Polarization Reversal of Alfvén Waves in a Nonaxisymmetric Region of a Quadrupole-Anchored Tandem Mirror

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Mode-selective excitation of Alfvén waves is made by use of azimuthally rotating, radio-frequency antennas in the central cell of the GAMMA 10 tandem mirror. It is found that the wave polarization reverses from right handed in the central cell to left handed in the anchor cell and vice versa. Ions in the anchor cell are significantly heated by a fast wave excited in the central cell and not by a slow wave. The polarization reversal and the wave-mode-dependent ion heating are interpreted in terms of a linear mode conversion which is caused by a mode coupling between fast and slow Alfvén waves via a spatial modulation of the magnetic field in the quadrupole-field region.

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It is of crucial importance to clarify propagation characteristics of Alfvén waves in an axially nonuniform and nonaxisymmetric magnetic field for understanding mechanisms of radio-frequency (RF) wave heating, RF stabilization of plasmas,[1,2] RF-induced radial transport,[3] and Alfvén wave current drive[4] in magnetic confinement devices. Fast (compressional) and slow (torsional) Alfvén wave propagation in a single, axisymmetric mirror field has been investigated for various azimuthal mode numbers, i.e., $m=0$ mode,[5,6] $m=+1$ mode,[6] and both $m=+1$ and -1 modes.[7] Little has been done with regard to the wave-propagation characteristics in a multiple mirror, especially with noncircular flux-tube cross section. In the TMX tandem mirror device which has elliptical mirror throats, it has been observed that a slow wave, generated spontaneously by the Alfvén ion-cyclotron instability in the end cell, excites a fast wave in the central cell.[8]

This Letter presents the first controlled experiments on a polarization reversal of the Alfvén waves propagating through a multiple mirror with an elliptical flux tube and a proposal of a mode conversion model which is based on mode coupling between fast and slow waves via a spatial modulation of the magnetic field. The model can interpret consistently the experimental results in GAMMA 10 and possibly those in TMX.

GAMMA 10 is a minimum-B anchored tandem mirror with outboard plug and thermal barrier in axisymmetric end mirrors.[9] In Fig. 1(a) the magnetic-field configuration of GAMMA 10 is shown with the flux density of 0.405 T at the central-cell midplane. The mirror ratios at the central, anchor, and plug/barrier cells are 4.9, 3.3, and 6.1, respectively. There are nonaxisymmetric transition regions at both sides of the quadrupole anchor cell to recircularize the flux-tube cross section. An axisymmetric choke coil is provided at each end of the central cell in order to reduce passing particles to the nonaxisymmetric region. The lengths of the central cell, the transition region, and the anchor cell are 5.6, 1.6, and 1.6 m, respectively. The diameters of the central-cell vacuum vessel and the plasma limiter are 100 and 36 cm, respectively, at the midplane. The ellipticity of the cross section in the transition region varies smoothly from unity to 50 at the maximum and again to unity at the anchor midplane.

So-called Nagoya type-III (Refs. 10 and 11) and type-II (double-half-turn) antennas are provided near both ends of the central solenoid. By controlling the relative phase of RF current flowing through four elements of the type-III antennas, an azimuthal mode number m of the excited wave magnetic field $b \sim \exp[i(m\theta + k_z z - \omega t)]$ can be selected among $m=+1$ (right handed, electron diamagnetic direction), $m=-1$ (left handed), and $m=\pm 1$ (nonrotating). The type II cannot drive the rotating field at present, so that the type III is used in most of the present experiments. Wave frequency relative to the ion-cyclotron frequency ω/ω_{ci} is changed by varying the magnetic-field intensity with the oscillator frequency of 9.6 MHz fixed. The radiation power of the type-III antenna is typically 200 kW in total with the duration of 50 ms.

