Self-Induced Phase Turbulence and Chaotic Itinerancy in Coupled Laser Systems

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Spatiotemporal dynamics of multiple coupled laser systems are predicted as an example of distinct phenomena in high-dimensional nonlinear systems far from thermal equilibrium. Generic dynamics, including a self-induced phase turbulence which results in *emitter partition noise*, chaotic itinerancy between ruins of local attractors (supermodes) resulting from *phase hopping*, as well as superslow relaxation into equilibria, are found. It is shown theoretically that such inherent dynamics result from the destruction of phase locking of multiple emitters due to population dynamics.

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Spatiotemporal dynamics of coupled nonlinear systems far from thermal equilibrium is an attractive current issue in physics, engineering, chemistry, biology, and other fields.[1,2] One of the simplest examples is the phase-locking problem of two coupled nonlinear oscillators. When locking is successful, the two oscillators are synchronized and act as one, with a unique frequency and with well-defined phase relationships between the oscillators. When synchronization fails, beat oscillations take place. If many nonlinear elements are coupled to each other, much more complex behavior is expected. In particular, in high-dimensional chaotic systems, many equilibria (spatial structures) coexist in the stationary state. The important general question then arises: *What kind of spatiotemporal dynamics take place when coexisting patterns become dynamically unstable?* In this Letter, we consider spatially coupled multiple laser systems as promising candidates for investigating this fundamental problem in high-dimensional chaotic systems. Ripper and Paoli observed phase coupling between twin-waveguide laser diodes (LD) due to an evanescent field.[3] Otsuka extended their idea to high-power *multiple* coupled-waveguide lasers (CWL),[4] and pointed out the interesting analogy between CWL and FM-mode-locked lasers in stationary states.[5] This CWL concept has been successfully demonstrated with laser diodes,[6] and most recently, Wang and Winful reported numerical simulations of the nonlinear dynamics of such LD arrays.[7,8] They showed that irregular and undamped spiking behavior results from the peculiar amplitude-phase coupling nonlinearity (so-called α parameter) in LD stemming from the carrier-dependent refractive index. However, the α nonlinearity is peculiar to LD, reflecting the band structure, and in most lasers, $\alpha=0$. Therefore, in this paper $\alpha=0$ CWL are studied to extract generic features of coupled laser systems.

In the conceptual model of CWL (Fig. 1), each emitter supports a single transverse and longitudinal mode with a frequency ω, when isolated from its neighbors. In the following analysis, we assume lasers for which the adiabatic elimination of polarization is valid, but for which the population dynamics should be included. In this case, the temporal evolution of the complex field amplitude $\tilde{E}_i$, assuming $\hat{E}_i(t)=\tilde{E}_i(t)\exp(j\omega t)$, and of the population inversion N_i in the ith emitter is described by the coupled equations

$$\dot{\tilde{E}}_i=\tfrac{1}{2}[G(N_i)-1/\tau_p]\tilde{E}_i+jq[\tilde{E}_{i+1}+\tilde{E}_{i-1}]\,, \qquad (1)$$

$$\dot{N}_i=P-N_i/\tau-G(N_i)\,|\tilde{E}_i|^2\,, \qquad (2)$$

where $G(N_i)=G(N_{\rm th})+(\partial G/\partial N)(N_i-N_{\rm th})$ is the gain ($N_{\rm th}$ is the threshold population inversion), τ_p is the photon lifetime, τ is the population lifetime, $q=\kappa c/n$ (κ is the coupling coefficient), and P is the pump power. Equation (1) is equivalent to the FM laser and active FM-mode-locking equation[5] with zero detuning, if $\tilde{E}_i$ is regarded as the longitudinal-mode field, and has been extensively studied for gas lasers in which population dynamics can be neglected.[9] If population dynamics [Eq. (2)] is introduced, however, drastic changes occur in the

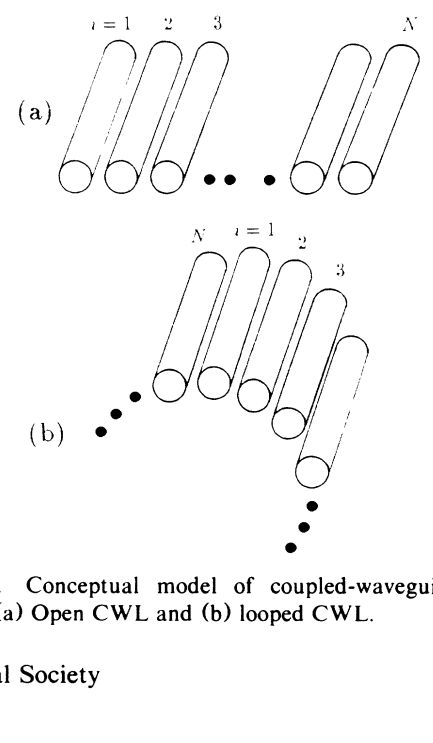

FIG. 1. Conceptual model of coupled-waveguide lasers (CWL). (a) Open CWL and (b) looped CWL.

dynamics.

