Linear Temperature Dependence of Resistivity as Evidence of Gauge Interaction

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(Received 22 March 1990)

We show that the gauge character of interactions, which naturally appears in strongly correlated electron systems in the absence of long-range magnetic order, results in a linear T dependence of the resistivity.

PACS numbers: 72.10.Bg, 11.10.Lm, 74.65.+n, 74.70.Vy

Apart from the high value of T_c the most striking difference between the copper oxides and the usual metals is revealed by their normal-state properties. One of the most famous and reliable of their normal-state anomalies is the linear T resistivity observed in a wide temperature range ($\gtrsim 600$ K).[1]

As was pointed out by Anderson[2] and supported later by numerous evidences these materials are close to a Mott-Hubbard metal-insulator transition, with a small number of holes introduced by doping being responsible for charge transport. Anomalous properties of the normal state indicate that these holes cannot be described by a Fermi-liquid theory. In fact, they should not be described by a Fermi-liquid theory unless there is a long-range magnetic order. The reason is that the strong on-site repulsion results in long-range forces between holes, *rendering* the Fermi-liquid description impossible. These forces are gauge fields.[3,4] The gauge fields always appear when some states are completely excluded from the Hilbert space, such as states with double occupation, which are excluded in strongly correlated electron systems. The long-range gauge forces become important in the absence of magnetic order. In this Letter we assume its absence and show that gauge forces result in the linear T dependence of resistivity. The correlated electron system can be formally described by the theory of Fermi and Bose fields coupled with the gauge potential. In this formalism the Bose field is responsible for the linear T dependence of resistivity. Thus, this theory is reminiscent of the original Anderson explanation of linear resistivity which turned out to be essentially true.[5]

The motion of holes impeded by the constraint of no double occupation results in a long-range interaction which is assumed to be the most important interaction in the problem. Thus, we can simplify the problem by leaving only this interaction and neglecting any local interactions (such as antiferromagnet local exchange, etc.):

$$H=\sum_{i,j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha}, \tag{1}$$

where $c_{i\alpha}$ is an electron at site i with spin α satisfying the constraint

$$n_i=\sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1 \tag{2}$$

and the total number of electrons is close to one per site, $(1/N)\sum_i n_i = 1-x$, with $x \leq 1$ the doping. This is just the Nagaoka problem.

This problem is, in fact, a gauge-field theory. The constraint can be solved by representing the electrons as a product:

$$c_{i\alpha}=\psi_i^{\dagger} Z_{i\alpha}, \tag{3}$$

where ψ_i is the spinless Fermi field of holes, whereas the complex spinor field Z is a CP^1 representation of the spin operator,

$$\mathbf{S}_i \equiv c_{i\alpha}^{\dagger}\sigma_{i\alpha\beta}c_{i\beta}=Z_{i\alpha}^{*}\sigma_{i\alpha\beta}Z_{i\beta}\,|Z|^2=1-\psi^{\dagger}\psi .$$

The ansatz (3) ensures the constraint (2) since $n_i=1-\psi_i^{\dagger}\psi_i$, and introduces new gauge degrees of freedom:

$$\psi_i \to \psi_i e^{ia_i},$$

$$c_i \to c_i,$$

$$Z_i \to Z_i e^{-ia_i}.$$

In new variables the Hamiltonian becomes

$$H=-\sum t_{ij}\psi_i^{\dagger}\psi_i Z_j^{*} Z_i . \tag{4}$$

The well-known Nagaoka theorem[6] states that one hole in the system chooses the ferromagnetic ground state $Z_i=\text{const}$, $Z_i^{*}Z_j=1$, so that the hole becomes a free particle. At a macroscopic concentration of holes the ferromagnetic correlations should persist at small scales less than $x^{-1/2}$ (no antiferromagnetic exchange). Thus, at small doping $x \ll 1$ one can use the continuum limit of the Hamiltonian (5):

$$H=-\frac{1}{2m}\int [\psi^{\dagger}(\nabla - i\mathbf{a})^2\psi+\gamma|\psi|^2(\nabla\mathbf{S})^2],$$

where $\mathbf{a}=iZ^*\nabla Z$ is the gauge field produced by the fluctuations of the vector field $\mathbf{S}$; its flux is the density of the Skyrmions—topological excitations:

$$F=(1/8\pi^2)\epsilon_{\mu\nu}\mathbf{S}\cdot[\partial_\mu\mathbf{S}\times\partial_\nu\mathbf{S}].$$