For plasma production, dominant use is made of the central-cell RF power for ionizing puffed hydrogen gas and heating the plasma in the central cell and trapping into the anchor cells.[12] This RF startup mode has a capability of building up quickly a central-cell plasma with a relatively high density of $(0.3-1.0)\times 10^{13}$ cm^{-3} (on axis) and a high averaged ion temperature of 0.1-3.2 keV, depending on experimental conditions such as the excited wave mode and the location of the cyclotron resonance layer. In the current experiments no use is made of electron-cyclotron heating (ECH) and neutral-beam (NB) sources, which are conventionally used to produce plug/barrier potentials in the end cells.[13]

Magnetic probes are inserted radially to measure radial profiles of three components b_r, b_θ, b_z of the wave field. To determine axial wave number k_z and azimuthal mode number m, a pair of the probes aligned along the field

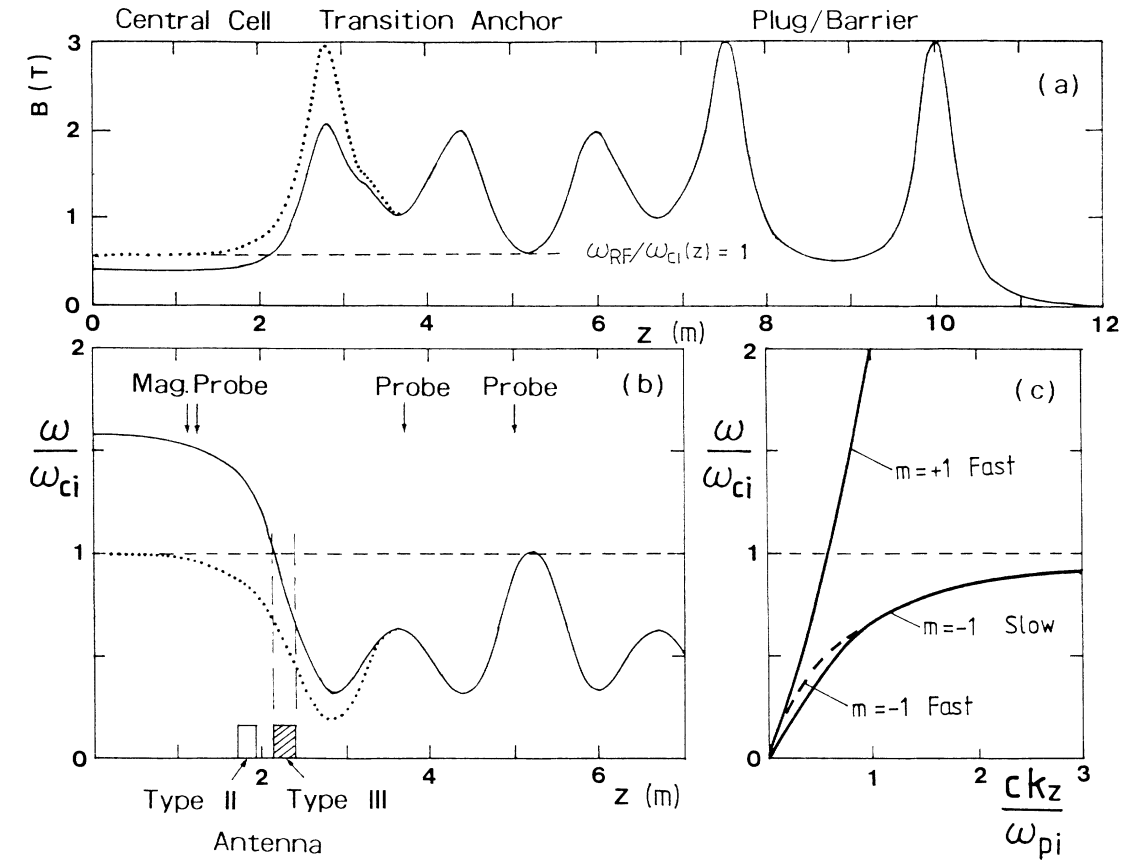

FIG. 1. (a) Axial profile of magnetic-field intensity of GAMMA 10. (b) Axial profile of normalized frequency ω/ω_{ci}. Location of RF antennas and magnetic probes are indicated by arrows. (c) Schematic of Alfvén wave-dispersion curves for delineating the propagation region of each mode in combination with (b).

