Chiral Weak Dynamics

T. Morozumi
Rockefeller University, 1230 York Avenue, New York, New York 10021

C. S. Lim
CERN, Geneva, Switzerland

A. I. Sanda
Rockefeller University, 1230 York Avenue, New York, New York 10021
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We derive a chiral weak Lagrangian which interpolates the QCD-corrected four-fermion weak-interaction Lagrangian to low energies. This is done using the Nambu–Jona-Lasinio model as a guide. The derivation treats SU(3) breaking from the current-quark mass on an equal footing with the spontaneous chiral-symmetry breaking. It leads naturally to an additional factor of 2–4 enhancement of $\Delta I = \frac{1}{2}$ $K \to \pi\pi$ decay amplitudes.

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Chiral symmetry has proven useful in describing strong interactions of π and K mesons at low energies. In contrast, the standard model of electroweak interaction is applicable only above an energy scale where the quark description is valid. In this Letter, we attempt to link these two descriptions. We obtain a Lagrangian for meson interactions at low energies which incorporates both the chiral symmetry of the strong interaction and the electroweak interaction.

We first point out that the usual nonlinear chiral Lagrangian used by most authors is not well suited for describing weak interactions. Some K-decay amplitudes are proportional to the difference in the pseudoscalar decay constants $F_K - F_\pi$. This vanishes to lowest order in the conventional chiral perturbation theory. Thus the result depends on how an additional $SU(3)_{L+R}$ breaking is introduced by hand.[1] Here we shall start with a chiral Lagrangian which gives a correct value for F_K/F_π at the lowest order.

It is easy to understand why the traditional chiral Lagrangian may not be suitable for describing all processes. This Lagrangian was constructed so that current-algebra results could be easily obtained.[2] Originally, this was the top priority as the underlying strong-interaction dynamics was not known. We are now confident that QCD will manifest itself in the form of a chiral Lagrangian at low energies. It thus makes sense to ask if an alternative chiral Lagrangian which is closer in line with QCD can be constructed.

A low-energy effective theory which reflects many properties of QCD is that of Nambu and Jona-Lasinio.[3] A chiral Lagrangian has been derived using this effective low-energy theory as a guide.[4] Following these references, we write

$$L_{\rm NJ} = \bar{q}(i\partial\!\!\!/ - m^0)q + 2G_1 \sum_a \left[\left(\bar{q}_\alpha \frac{\lambda_a}{2} q_\alpha \right)^2 + \left(\bar{q}_\alpha \frac{\lambda_a}{2} i\gamma_5 q_\alpha \right)^2 \right], \qquad (1)$$

where λ_a is a flavor U(3) matrix and the color index α is summed over (below, we shall drop the color index). The diagonal current-quark mass matrix m^0 breaks the $SU(3)_{L+R}$ symmetry.

To derive a chiral Lagrangian, introduce pairs of scalar and pseudoscalar auxiliary fields (S_a, σ_a) and (P_a, π_a), respectively, with

$$1 = \int \prod_a DS_a\, D\sigma_a\, DP_a\, D\pi_a \exp\left\{ i\int d^4x \left[S_a \left(\sigma_a - \bar{q}\frac{\lambda_a}{2} q \right) + P_a \left(\pi_a - \bar{q}\frac{\lambda_a}{2} i\gamma_5 q \right) \right] \right\}. \qquad (2)$$

With this constraint, we obtain

$$Z_{\rm strong} = \int \prod_a Dq\, D\bar{q}\, DS_a\, DP_a \exp i\int d^4x\, L_f(S_a, P_a) \int D\sigma_a\, D\pi_a \exp\left\{ i\int d^4x [(S_a - m_a^0)\sigma_a + P_a\pi_a + L_{\rm st}(\sigma_a, \pi_a)] \right\}, \qquad (3)$$

where

$$L_f = \bar{q} i\partial\!\!\!/ q - S_a \bar{q}\frac{\lambda_a}{2} q - P_a \bar{q}\frac{\lambda_a}{2} i\gamma_5 q, \qquad (4)$$

$$L_{\rm st} = 2G_1 \sum_a (\pi_a^2 + \sigma_a^2), \quad m_a^0 = {\rm Tr}(m^0 \lambda_a). \qquad (5)$$

Now integrations over σ_a and π_a can be performed trivially by completing the squares.

