Clean Test of Electroweak Quantum Effects Independent of Low-Energy Neutral-Current Processes

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Electroweak quantum effects are studied in the M_W-M_Z relation by using recent data of ALEPH, DELPHI, L3, OPAL, and Mark II Collaborations on M_Z, and those of CDF and UA2 Collaborations on M_W. It is shown that these data demand the existence of a $G_F m_t^2$-type correction in addition to the well-known logarithmic terms, which gives new phenomenological support to the electroweak theory independent of the neutral-current data.

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The two high-energy e^+e^- colliders, the SLAC Linear Collider (SLC) and CERN's LEP, have given us lots of surprisingly precise information on the Z boson. Precision tests of the standard $SU(2)\times U(1)$ electroweak theory[1] have been thereby performed at the quantum-correction level,[2-5] though comprehensive analyses of neutral-current processes and weak bosons already demanded the inclusion of the radiative corrections before SLC and LEP.[6-8] More concretely, the data on M_Z and those on M_W (CDF+UA2) not only improved the above-mentioned comprehensive analyses,[3-5] but also enabled a clean test which depends only on M_W and M_Z, i.e., a test through the M_W-M_Z relation.[2]

In the electroweak quantum corrections, the leading-logarithmic (LL) terms $[\alpha\ln(m_f/M_{W,Z})]^n$ (where m_f is a fermion mass) are quite important, as is well known. Such corrections are familiar to us also in QCD and QED. We can reproduce these corrections using the QED running coupling constant $\alpha(M_Z)$ instead of α^{expt} $(=1/137.036)$, which is justified by the renormalization-group analysis.[9] In addition to these LL corrections, a term proportional to $G_F m_t^2$ is also known to appear in the corrections if the top quark is very heavy.[10-12] Therefore, the recent CDF result, $m_t \gtrsim 78$ GeV,[13] implies that the top-quark effect is non-negligible too.

In theories like QCD or QED, the decoupling theorem holds and virtual heavy-particle effects are all suppressed by the inverse power of their masses.[14,15] That is, the appearance of the $G_F m_t^2$ terms is one of the characteristic features of the electroweak theory, in contrast to the LL-type corrections which are very universal. Therefore, it is quite meaningful as a precision test of the electroweak theory to examine whether the data really demand a $G_F m_t^2$-type correction or can be reproduced without it (i.e., by the LL terms alone). In fact, some constraint has been derived on m_t $(40\text{-}80 \lesssim m_t \lesssim 200$ GeV) through this correction.[3-8] This is a very interesting result, although the fact that such a constraint can be obtained from the data by assuming the electroweak theory at the quantum-correction level does not necessarily mean that the data actually demand the $G_F m_t^2$-type correction, if not independent.

In the above analyses, however, the neutral-current processes, especially deep-inelastic νN scattering, play a quite important role, which depends inevitably on strong-interaction effects. Therefore, it is significant as a clean precision test of the theory to make an investigation independent of those neutral-current experiments. A previous analysis[2] has been made under this consideration. In this Letter, I shall make a further analysis following the same policy. What I want to show is that the recent data of the ALEPH, DELPHI, L3, OPAL, and Mark II Collaborations on M_Z (Refs. 16 and 17) and those of the CDF and UA2 Collaborations on M_W (Refs. 18 and 19) really demand the existence of the $G_F m_t^2$-type term [plus other $O(\alpha)$ corrections].

Let us start our discussion by briefly summarizing the M_W-M_Z relation. As is well known, the electroweak theory has five kinds of independent parameters: α, M_W, M_Z, m_f, and m_ϕ (the Higgs-boson mass). The muon decay width has been measured very precisely (usually expressed by G_F), and gives a strong constraint on the above parameters through

$$G_F(\alpha,M_W,M_Z,m_f,m_\phi)=G_F^{\text{expt}}. \qquad (1)$$

Here the left-hand side is a function of the five parameters as is expressed, but its behavior is mainly controlled by α, M_W, and M_Z. So, once we use $\alpha=\alpha^{\text{expt}}$, Eq. (1) gives a relation between M_W and M_Z which depends also on m_f and m_ϕ to a certain extent. This is what I call "the M_W-M_Z relation," and is usually expressed as a value of M_W calculated from α, G_F, M_Z, m_f, and m_ϕ. This relation has been studied in detail and improved by several authors[20] since its importance was pointed out.[21] Now we can thereby predict M_W with a theoretical uncertainty less than $\sim$40 MeV (see, e.g., Ref. 22).