lines and an array of the probes in the azimuthal direction are used in the central cell, respectively. Wave number and wave polarization are determined from fast-Fourier-transform (FFT) cross-power spectral analyses. In Fig. 1(b) the wave frequency normalized by the ion-cyclotron frequency $\omega/\omega_{ci}(z)$ is plotted along the axial position z. Locations of the antennas and magnetic probes are indicated in the same figure. In Fig. 1(c) a schematic of the dispersion relations for the fast and slow Alfvén waves is shown in order to have a qualitative idea of the wave-propagating region.

Dispersion characteristics are plotted in Fig. 2(a) for $m=+1$ and -1 waves excited in the central cell by use of the rotating antennas. The axial wave number k_z is normalized by the local ion dispersion length $c/\omega_{pi}(r)$, where c and ω_{pi} are the speed of light and ion plasma frequency, respectively. The central-cell density is (1×10^{12})–(1×10^{13}) cm^{-3} on the axis and the radial profile is a Gaussian with a half-maximum diameter of 15–20 cm. The solid lines in Fig. 2(a) are theoretical dispersion curves calculated under the assumption of a cold, homogeneous cylindrical plasma with the parameters listed in Fig. 2(a). Experimental results agree well with the cold plasma dispersion.

Radial profiles of wave field components b_r and b_θ and phase difference $\Delta\Theta$ between b_r and b_θ are shown in Figs. 2(b) and 2(c) for $m=+1$ excitation. From the phase difference, it is confirmed that the excited fast wave is right-hand circularly polarized (RHP) in the core region, left-hand circularly polarized (LHP) in the edge region, and linearly polarized with radius of 13 cm. This profile is consistent with the theoretical prediction for a cold, homogeneous cylindrical plasma. For $m=-1$ excitation at $\omega/\omega_{ci}<1$ with the magnetic field shown by the dotted line in Fig. 1(a) the excited slow wave is LHP in the core and RHP in the edge, as theoretically predicted. It is also confirmed that the probe located beyond the ion-cyclotron resonance layer cannot detect the slow wave, since the wave damps away through the resonance layer.

In the transition-anchor region with a nonaxisymmetric and nonuniform magnetic field, radial profiles of the wave components b_r and b_θ and the phase difference are measured. Azimuthal and axial wave numbers cannot be measured due to the spatial difficulty of installing an array of magnetic probes. In both regions the wave

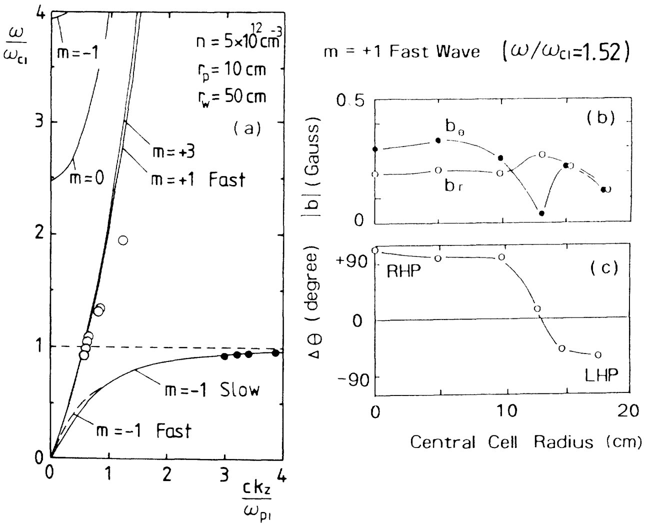

FIG. 2. (a) Theoretical dispersion curves of Alfvén waves in a cold, uniform cylindrical plasma with vacuum boundary layer (solid and dashed lines) and experimental results (open and solid circles). (b) Radial profiles of wave components b_r and b_θ and phase difference $\Delta\Theta$ for an $m=+1$ fast wave excited in the central cell. $\Delta\Theta=+90°$ and $-90°$ correspond to right-hand circularly polarized and left-hand circularly polarized, respectively.