The generating functional becomes

$$Z_{\rm strong}=\int DM\,DM^\dagger \exp\left\{-i\int d^4x \frac{1}{4G_1}{\rm Tr}[(M-m^0)^\dagger(M-m^0)]\right\}Z_f(M^\dagger M)\,, \tag{6}$$

where $M=S+iP$, $S=\sum_{a=0}^8(\lambda_a/2)S_a$, and $P=\sum_{a=0}^8(\lambda_a/2)P_a$.

The fermionic determinant can be written in the form

$$Z_f(M^\dagger M)=\int DM\,DM^\dagger \exp\left\{i\int d^4x[Z_\sigma^{-1}{\rm Tr}(\partial_\mu M\,\partial^\mu M^\dagger)+H(W^2,W^3,W^4)]\right\}, \tag{7}$$

where Z_σ represents a logarithmically divergent factor generated from performing the integration over quark loops. The chiral invariance implies that H is a function of $W^2={\rm Tr}(M^\dagger M)$, $W^3={\rm Tr}(M^\dagger MM^\dagger M)$, $W^4=\det M+\det M^\dagger$, and appropriate higher-order derivative terms.[5]

If we take a specific form for H,

$$H=\mu_0^2{\rm Tr}(M^\dagger M)-\lambda{\rm Tr}(M^\dagger MM^\dagger M)\,, \tag{8}$$

the Lagrangian obtained from Eqs. (6) and (7) reduces to the σ model. Nonvanishing $\langle M\rangle$ implies spontaneous chiral-symmetry breaking. In the present formalism $\langle M\rangle$ must be interpreted to be the consitutent-quark mass matrix [see Eq. (4)]:

$$\langle M\rangle=\begin{bmatrix} m_u & 0 & 0\\ 0 & m_d & 0\\ 0 & 0 & m_s\end{bmatrix}. \tag{9}$$

Also, after the field renormalization, one can derive an expression for the pseudoscalar-octet (π_{ij}) decay constant:

$$F_{ij}=\frac{m_i+m_j}{Z_\sigma^{1/2}}\,. \tag{10}$$

This leads to a relation

$$\frac{F_K}{F_\pi}=\frac{m_u+m_s}{m_u+m_d}\approx 1.3\,, \tag{11}$$

which is in agreement with the experimental value of 1.2.

In the conventional approach,[6] the spontaneous breaking of $SU(3)_L\times SU(3)_R$ symmetry is considered first. Then explicit $SU(3)_{L+R}$-symmetry breaking from m_0 is considered as a perturbation. In the present approach, as evidenced by Eq. (9), the formalism dictates the manner in which these symmetry-breaking effects must be treated.

For example, consider the constraint

$$S_a+4G\sigma_a-m_a^0=0\,, \tag{12}$$

which is the equation of motion for σ_a derived from Eq. (3). Taking the vacuum expectation value $\langle M_{ij}\rangle=m_i\delta_{ij}$, $\langle\sigma_{ij}\rangle=\frac{1}{2}\langle\bar q_i q_i\rangle\delta_{ij}$, we obtain

$$m_i=m_i^0-2G_1\langle\bar q_i q_i\rangle\,, \tag{13}$$

a Nambu-Jona-Lasinio gap equation. This constraint treats $SU(3)_{L+R}$-symmetry breaking from m_0 and the spontaneous $SU(3)_L\times SU(3)_R$ breaking from $\langle\bar q_i q_i\rangle$ on exactly the same footing.

Also, Eq. (10) can be written as

$$F_{ij}\sim m_i^0+m_j^0-2G_1\langle\bar q_i q_i+\bar q_j q_j\rangle\,. \tag{14}$$

Again, $SU(3)_{L+R}$ breaking from m^0 enters directly into that of F_{ij}.

The presence of SU(3) breaking makes the analysis more complicated. Since $F_K\neq F_\pi$, we cannot set $M^\dagger M=1$. For this reason the fermionic determinant will, in general, contain more terms compared to that for the linear case.