The great advantages of using this relation are as follows: (1) Only directly measurable quantities are used, so the analysis becomes very clear; and (2) it has the least dependence on the complicated hadron physics, like the parton distribution functions in the nucleon, QCD corrections to quark lines, and estimates of various hadron matrix elements, etc., which takes an important role in many other weak processes. That is why I have used, and also use here, only the M_W-M_Z relation.

The tree-level relation is derived easily from Eq. (1) as

$$M_W^2=\frac{1}{2}M_Z^2\left[1+\left(1-\frac{2\sqrt{2}\pi\alpha}{M_Z^2G_F}\right)^{1/2}\right], \tag{2}$$

and all we have to do in order to take account of the LL corrections is replace α by $\alpha(M_Z)$ as was mentioned already.[9] Since the present experimental lower bound on the top-quark mass is ~ 78 GeV by CDF,[13] we can calculate $\alpha(M_Z)$ with the well-known fermions as

$$\alpha(M_Z)=\frac{\alpha}{1+(2\alpha/3\pi)\sum_{f(\neq t)}Q_f^2\ln(m_f/M_Z)}, \tag{3}$$

where the sum is over both flavors and colors, and Q_f is the electric charge in $|e|$ units. That is, M_W is uniquely calculable (independent of m_t and m_ϕ) at the LL-correction level.

By using the combined data of four collaborations at LEP[16] and the Mark II Collaboration at SLC,[17]

$$M_Z^{\text{expt}}=91.157\pm0.032\text{ GeV},$$

and the W-boson mass is calculated as

$$M_W^{(0)}=80.90\pm0.04\text{ GeV}\quad(\text{tree level}), \tag{4a}$$

$$M_W^{(\text{LL})}=79.75\pm0.04\pm0.02\text{ GeV}\quad(\text{LL level}). \tag{4b}$$

Here, concerning the quark masses, I have used $m_u=m_d=0.040$ GeV, $m_s=0.10$ GeV, $m_c=1.5$ GeV, and $m_b=4.7$ GeV. They were derived by fitting the free-quark calculations of the renormalized photon self-energy with its numerical estimate which uses a dispersion relation and experimental data of $\sigma(e^+e^-\to\text{hadrons})$.[23] The second error in $M_W^{(\text{LL})}$ comes from this numerical estimate. It is remarkable that not only $M_W^{(0)}$ but also $M_W^{(\text{LL})}$ cannot reproduce the combined data of CDF and UA2,[18,19]

$$M_W^{\text{expt}}=80.24\pm0.37\text{ GeV},$$

which means that the data demand some other corrections.

Let us take account of the other corrections, including a term proportional to $G_Fm_t^2$ and other (small) $O(\alpha)$ corrections, which are characteristic of the electroweak theory as stressed already. The complete set of corrections, i.e., these plus the leading-log terms, is usually expressed by Δr (Sirlin's notation[24]). Recently, the following has been proposed as a formula which sums up both the leading-log and the $G_Fm_t^2$ terms (and also some finite terms) to all orders in perturbation (Consoli, Hollik, and Jegerlehner in Ref. 20):

$$M_W^2=\frac{\rho}{2}M_Z^2\left\{1+\left[1-\frac{2\sqrt{2}\pi\alpha}{\rho M_Z^2G_F}\left(\frac{1}{1-\Delta\alpha}+\Delta r_{\text{rem}}\right)\right]^{1/2}\right\}, \tag{5}$$

where

$$\Delta\alpha\equiv-\frac{2\alpha}{3\pi}\sum_{f(\neq t)}Q_f^2\left\{\ln\left(\frac{m_f}{M_Z}\right)+\frac{5}{6}\right\},$$

$$\rho\equiv\left(1-\frac{3\sqrt{2}G_Fm_t^2}{16\pi^2}\right)^{-1},$$

$$\Delta r_{\text{rem}}\equiv\Delta r-\Delta\alpha+\frac{M_W^2}{M_Z^2-M_W^2}\left(1-\frac{1}{\rho}\right).$$