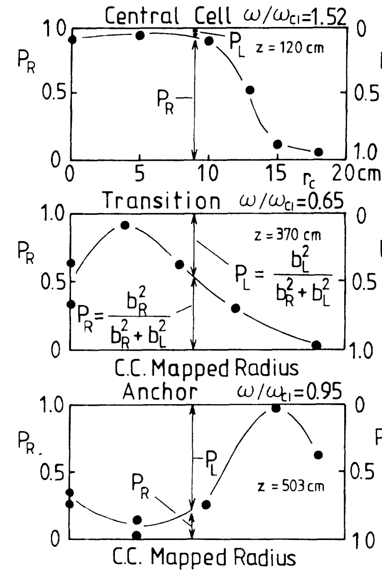

FIG. 3. Axial variation of wave polarization for $m=+1$ fast-wave excitation in the central cell. Polarization reverses from right handed in the central-cell core to left handed in the anchor-cell core. Here, $b_R=(b_r-ib_\theta)/2$ and $b_L=(b_r+ib_\theta)/2$. Polarization fraction is defined by $P_R=b_R^2/(b_L^2+b_R^2)$ and $P_L=1-P_R=b_L^2/(b_L^2+b_R^2)$.

amplitudes are maximum not in the edge region but in the core region. This indicates that not a surface wave but a body wave is dominantly excited and propagates in the transition-anchor region as does in the central cell. In order to see clearly the wave polarization, wave fields b_r and b_θ measured in cylindrical coordinates $(\hat{\mathbf{e}}_r,\hat{\mathbf{e}}_\theta)$ are transformed to b_L,b_R in rotating coordinates $(\hat{\mathbf{l}},\hat{\mathbf{r}})$:

$$\begin{aligned}\mathbf{b}&=b_r\hat{\mathbf{e}}_r+b_\theta\hat{\mathbf{e}}_\theta\\&=\tfrac{1}{2}(b_r+ib_\theta)(\hat{\mathbf{e}}_r-i\hat{\mathbf{e}}_\theta)+\tfrac{1}{2}(b_r-ib_\theta)(\hat{\mathbf{e}}_r+i\hat{\mathbf{e}}_\theta)\\&=b_L\hat{\mathbf{l}}+b_R\hat{\mathbf{r}}\,.\end{aligned}$$

Here $\hat{\mathbf{l}}$ and $\hat{\mathbf{r}}$ are right-hand $(\hat{\mathbf{e}}_r-i\hat{\mathbf{e}}_\theta)$ and left-hand $(\hat{\mathbf{e}}_r+i\hat{\mathbf{e}}_\theta)$ rotation vectors, respectively. Then, LHP component b_L and RHP component b_R are represented as $\frac{1}{2}(b_r+ib_\theta)$ and $\frac{1}{2}(b_r-ib_\theta)$, respectively, both of which are calculated from the measured b_r,b_θ and phase difference.

In Fig. 3 radial profiles of the wave polarization fraction defined by $P_R=b_R^2/(b_L^2+b_R^2)$ and $P_L=1-P_R=b_L^2/(b_L^2+b_R^2)$ are shown in the central cell, in the transition, and in the anchor region. $P_R>0.5$ means that the RHP component dominates over the LHP. $P_R=P_L=0.5$ means that the wave is linearly polarized. In case of the $m=+1$ wave excitation, the central-cell wave is dominantly RHP in the core and LHP in the outer region. In the transition region adjacent to the axisymmetric choke coil, the wave field is still RHP, not circularly but elliptically polarized in the core. On the contrary, in the anchor, the wave is dominantly LHP in the core; that is, the polarization almost reverses. In case of the $m=-1$ wave excitation, the polarization also reverses from LHP in the central-cell core to RHP in the anchor-cell core and from RHP to LHP in the outer region. In both cases frequency spectra of the propagating waves have a single peak at the excitation frequency without any sideband.