The coefficient $\gamma>0$ describes the strength of the hole-spin wave repulsion; for usual lattices $\gamma\sim 1$. Thus, the interaction between holes and spins is naturally divided into two parts: contact repulsion and nonlocal interaction with Skyrmions mediated by the gauge field. The first part stabilizes the ferromagnetic state at short scales, whereas the second is responsible for the large-distance behavior.

This theory can be formally rewritten by introducing the independent auxiliary gauge field $(a_0,\mathbf{a})$,

$$L=\psi^\dagger(i\partial_0-a_0)\psi-\frac{1}{2m}\psi^\dagger(i\nabla-\mathbf{a})^2\psi + Z^*(i\partial_0-a_0)Z-\frac{\gamma}{2m}\psi^\dagger\psi Z^*(i\nabla-\mathbf{a})^2 Z. \quad (5)$$

The gauge field a_μ plays the role of Lagrange multipliers ensuring the constraints resulting from the local constraint (2):

$$j_\psi+j_Z=0,\quad \rho_\psi+\rho_Z=0, \quad (6)$$

where $j_{\psi,Z}$ is the current of holes or spin excitations and ρ is their density. The physical electromagnetic current $j_{\text{phys}}=(1/2m)c^\dagger\nabla c$ can be regarded as the hole or spin current according to the relation (6): $j_{\text{phys}}=j_\psi=-j_Z$.

We do not know the ground state of the Lagrangian (5), but we assume that at some small doping the long-range magnetic order disappears and the excitations in the normal state can be described as spinless holes and spin-carrying bosons Z interacting via gauge field a. It implies that these bosons do not form a Bose condensate which would be destroyed by the gauge interaction. We do not know the renormalized physical parameters (such as the bosonic and fermionic mass density of states, etc.). Instead we introduce them phenomenologically. We assume that all properties of the Bose field are governed by a single energy scale $E_c\ll\epsilon_F$.

The gauge field a_μ becomes dynamical when we take into account the radiative corrections produced by fermions and bosons. In the normal state the interaction between the gauge field and charged fields can be treated perturbatively. The leading contribution to the propagator of the gauge field is given by the fermionic and bosonic loop. As usual the electric field is screened at small scales and its effect can be neglected. This is not the case for transverse photons, which mediate the long-range interaction between particles. The physical reason for this effect is that the state magnetic field is not screened in the charged normal gas, whereas Landau damping provides effective screening for the magnetic field. The importance of this effect for the low-temperature properties of the normal metal was pointed out by Reizer.[7]

In the Coulomb gauge $a_0=0$, the fermionic contribution to the transverse field propagator is

$$(D_F)_{\mu\nu}=D_F(\delta_{\mu\nu}-q_\mu q_\nu/q^2),$$
$$D_F^{-1}=i\omega\lambda_F+\chi_F q^2, \quad (7)$$

where $\chi_F\sim m_F^{-1}$ is the diamagnetic susceptibility of the charged Fermi gas of holes, and λ_F describes the strength of Landau damping. In the clean metal at low temperatures $\lambda_F\sim\sqrt{x}/q$. At small q the singular behavior of λ_F is cut off by the mean free path $l=\tau_{\text{tr}}v_F$. The transport relaxation rate $\tau_{\text{tr}}^{-1}=\tau_{\text{imp}}^{-1}$ in the dirty metal. In the clean metal it was evaluated by Reizer and Lee in a different context.[7,8] In 2D it is $\tau_{\text{tr}}^{-1}\sim(T/\epsilon_F)^{4/3}\epsilon_F$ or $(\omega/\epsilon_F)^{4/3}\epsilon_F$.[8] Dressing of the electron bubble (7) by the gauge field does not result in any singular contributions to D_F^{-1}: damping is cut off at small q, and the magnetic susceptibility χ does not acquire singular contributions as well. In the limit $q/\omega\to 0$, $\omega\to 0$, the damping coefficient becomes $\lambda_F^{-1}(\epsilon_F\tau_{\text{imp}})^{-1}+(T/\epsilon_F)^{4/3}$.

