

## Jet Inclusive Production to $O(\alpha_s^3)$ : Comparison with Data

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Previous calculations of  $O(\alpha_s^3)$  corrections to inclusive jet production in hadron collisions are extended to finite-size jet cones and successfully compared with UA2 and CDF experimental data.

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A precise determination of the inclusive jet cross section in hadronic collisions at high energies is very important in order to provide decisive tests of parton-parton dynamics within the standard model and eventual evidence of new physics. Recent calculations<sup>1,2</sup> of  $O(\alpha_s^3)$  corrections have been performed for jet cross sections, which start to be properly defined from that order of perturbation theory, and show a reduced theoretical sensitivity to the renormalization and factorization scales, as compared to the Born  $O(\alpha_s^2)$  terms. Those results have been obtained either for all subprocesses in a fully analytical form limited to small jet-cone sizes  $\delta \ll 1$ ,<sup>1</sup> or for the pure-gluon case for finite-size cones up to  $\pi/3$ .<sup>2</sup> On the experimental side, the UA2<sup>3</sup> and CDF<sup>4</sup> Collaborations have given results up to jet transverse momentum  $p_T \approx 200$  GeV/c, with various jet algorithms and kinematical cuts.

The aim of the present Letter is to extend the analysis of Ref. 1 to a more general configuration which takes into account both the experimental definition of the jet algorithm and finite-size effects. Our findings are then compared to the experimental data.<sup>3,4</sup>

We briefly discuss the method used. We start from

$$d\sigma_{\text{num}}(\Delta - \delta) \equiv d\sigma(p_1, p_2 \in C_{\Delta - \delta}; \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}_J) \\ + d\sigma(p_1 \in C_\delta, p_2 \in C_{\Delta - \delta}; \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}_J) - d\sigma(p_1 \in C_\delta, p_2 \in C_{\Delta - \delta}; \mathbf{p}_1 = \mathbf{P}_J) + \text{permutations},$$

where  $C_\delta$  and  $C_{\Delta - \delta}$  are the cone of semiaperture  $\delta$  and the crown between the cones  $C_\delta$  and  $C_\Delta$ , respectively.

The numerical computation is performed by integrating over four variables using VEGAS. A more detailed discussion on the integration procedure will be given elsewhere.<sup>7</sup>

In Table I we show the dependence of the resulting  $O(\alpha_s^3) + O(\alpha_s^2)$  cross section on the cone size  $\Delta$  for the subprocesses  $gg \rightarrow \text{jet} + X$  and  $q_i \bar{q}_i \rightarrow \text{jet} + X$ , as compared to the analytical results of Ref. 1, valid in principle for  $\delta \ll 1$ . Both the analytical and numerical predictions are precise up to  $\sim 2\%$ . The validity of the analytical

the analytical results of Ref. 1, corresponding to the Furman<sup>5</sup> definition of the jet cone of semiaperture angle  $\delta$ , for  $\delta \ll 1$ , obtained from the basic results of Ellis and Sexton<sup>6</sup> of virtual and real parton-parton subprocesses in  $n$  dimensions. Then we add a further contribution, which is free of infrared and collinear singularities, and therefore can be calculated for  $n=4$ , which takes care of the finite cone size and the jet algorithm.

We first consider the case, *à la* Furman,<sup>5</sup> where the jet momentum  $P_J$  is well defined and it comes from the vectorial sum of hadron momenta contained within a cone of size  $\Delta$ , with no restrictions on  $\Delta$ . This situation is realized in the UA2 jet algorithm.<sup>3</sup> Then the corresponding jet cross section  $d\sigma(\Delta)$  can be schematically written as

$$\frac{E_J d\sigma(\Delta)}{d^3P_J} = \frac{E_J d\sigma_{\text{an}}(\delta)}{d^3P_J} + \frac{E_J d\sigma_{\text{num}}(\Delta - \delta)}{d^3P_J}, \quad (1)$$

where  $d\sigma_{\text{an}}(\delta)$  is the analytical contribution already given in Ref. 1 for small  $\delta$ , and  $d\sigma_{\text{num}}(\Delta - \delta)$  is the complementary contribution computed by performing the numerical integration of the matrix elements squared. More precisely one obtains ( $p_{1,2}$  are two generic final partons)

approximation up to  $\delta \leq 0.7-0.8$  is striking. This might be understood by observing that the neglected terms in the calculation are of order  $\delta^2/4$ , since they all come from the expansion of  $(1 - \cos\delta)/(1 + \cos\delta)$ .