The anchor ions are found to be heated up to a temperature (600 eV, for example) a few times higher than the central-cell temperature (150 eV) in case of the $m=+1$ fast wave excitation at $\omega/\omega_{ci}=1.52$. No threshold power for the anchor ion heating has been found in the range of the RF oscillator output power of 30 to 300 kW. The $m=-1$ slow wave excitation at $\omega/\omega_{ci}<1$ shows no anchor ion heating within experimental errors. As shown in Fig. 1(c), the $m=+1$ fast wave is an allowed eigenmode for $\omega/\omega_{ci}<m_i/m_e$ (ion-to-electron mass ratio) in a cylindrical plasma with a vacuum boundary[14] as well as the $m=-1$ slow wave for $\omega/\omega_{ci}<1$. Then both the $m=+1$ fast and the $m=-1$ slow waves should be able to propagate through the nonuniform transition region and the slow wave should be able to heat the anchor ions, though part of these waves may be reflected at the steep-gradient region of the magnetic field.[5]

These experimental findings suggest that the $m=+1$ fast Alfvén wave is mode converted to an $m=-1$ slow wave and vice versa with very high conversion efficiency in a short distance comparable to one wavelength. A possible mechanism of the mode conversion is proposed as follows. The flux tube in the transition and the anchor regions has an elliptical cross section due to a quadrupole-field component with an azimuthal-angle dependence of nearly $\cos 2\theta$. The quadrupole-field modulation of the magnetic field can be supposed as a virtual mode with dominant Fourier components of $m_0 = \pm 2$ at the frequency $\omega_0 = 0$. Then the fast or slow wave with mode m_1 propagating through these regions will be spatially modulated to excite waves with $m_2 = m_1 \pm m_0$. Furthermore, an axial wave number k_1 is also spatially modulated by $\pm k_0$ to produce $k_2 = k_1 \pm k_0$, where $\pm k_0$ is associated with the axially nonuniform magnetic field. Subject to these matching conditions of both azimuthal and axial wave numbers together with a frequency matching condition $\omega_2 = \omega_1 \pm \omega_0$, the $m_1 = +1$ fast [$m_1 = -1$ slow] wave could mode convert through the azimuthal modulation of $m_0 = \pm 2$ to $m_2 = -1$ and $+3$ slow [$m_2 = +1$ and -3 fast] waves, as shown schematically in Fig. 4(a) [4(b)]. Here, the $m_2 = +3$ slow wave can be hardly excited, since slow waves with any positive m have never been observed. The $m_2 = -3$ fast wave is predicted to be cut off at $\omega/\omega_{ci} < 4$, as shown schematically in Fig. 4(b). Then, the $m_1 = +1$ fast ($m_1 = -1$ slow) wave will resonantly mode convert to the $m_2 = -1$ slow ($m_2 = +1$ fast) wave, respectively. The $m_2 = -1$ slow ($m_2 = +1$ fast) wave is a body wave with a peaked radial profile of the amplitude with LHP (RHP) in the core. The $m_2 = -1$ slow wave with LHP in the core of the anchor can heat ions on the cyclotron resonance layer. This proposed model of the mode conversion can explain consistently the experimental results of the polarization reversal and the anchor ion heating by fast waves excited in the central cell.