The total propagator of the gauge field is the sum of the Fermi and Bose contributions:

$$D^{-1}=D_F^{-1}+D_B^{-1}=i\omega\lambda+\chi q^2,$$
$$\lambda=\lambda_F+\lambda_B,\quad \chi=\chi_F+\chi_B. \quad (8)$$

The Bose contribution to the photon propagator does not result in as large a damping as the Fermi one and can be neglected, $\lambda\approx\lambda_F$. Its contribution to the diamagnetic susceptibility $\chi_B\sim m_B^{-1}$ is at most the same as the Fermi one, and it does not influence the estimates. However, the physical resistivity is governed by the bosons as we show below.

As we explained above, one can ascribe the physical electromagnetic charge to either holes or bosons. Choosing, say, the former prescription we express the conductivity σ through the correlator of j_ψ currents. In the Gaussian approximation over the gauge field we get

$$\langle j_\psi j_\psi\rangle=D_F^{-1}-D_F^{-1}(D_F^{-1}+D_B^{-1})^{-1}D_F^{-1} =(D_F+D_B)^{-1},$$

where D_F and D_B denote the hole and boson dressed bubbles. This equation is symmetric in bosons and fermions and is characteristic for all systems where interaction is carried by the gauge field.[9] Thus, the conductivity is

$$\sigma=(\lambda_B^{-1}+\lambda_F^{-1})_{q=0}^{-1}. \quad (9)$$

This equation neglects the contribution from the diagrams in which two hole bubbles are connected with more than one photon line. Physically, it means that we neglect the effect of the photon drag.

The fermion part λ_F is known.[8] Below we estimate λ_B.

As we see below the main contribution to the dissipation of bosons comes from the scattering by the low-energy photons $\omega\lambda \approx v_F q^2$ [see (7)], so that $\omega/v_F q \ll 1$. In this frequency range the radiative corrections to the vertex are nonsingular and small, since they contain the powers of the dimensionless parameter $m_F/m_B \ll 1$, which is proportional to the doping concentration. We will neglect these corrections. This justifies the use of the kinetic equation approach. However, the corrections can renormalize one-particle properties (such as mass, relaxation time, etc.) substantially.

The renormalization of the vertex becomes very important in the opposite limit, $\omega/v_F q \gg 1$, which changes the interaction with an external electromagnetic field.

In the framework of the kinetic equation the problem is reduced to the evaluation of transport relaxation time:

$$\lambda_B = (e^2 n_B/m_B)\tau_{tr}, \tag{10}$$

where m_B is the renormalized mass of bosons and n_B is an effective number of bosons involved in scattering.

The transport relaxation time is given by the standard expression:

$$\tau_{tr}^{-1}(\epsilon,p) = \int \frac{dp^2}{(2\pi)^2}\frac{d\omega}{2\alpha}\,\mathrm{Im}G_B(\epsilon-\omega p') \times \mathcal{J}_\mu(p)\,\mathrm{Im}D^B_{\mu\nu}(\omega q)\mathcal{J}_\mu(p) \times \left(\coth\frac{\omega}{2T}+\coth\frac{\epsilon-\omega}{2T}\right)(1-\cos\theta_{p,p'}). \tag{11}$$