We now compare our results with the UA2 data,<sup>3</sup> which correspond to a value  $\Delta = \arccos(0.2) = 1.37$ . We have used two different sets of structure functions to have an idea of the theoretical uncertainty. The first choice, hereafter denoted by ACGG1, corresponds to the Diemoz *et al.*<sup>8</sup> parametrization, with  $\Lambda = 160$  MeV and  $m_t = 60$  GeV, modified in order to be consistent with our

TABLE I. Inclusive jet cross section  $p_t^4 E d\sigma/d^3p$  (nb  $\text{GeV}^{-2}$ ) vs the cone size  $\Delta$  resulting from the analytical calculation (Analyt.), the combined analytical and numerical one (Numeric.), and the Born term (one loop) only (Born), for  $\sqrt{s} = 1.8$  TeV,  $\mu = p_t$ ,  $\theta = 90^\circ$ , and various  $x_i = 2p_i/\sqrt{s}$ .

$x_i$	Process	$\Delta$	Analyt.	Numeric.	Born
0.1	$gg$	0.1	$0.177 \times 10^5$	$0.177 \times 10^5$	$0.539 \times 10^5$
		0.3	$0.375 \times 10^5$	$0.369 \times 10^5$	$0.539 \times 10^5$
		0.5	$0.467 \times 10^5$	$0.480 \times 10^5$	$0.539 \times 10^5$
		0.7	$0.527 \times 10^5$	$0.542 \times 10^5$	$0.539 \times 10^5$
		0.9	$0.573 \times 10^5$	$0.607 \times 10^5$	$0.539 \times 10^5$
		1.1	$0.609 \times 10^5$	$0.677 \times 10^5$	$0.539 \times 10^5$
		1.3	$0.639 \times 10^5$	$0.702 \times 10^5$	$0.539 \times 10^5$
0.3	$gg$	0.1	$0.164 \times 10^2$	$0.164 \times 10^2$	$0.804 \times 10^2$
		0.3	$0.515 \times 10^2$	$0.494 \times 10^2$	$0.804 \times 10^2$
		0.5	$0.670 \times 10^2$	$0.666 \times 10^2$	$0.804 \times 10^2$
		0.7	$0.776 \times 10^2$	$0.771 \times 10^2$	$0.804 \times 10^2$
		0.9	$0.855 \times 10^2$	$0.887 \times 10^2$	$0.804 \times 10^2$
		1.1	$0.918 \times 10^2$	$0.991 \times 10^2$	$0.804 \times 10^2$
		1.3	$0.972 \times 10^2$	$0.101 \times 10^3$	$0.804 \times 10^2$
0.1	$q\bar{q}$	0.1	$0.705 \times 10^4$	$0.705 \times 10^4$	$0.829 \times 10^4$
		0.3	$0.817 \times 10^4$	$0.817 \times 10^4$	$0.829 \times 10^4$
		0.5	$0.869 \times 10^4$	$0.886 \times 10^4$	$0.829 \times 10^4$
		0.7	$0.904 \times 10^4$	$0.934 \times 10^4$	$0.829 \times 10^4$
		0.9	$0.929 \times 10^4$	$0.989 \times 10^4$	$0.829 \times 10^4$
		1.1	$0.950 \times 10^4$	$0.106 \times 10^5$	$0.829 \times 10^4$
		1.3	$0.966 \times 10^4$	$0.110 \times 10^5$	$0.829 \times 10^4$
0.3	$q\bar{q}$	0.1	$0.411 \times 10^3$	$0.411 \times 10^3$	$0.555 \times 10^3$
		0.3	$0.505 \times 10^3$	$0.501 \times 10^3$	$0.555 \times 10^3$
		0.5	$0.549 \times 10^3$	$0.551 \times 10^3$	$0.555 \times 10^3$
		0.7	$0.578 \times 10^3$	$0.590 \times 10^3$	$0.555 \times 10^3$
		0.9	$0.600 \times 10^3$	$0.630 \times 10^3$	$0.555 \times 10^3$
		1.1	$0.617 \times 10^3$	$0.650 \times 10^3$	$0.555 \times 10^3$
		1.3	$0.630 \times 10^3$	$0.656 \times 10^3$	$0.555 \times 10^3$