Quantitatively, the length of the transition region of about 3 m corresponds to the normalized modulation wave number $ck_0/\omega_{pi} \sim 0.35$ for a plasma density of 5×10^{12} cm^{-3}. The normalized wave numbers for fast and slow waves are predicted in Fig. 2(a) to be 0.35 and 0.7 for an averaged wave frequency $\omega/\omega_{ci} \sim 0.5$ in the transition region. Then the matching conditions of wave number and frequency in Figs. 4(a) and 4(b) are satisfied. Actually, both ω/ω_{ci} and ck_0/ω_{pi} in the transition region have broad spectra due to the strongly nonuniform magnetic field and the slow wave has a variety of eigenmodes in a radially nonuniform plasma.[15] This causes a large redundancy for satisfying the matching conditions for a wide range of the plasma density, as observed in GAMMA 10.

The present model on mode conversion is based on a local theory, which could not be generally applied for cases where the scale length of the field gradient is comparable to the wavelength, as is in the present experiments. Nonlocal formulation on coupled Alfvén wave propagation in a quadrupole magnetic field has been done to validate the proposed idea of the mode conversion mechanism.[16] Approximate coupled solutions for a weak periodic quadrupole-field modulation shows that a linear mode conversion between fast and slow waves occurs in a short conversion distance with the converted wave amplitude proportional to the quadrupole perturbation and with a high conversion efficiency, as observed in the experiments. Numerical calculations of the coupled differential equations are in progress to clarify the steep field-gradient effect and the strong quadrupole-field effect on the mode conversion, although the nonaxisymmetric, quadrupole perturbation is predicted to be essential to the mode coupling between fast and slow waves.

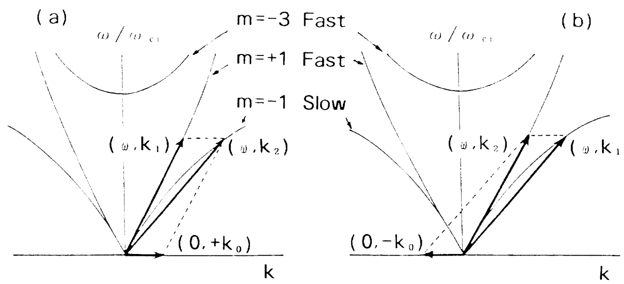

FIG. 4. Schematic of mode conversion between fast and slow waves via spatial modulation ($\pm m_0$ and $\pm k_0$). (a) Conversion from $m_1 = +1$ fast to $m_2 = -1$ slow Alfvén wave. (b) Conversion from $m_1 = -1$ slow to $m_2 = +1$ fast wave.

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[1]J. R. Ferron *et al.*, Phys. Rev. Lett. **51**, 1955 (1983).
[2]Y. Yasaka and R. Itatani, Phys. Rev. Lett. **56**, 2811 (1986).
[3]S. Riyopoulos, T. Tajima, T. Hatori, and D. Pfirsch, Nucl. Fusion **26**, 627 (1986).
[4]T. Ohkawa *et al.*, Nucl. Fusion Suppl. **1**, 681 (1989).
[5]N. F. Cramer, Nucl. Fusion **11**, 285 (1971).
[6]Y. Amagishi, A. Tsushima, and M. Inutake, Phys. Rev. Lett. **48**, 1183 (1982).
[7]T. Intrator *et al.*, Nucl. Fusion **29**, 377 (1989).
[8]G. R. Smith, T. A. Casper, and M. J. Garber, Nucl. Fusion **23**, 1381 (1983).
[9]M. Inutake *et al.*, Phys. Rev. Lett. **55**, 939 (1985).
[10]T. Watari *et al.*, Phys. Fluids **21**, 2076 (1978).
[11]Y. Ohsawa *et al.*, Phys. Rev. Lett. **40**, 1246 (1979).
[12]M. Ichimura *et al.*, Nucl. Fusion **28**, 799 (1988).
[13]T. Cho *et al.*, Phys. Rev. Lett. **64**, 1373 (1990).
[14]F. J. Paoloni, Phys. Fluids **18**, 640 (1975).
[15]K. Appert, J. Vaclavik, and L. Villard, Phys. Fluids **27**, 432 (1984).
[16]H. Hojo *et al.*, in Proceedings of the Japan Physical Society Meeting, Osaka, 1990 (to be published).