This formula is consistent with the explicit two-loop calculations.[25] I use it in this Letter. All the necessary formulas to compute Δr are found, e.g., in Refs. 24 and 26-29. The corrected M_W for $M_Z^{\text{expt}}=91.157\pm0.032$ GeV, $m_t=140$ GeV, and $m_\phi=100$ GeV becomes

$$M_W=80.21\pm0.04\pm0.04\text{ GeV}, \tag{6}$$

where the second error comes from the so-called scheme dependence, which I have studied according to Jegerlehner's procedure,[20,22] and QCD effects in heavy-quark loops (Kniehl, Kühn, and Stuart in Ref. 20), in addition to the ±0.02 in Eq. (4b). This M_W is in full agreement with $M_W^{\text{expt}}=80.24\pm0.37$ GeV.

Concerning the second error in Eq. (6) matter under the present circumstance that neither m_t nor m_ϕ is known. In fact, the central value of M_W changes for different m_t and m_ϕ as shown in Table I. In this analysis it is important that the fully corrected M_W can explain M_W^{expt} within the present experimental and phenomenological constraints on m_t and m_ϕ, i.e., $80\lesssim m_t\lesssim 200$ GeV[3-7,13] and $m_\phi\gtrsim 24$ GeV,[30] while $M_W^{(0)}$ and $M_W^{(\text{LL})}$ cannot. Note that in the table it becomes difficult even for M_W to reproduce M_W^{expt} if the top quark were lighter than ~ 80 GeV or heavier than ~ 200 GeV.

Before closing the analysis I wish to mention that my calculations of Δr are numerically in full agreement with those of Sirlin,[24] Jegerlehner,[22] and Hollik.[28]

Finally, I give a brief comment about another important quantity of the Z boson which has also been measured precisely at SLC and LEP, i.e., the Z width. If the total Z width Γ_Z is taken into the analysis, some additional ambiguity appears from the QCD corrections to the hadronic final state (see, e.g., Ref. 28 as a review). That is why I have not used Γ_Z in the present analysis, but it does not mean that the analysis of the Z width is

TABLE I. The fully corrected M_W for $M_Z^{\text{expt}}=91.157$ GeV, and different m_t and m_ϕ. All values here are in GeV units.

m_ϕ \ m_t	30	60	80	150	200	230
50	79.77	79.74	79.88	80.30	80.66	80.92
100	79.74	79.71	79.85	80.27	80.63	80.89
500	79.64	79.61	79.75	80.18	80.54	80.79
1000	79.59	79.56	79.70	80.13	80.49	80.74

uninteresting. For example, we can thereby get independent information on m_t. In fact, according to my brief computations, the present data $\Gamma_Z^{\rm expt}=2.538\pm 0.028$ GeV [31] demand a very heavy top quark, $m_t \sim 250$ GeV (anyway $m_t \gtrsim 200$ GeV). The data of Γ_Z are still somewhat unstable in contrast to $M_{W,Z}^{\rm expt}$, so it is premature here to draw any dramatic conclusions from this result, though it seems to be in contradiction to other analyses. It shows, however, that we should keep paying attention to Γ_Z too.

In conclusion, I have calculated the W-boson mass at the tree, leading-log, and full correction levels in the framework of the standard SU(2)×U(1) electroweak theory using $M_Z^{\rm expt}$ measured recently at LEP and SLC. Among them, only the fully corrected M_W reproduces $M_W^{\rm expt}$ by CDF and UA2 for reasonable m_t and m_ϕ. This result means that the new data demand the existence of the $G_F m_t^2$-type term in the electroweak radiative corrections. Therefore, it gives new phenomenological support, independent of the neutral-current processes, to the electroweak theory, although the final test in this strategy is possible only when m_t and m_ϕ are determined experimentally.

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