Here $\theta_{pp'} = \angle(p,p')$ is a scattering angle, $\mathbf{q}=\mathbf{p}-\mathbf{p}'$, and $\mathcal{J}_\mu = \mathcal{J}(q,\omega)p_\mu$ is a bosonic vertex. The angular factor in Eq. (11) describes the renormalization of the electromagnetic vertex, which appears at $\omega \gg v_F q$ and dramatically changes it. It suppresses nonphysical processes with infrared photons. Neglect of this factor simply means the loss of the gauge invariance; without this factor the integral diverges at finite temperatures. This remarkable property of the kinetic of gauge theories was pointed out by Lee.[8]

The main contribution to the relaxation rate (11) comes from the scattering by photons with energy $\omega_0 = \chi q^2/\lambda$. These energies are small, $\omega_0 \ll v_F q$, so that we can replace $\mathcal{J}_\mu$ in (11) by its value at $q \to 0$, $\omega/q v_F \to 0$: $\mathcal{J}_\mu = \mathcal{J}(\epsilon,p)p_\mu$. At low temperatures these energies become lower than the temperature:

$$\omega_0/T \equiv \chi q^1/\lambda T \sim (m_B/m_F)(Tm_B)^{1/2} \ll 1. \tag{12}$$

In this temperature range the factor $\coth(\omega/2T) + \coth[(\epsilon-\omega)/2T]$ is reduced to $2T/\omega$. Then the integral over ω is performed using the analyticity of $G_B^R(\epsilon)$ in the upper half plane:

$$\tau_{tr}^{-1}(\epsilon,p) = \int dp'\,d\theta\,p'\mathcal{J}^2(p)\,\mathrm{Im}G_B(\epsilon + i\lambda^{-1}\chi q^2, p') \times T\chi^{-1} f(p',p), \tag{13}$$

where

$$f(p,p') = (1-\cos\theta_{p,p'})[1-(pq)^2/p^2q^2]p^2/q^2.$$

We do not need to know the specific properties of G_B in order to evaluate Eq. (13). The reason is that G_B depends weakly on the angle $\theta_{p,p'}$ since $\omega_0 \ll T \sim \epsilon \sim m_B^{-1} \times (p')^2$ and, as we show below, $\omega_0 \ll \tau_{tr}^{-1}$ as well. Integrating over the angle θ we get

$$\tau_{tr}^{-1} = T\chi^{-1}\int \frac{p'\,dp'}{(2\pi)^2}\,\mathrm{Im}G_B(\epsilon,p')\mathcal{J}^2(p')\phi_1(p'/p),$$

$$\phi_1(a) \equiv a^2\int \frac{d\theta}{2\pi}\frac{(1-\cos\theta)\sin^2\theta}{(1-2a\cos\theta+a^2)^2} = \frac{1}{2}\frac{\min(p^2,p'^2)}{p^2+p'^2}.$$

The function $\mathrm{Im}G_B(\epsilon,p')$ is sharply (order of τ_{tr}^{-1}) peaked around $\epsilon = p'^2/m_B$; therefore $\phi_1(p'/p)$, being a smooth function of p', can be replaced by $\phi_1(1)$, yielding

$$\tau_{tr}^{-1} = \tfrac{1}{4}TJ^2(\epsilon)\chi^{-1}v_B(\epsilon),$$

where $v_B(\epsilon)$ is the boson density of states. Assuming that $\mathcal{J}(\epsilon)$ and $v_B(\epsilon)$ do not depend on ϵ in the range $E_c < \epsilon < \epsilon_F$, we finally get

$$\lambda_B = \left(\frac{4\chi}{\mathcal{J}^2 v_B}\frac{n_B}{m_B}\right)T^{-1}.$$

This result is not sensitive to the photon damping λ and, thus, to additional nonmagnetic mechanisms of dissipation. The damping λ appears only in the condition (12), which makes this analysis valid. According to our assumption, parameters of the Bose system $m_B, n_B, \mathcal{J}, v$ are determined by a single energy scale E_c, which yields the estimate

$$\lambda_B \sim \epsilon_F/T.$$

This damping is smaller than the damping of the gauge field generated by fermions, $\lambda_B \ll \lambda_F$, which justifies our assumption $\lambda = \lambda_B + \lambda_F \simeq \lambda_F$. On the contrary, according to (9) the resistivity is governed by the Bose contribution,

$$R \sim (\hbar/e^2)T/\epsilon_F. \tag{14}$$

The temperature dependence of the resistivity is not sensitive to nonmagnetic scattering mechanisms (such as impurities, phonons, etc.). This is not true for a frequency-dependent conductivity $\sigma(\epsilon)$ at $\epsilon \gg T$, which depends crucially on the photon damping λ.

