which there are a definite number of electrons in each well. In this case, it is known[6] that phase transitions can occur as a function of $d/l$ between different generalized Laughlin[13] states of the type proposed by Halperin.[5] It is clear, then, that the phase diagram in the fractional regime will contain regions of FQHE's based on different ground states.

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