factorization scheme, which absorbs the kinematical terms becoming large near the boundary of the phase space into the finite next-to-leading corrections to the structure functions. In Ref. 1 this corresponds to the choice  $CQ=1$ . The second set corresponds to the Martin, Roberts, and Stirling<sup>9</sup> parametrization, denoted by MRS2, characterized by a hard glue with  $\Lambda=250$  MeV and the modified minimal-subtraction factorization scheme. Both sets of structure functions include next-to-leading terms in the evolution programs and reflect two extreme descriptions of the glue distribution. As discussed in Ref. 1 the scale dependence induces an uncertainty which ranges between  $\approx 10\%$  at  $x_i \approx 0.1$  and  $\approx 30\%$  at  $x_i \approx 0.6$ . Needless to recall that the Born terms only, with  $\alpha_s(\mu^2)$  to one loop, lead to an ambiguity of order of 2.

Our results are shown in Fig. 1, for  $\mu^2 = M^2 = p_t^2$  and compared with the UA2 data.<sup>3</sup> The parametrization ACGG1 clearly describes the data better. However, the systematic uncertainty cannot rule out the MRS2 set.

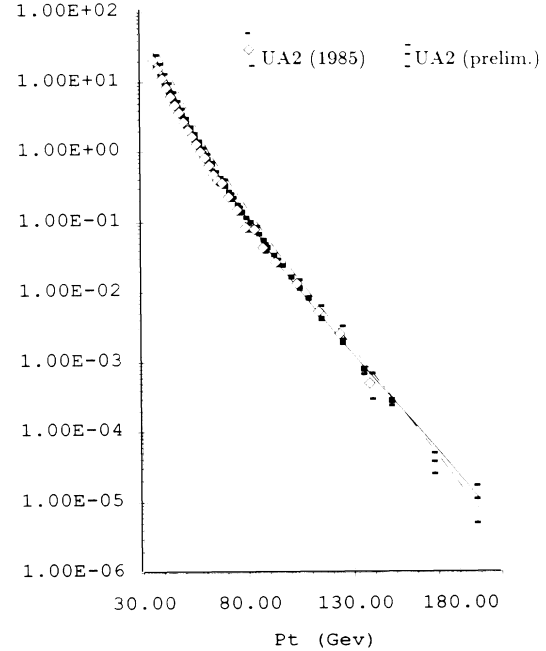


FIG. 1. Inclusive jet cross section  $(d^2\sigma/dp_t d\eta)_{\eta=0}$  (nb  $\text{GeV}^{-1}$ ) vs  $p_t$ , for  $\sqrt{s} = 630$  GeV,  $\mu = p_t$ , using the parametrizations of the structure functions ACGG1 (solid line) and MRS2 (dash-dotted line). The data are from the UA2 Collaboration (Ref. 3). Statistical errors only are shown.

The mass-scale uncertainty, reported above, is not shown in the figure. The agreement is excellent.