There is also another crucial physics difference between the two schemes.[5] Consider the mass of the σ meson:

$$M_\sigma^2=\frac{F_\pi^2(M_K^2-M_\pi^2)}{(2F_K-F_\pi)(F_K-F_\pi)}+M_\pi^2 \tag{15}$$

at the tree level. Previously, a lack of clear evidence for a resonance in the S-wave $\pi\pi$ scattering motivated the choice of $M_\sigma\to\infty$; i.e., $F_K=F_\pi$ in the nonlinear limit. In fact, however, the S-wave phase shift is consistent with the presence of a σ pole. Recent experimental data on the $\pi\pi$ phase shift measured in the reactions $e^+e^-\to\pi^+\pi^-e^+e^-$ and $PP\to\pi^+\pi^-PP$, as well as the NN phase shift, also require the presence of the σ meson with a broad width.[7]

Thus the scalar mesons contribute to the dynamics of the present approach.

Now consider the weak-interaction Lagrangian,[8]

$$L_W=\frac{G_F}{\sqrt{2}}s_1c_1c_3\sum_i R_i(\mu)Q_i\,, \tag{16}$$

where we take R_i to be defined at a renormalization scale $\mu=0.8$ GeV.

One might obtain the chiral weak Lagrangian by rewriting Q_i in terms of meson operators:[1]

$$\begin{aligned} V_{ij}^\mu&=\bar q_i\gamma^\mu q_j=-i([M,\partial^\mu M^\dagger]+[M^\dagger,\partial^\mu M])_{ji}\,,\\ A_{ij}^\mu&=\bar q_i\gamma^\mu\gamma^5 q_j\\ &=-i(-\{M,\partial^\mu M^\dagger\}+\{M^\dagger,\partial^\mu M\})_{ji}\,,\\ \bar q_{Ri}q_{Lj}&=-(1/4G_1)(M-m^0)_{ji}\,,\end{aligned} \tag{17}$$

where these identities were obtained from Noether's theorem. Here m^0, G_1, and M are renormalized quantities. Note that the theorem does not fix the currents uniquely. The term with m^0 is added to the expression for $\bar{q}_i q_j$ so that it is consistent with Eq. (12).

For the Q_6 operator, we have shown that the above approach yields the correct answer. Note that Q_6 is identical in structure to the interaction term of the Nambu- Jona-Lasinio Lagrangian given in Eq. (1). Thus the "derivation" of the strong chiral Lagrangian can be used to "derive" the bosonized form of Q_6.

For completeness we record here the chiral weak Lagrangian,

$$L=\mathrm{Tr}(\partial_\mu M \partial^\mu M^\dagger)+\mu^2\mathrm{Tr}(M^\dagger M)-\lambda\mathrm{Tr}(M^\dagger M M^\dagger M)+\frac{1}{4G_1}\mathrm{Tr}[m^0(M+M^\dagger)]+\frac{G_F}{\sqrt{2}}s_1c_1c_3\sum_{i=1}^{6}R_iQ_i+\mathrm{H.c.}\,, \tag{18}$$

$$\begin{aligned}&Q_1=(V-A)_{23}(V-A)_{11},\quad Q_2=(V-A)_{13}(V-A)_{21},\quad Q_3=(V-A)_{23}\mathrm{Tr}(V-A)\,,\\ &Q_4=[(V-A)(V-A)]_{23},\quad Q_5=(V-A)_{23}\mathrm{Tr}(V+A),\quad Q_6=-(1/2G_1^2)[(M-m^0)(M-m^0)^\dagger]_{23}\,,\end{aligned} \tag{19}$$

where V and A are given in Eq. (17). The contraction over the Lorentz index μ is understood. Here we have dropped terms resulting from Fierz transformation of four-quark operators Q_1–Q_6. They are present in general but do not give a substantial contribution at the present level of consideration. We have evaluated amplitudes for $K\to\pi\pi$ decay. Using the notation

$$\begin{aligned}\langle\pi^+\pi^-|Q_2|K^0\rangle&=-\langle\pi^0\pi^0|Q_1|K^0\rangle\\&=\sqrt{2}\langle\pi^+\pi^0|Q_1|K^+\rangle\\&=\sqrt{2}\langle\pi^+\pi^0|Q_2|K^+\rangle=X\,,\\ \langle\pi^+\pi^-|Q_6|K^0\rangle&=\langle\pi^0\pi^0|Q_6|K^0\rangle=Y\,,\end{aligned} \tag{20}$$

our result is

$$\begin{aligned}X&=i\sqrt{2}F_\pi(M_K^2-M_\pi^2)\,,\\ Y&=-4i\sqrt{2}\frac{M_K^4}{(m_s^0)^2}(F_K-F_\pi)\frac{F_K}{3F_\pi-2F_K}\,,\end{aligned} \tag{21}$$