We start with the dirty limit in which λ_F saturates at $\lambda_{imp} = \tau_{imp}\epsilon_F$ for small $q < x^{1/2}(\tau_{imp}\epsilon_F)^{-1}$. Since the main contribution to resistivity comes from bosons with $p^2/m_B \sim T$, at sufficiently low temperatures $T < x(\tau_{imp}\epsilon_F)^{-2}m_B^{-1}$, and then λ can be replaced by λ_{imp}.

At $\epsilon \gg T$ the photon frequencies are large, $\omega \sim \epsilon$, so that the factor $\coth(\omega/2T) + \coth[(\epsilon-\omega)/2T]$ in (11) is reduced to $\mathrm{sgn}\,\omega + \mathrm{sgn}(\epsilon-\omega)$; one can neglect the χq^2 term in the photon Green's function since $\lambda\omega \sim \lambda q^2 m_B^{-1}$

$\gg \chi q^2$. The result is

$$\tau_{tr}^{-1} = \int \frac{dp'\,d\omega}{(2\pi)^2} \mathcal{J}^2 \,\mathrm{Im} G_B(\epsilon-\omega, p')(\lambda\omega)^{-1}[\mathrm{sgn}\omega + \mathrm{sgn}(\epsilon-\omega)]\phi_2(p'/p)p^2 \,,$$

$$\phi_2(a) = \int \frac{d\theta}{2\pi} \frac{(1-\cos\theta)\sin^2\theta}{1-2a\cos\theta + a^2} = \tfrac{1}{2}(1-\tfrac{1}{2}a) \,.$$

Proceeding as above we replace $\phi(p'/p)p^2$ by its value of $p'^2 = 2\epsilon/m_B \approx p^2$ and perform the integral over p'. The remaining integral over ω gives, with logarithmic accuracy,

$$\tau_{tr}^{-1} = [(1/4\pi)\mathcal{J}^2 \lambda_{imp}^{-1} \nu_B(\epsilon) m_B \ln(\lambda/\chi m_B)]\epsilon \,.$$

Assuming again that all parameters of the Bose system are governed by a single energy scale, we estimate the correction to residual resistivity:

$$\delta R \sim (\tau_{imp}\epsilon_F)^{-1}\epsilon/E_c \,. \qquad (15)$$

In the clean case the main contribution to the scattering rate $\tau_{tr}^{-1}(\epsilon)$ comes from photons with $\lambda\omega \sim \chi q^2$. The estimate yields

$$\tau_{tr}^{-1} \sim (\epsilon/\epsilon_F)^{3/2} E_c \,,$$

which falls rapidly at $\epsilon \ll \epsilon_F$. Thus, in this case the frequency-dependent resistivity is governed by the fermions:

$$R \sim (\epsilon/\epsilon_F)^{4/3} \,.$$

The above analysis was based mainly on the weak assumption of the absence of a long-range magnetic order. The main physical results are a consequence of the gauge character of the interactions in strongly correlated electron systems and are presumably model independent.

When this work was in progress we became aware of the analogous results obtained by Nagaosa and Lee in a different framework.[10] Our work was substantially facilitated by discussions with P. A. Lee who explained their results prior to publication.

We acknowledge helpful conversations with P. W. Anderson, G. Kotliar, A. Larkin, M. Reizer, A. Ruckenstein, and C. Varma. This work is supported in part by NSF, under Grants No. DMR 88-18713 and No. DMR 8518163.

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