We now discuss the comparison with CDF data.<sup>4</sup> The corresponding jet algorithm leads us to define a jet as a deposition of transverse energy  $E_t$  inside a circle  $C_R$  of radius  $R \equiv (\Delta\eta^2 + \Delta\Phi^2)^{1/2} = 0.6$  in the rapidity-azimuthal-angle plane, irrespective of the jet direction inside  $C_R$ . Furthermore, the jet direction is varied in the range  $0.1 < |\eta| < 0.7$ . To approximate this experimental configuration we have considered within our analytical jet algorithm a cone of average value  $\delta \approx 0.55$ , and let its direction vary within the appropriate  $\eta$  range. This procedure is expected to be a rather good approximation of the CDF measurement, because of the weak (logarithmic) dependence on  $\delta$ , up to  $\delta \approx 0.8$ , as well as the reliability of the analytical results, as discussed above. It is clear that a more precise—but much longer—computation might be in principle achieved combining the analytical and numerical methods, appropriately modified to exactly match the CDF jet algorithm. We believe, however, that the improvement in the precision possibly so gained would be smaller than the theoretical ambiguity coming from the scale dependence and the choice of the structure functions.

Our results are shown in Fig. 2, using the ACGG1 structure functions and with  $\frac{1}{4} p_t \leq \mu = M \leq 2 p_t$ . The agreement is clearly quite satisfactory. Notice the strong reduction of theoretical sensitivity to the mass scales from  $O(\alpha_s^2)$  to  $O(\alpha_s^3)$ . As discussed previously, in

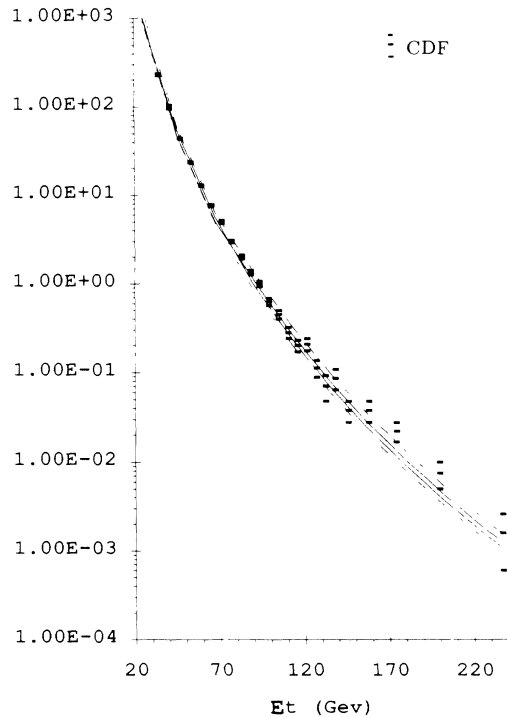


FIG. 2. Inclusive jet cross section  $\langle d^2\sigma/dE_t d\eta \rangle$  (nb GeV<sup>-1</sup>) vs  $E_t$  at  $\sqrt{s}=1.8$  TeV, for  $R=0.6$  averaged over  $0.1 < |\eta| < 0.7$ , for  $\mu = \frac{1}{2} p_t$  (upper curve) and  $\mu = 2 p_t$  (lower curve). The dotted band refers to the Born (one-loop) prediction. The data are from the CDF Collaboration (Ref. 4). Statistical errors only are shown.

connection to the UA2 data, the choice of the structure functions induces a further ambiguity, not explicitly reported in Fig. 2.

During the completion of this work we became aware of a paper of Ellis, Kunszt, and Soper,<sup>10</sup> where a similar conclusion is reached on the basis of a jet algorithm much closer to the CDF definition. On the other hand, our approach is more directly applicable to the UA2 configuration which cannot be analyzed in their case because of the limitation  $R = (\Delta\eta^2 + \Delta\Phi^2)^{1/2} < \pi/3$ . The

two approaches have been found in excellent agreement to  $\sim$  few % in some particular configurations (gluon-gluon scattering,  $\delta \ll 1$ ) where we could directly compare our results.

To conclude, we have shown that  $O(\alpha_s^3)$  corrections improve considerably the QCD predicting power, introducing a dependence on the jet-cone size, which is missing at the Born level, in agreement with data, and reduce the theoretical uncertainty to a level of  $\sim 30\%$ , which is comparable with the actual experimental accuracy.

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