where we have dropped small correction terms. This is identical to the vacuum saturation approximation except for the factor $F_K/(3F_\pi-2F_K)$. The importance of introducing SU(3) breaking properly is demonstrated by the fact that this additional factor enhances the $\Delta I=\frac{1}{2}$ amplitude by a factor of 2.2–4. The lower value corresponds to evaluating $F_K/(3F_\pi-2F_K)$ with $F_\pi=93$ MeV and $F_K=114$ MeV, and the upper value corresponds to rewriting the factor as $(m_u+m_s)/(4m_u-2m_s)$ with $m_u=300$ MeV and $m_s=500$ MeV. Obviously, because of large cancellation in the denominator, the enhancement factor is sensitive to the higher-order chiral corrections. We shall leave this correction to future publications.

We can trace the origin of the enhancement factor to the σ propagator in $K\to\sigma\to\pi\pi$ amplitude by writing

$$\frac{F_K}{3F_\pi-2F_K}=\frac{M_\sigma^2-M_\pi^2}{M_\sigma^2-M_K^2}\frac{F_K^2}{F_\pi^2}\,. \tag{22}$$

Here $M_\sigma^2-M_K^2=D_\sigma^{-1}(M_K^2)$. It is important to note that the resonance width $iM_\sigma\Gamma_\sigma$ is absent in the σ propagator. According to Watson's theorem, the final-state interactions for the $\pi\pi$ channel with isospin I can be taken into account by multiplying the amplitude by $e^{i\delta_I}$, where δ_I is the S-wave $(\pi\pi)_I$ phase shift. Any diagram which produces the absorptive part of the σ propagator can be included in the final-state interaction. In order to avoid double counting, the lowest-order tree-level σ propagator must be used. Our explanation of the $\Delta I=\frac{1}{2}$ rule requires $M_\sigma\simeq700$ MeV. The width can be $O(m_\sigma)$. While experimental data suggest the necessity for the σ meson, further experimental study to clearly establish the resonance is urged. Our enhancement can be also studied in lattice computations.

If our assumption that the Nambu–Jona-Lasinio model represents the low-energy QCD dynamics is correct, QCD requires us to treat the σ meson as a dynamical degree of freedom. This in turn leads to a linear σ model and the explanation of the $\Delta I=\frac{1}{2}$ rule.

The coefficient function R_1-R_6 has been evaluated by many authors.[9] In our numerical comparison, we shall use the result of Bardeen, Buras, and Gerard. Traditionally, the mass-independent renormalization-group equation is used in computing the QCD correction. Bardeen, Buras, and Gerard point out that the total Glashow-Iliopoulos-Maiani cancellation for the anomalous dimension at $Q^2\geq m_c^2$ assumed in the previous considerations is unreliable. A full analysis relaxing this assumption leads to R_6 which is 2–3 times larger than those of earlier references. The enhancement factor comes from integrating the c-quark contribution over the entire region between μ^2 and M_W^2 as opposed to integrating over only the region between μ^2 and m_c^2. This is a more precise way of treating the charm-quark contribution to R_6. We refer to the original references for details. Figure 1 shows our result for $|\langle\pi\pi(I=0)|\times H|K^0\rangle|$. There are two sets of curves corresponding to $\Lambda=0.4$ and 0.3 GeV. These parameters are chosen to be identical to those of Bardeen, Buras, and Gerard for ease of comparision. The shaded region corresponds to the variation of the $F_K/(3F_\pi-2F_K)$ factor. Our result for the $\Delta I=\frac{3}{2}$ amplitude $A(K^+\to\pi^+\pi^0)=2.5\times10^{-8}$ GeV is identical to that of Bardeen, Buras, and Gerard.

As is seen in Fig. 1, the additional enhancement of

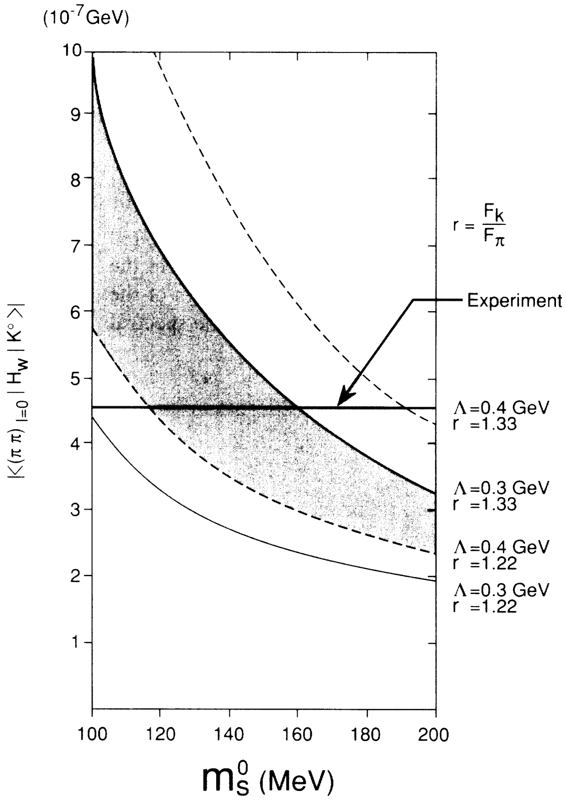

FIG. 1. Comparison of Eq. (21) with the experimental values for the $\Delta I=\frac{1}{2}$ $K\rightarrow\pi\pi$ decay amplitude. The two sets of curves correspond to $\Lambda=0.4$ and 0.3 GeV. These parameters are chosen to be identical to those of Bardeen, Buras, and Gerard (Ref. 9) for ease of comparison. Since F_K/F_π at $\mu=800$ MeV is not known precisely, we let the ratio vary from 1.22 to 1.33.

$F_K/(3F_\pi-2F_K)$ makes the theory consistent with experiment with admittedly large theoretical uncertainties.

In summary, we have chosen a specific form of chiral Lagrangian which is consistent with the Nambu–Jona-Lasinio model. We argued that the properties of low-energy QCD are reflected more closely in this version of the chiral Lagrangian. The key difference between the present approach and the conventional approach is that the spontaneous $SU(3)_L\times SU(3)_R$-symmetry breaking is treated in the presence of the current-quark mass which breaks $SU(3)_{L+R}$ at the tree level. A chiral weak Lagrangian was derived in this framework. As a first test, $K\rightarrow 2\pi$ amplitude has been computed. It was found that the $\Delta I=\frac{1}{2}$ amplitude receives additional enhancement by a factor of $F_K/(3F_\pi-2F_K)$. With this factor, the long-standing $\Delta I=\frac{1}{2}$ enhancement problem can be resolved.

Much work remains to be done along these lines. As pointed out by Bardeen, Buras, and Gerard,[10] meson evolution from $\mu=800$ MeV to $\mu=m_K$ must be computed before a serious comparison between theory and experiment can be made. Also, vector-meson dominance is an important part of low-energy physics. Both of these effects should be included in the chiral Lagrangian framework.[11]

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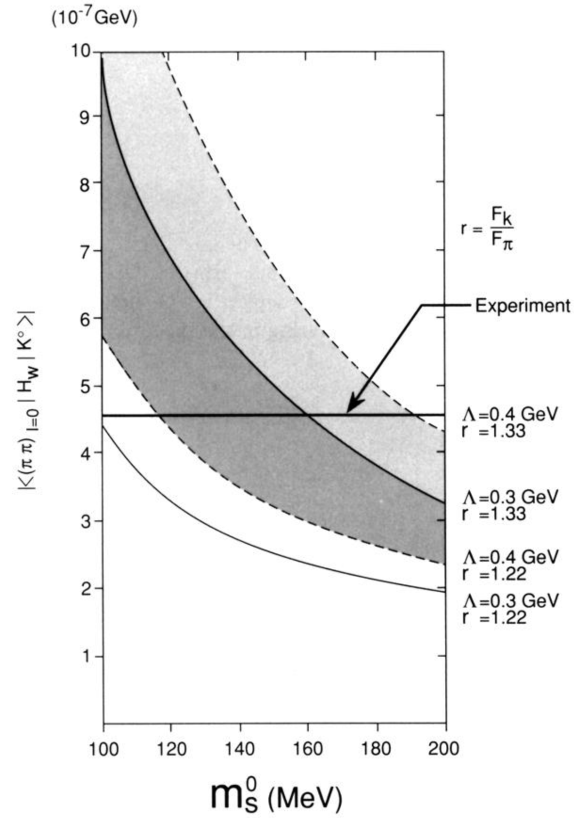

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