First, let us look for equilibria of Eqs. (1) and (2) with $E_{i+1}=E_i$ and $\phi_{i+1}-\phi_i\equiv\Delta\phi_i=$const independent of i, where $\tilde{E}_i=E_i\exp(j\phi_i)$. The steady-state solutions are different for open and looped CWL, although they coincide in the limit of $N\to\infty$. (In this limit, in-phase as well as π-out-of-phase modes with uniform field amplitudes whose far-field patterns correspond to positive and negative mode pulses in laser FM-mode locking[10] are obtained for both cases.)

In the open-CWL case [Fig. 1(a)], the population-inversion distribution becomes nonuniform across the emitters, i.e., $G(N_i)\neq 1/\tau_p$, and the system is found to have solutions which satisfy $\dot{E}_i=0$, $\dot{N}_i=0$ with $\bar{\phi}_{i+1}-\bar{\phi}_i\equiv\Delta\bar{\phi}_i=\pm\pi/2$. In short, steady-state values of $\bar{E}_i$ and $\bar{N}_i$ are obtained from the numerical analysis for arbitrary combinations of relative phase $\Delta\bar{\phi}_i$ between adjacent emitters. They are degenerate in lasing frequencies, since $\dot{\phi}_i=0$ for all the solutions.[11]

In the case of looped CWL [periodic boundary condition, Fig. 1(b)], the population-inversion distribution across all the emitters becomes uniform, i.e., $G(N_i)=1/\tau_p$, because of the coupling between the outermost emitters, and the supermodes given by $\bar{E}_{i+1}=\bar{E}_i$, $\Delta\bar{\phi}_i=2\pi l/N$, and $\Omega^l=\omega+2q\cos(2\pi l/N)$ ($l=0,1,2,\ldots,N-1$) are deduced as equilibria with different lasing frequencies Ω^l.

Next, let us consider the dynamical properties based on numerical simulations of Eqs. (1) and (2). Linear stability analysis shows that the equilibria for $N\gg 1$ are not always dynamically stable even if the α nonlinearity[8] is absent, while nonlinearity is required to induce instabilities in two-emitter ($N=2$) systems.[8,12] Indeed, all solutions for the open CWL are easily destroyed in weakly coupled regimes by an extremely small perturbation to the system. The temporal evolution of relative phases $\Delta\phi_{1,2}$ is shown in Fig. 2(a) for the open CWL ($N=5$). The chaotic evolution of relative phases persists, where the pulsation frequency is approximately given by the energy-transfer rate q.[7,8] Although the global instability scenario has not yet been clarified, various coexisting periodic solutions embedded in a chaotic sea are obtained separately by changing the initial relative phase condition. At the same time, negligibly small field-amplitude fluctuations approximately given by $(1/N)\sum_{i=1}^{N}\frac{1}{2}[G(N_i)-1/\tau_p]/q$ ($<10^{-3}\bar{E}_{\rm av}$) take place in every emitter, although total intensity $I=\sum_{i=1}^{N}|\tilde{E}_i|^2$ remains constant. In this sense, the present instabilities can be regarded as self-induced phase turbulence because the intensity fluctuation of individual emitters is negligibly small. Such turbulence results in *emitter partition phase noise* whose time scale is characterized by the energy-transfer rate q. This phase turbulence has been found to be suppressed when population dynamics are omitted, i.e., $\dot{N}_i=0$. This strongly suggests that the population dynamics play an essential role in this phase turbulence. This point will be discussed again later.

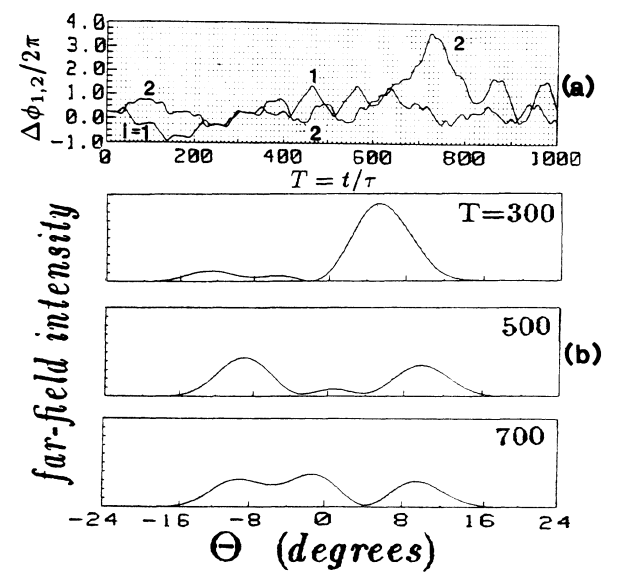

FIG. 2. (a) Temporal evolution of relative phases $\Delta\phi_{1,2}$ for $N=5$ open CWL, assuming the pump power $p\equiv\frac{1}{2}(\partial G/\partial N)\times N_{\rm th}\tau_p(P/P_{\rm th}-1)=0.25$, $q'=q\tau_p=10^{-4}$, and $\tau/\tau_p=2\times10^3$. Other relative phases also show chaotic pulsations. (b) Time-dependent far-field patterns corresponding to (a), which are [Fourier transform of the near field]$\times$[far-field intensity distribution for individual emitter $|E(\Theta)|^2$], assuming half-width of $|E(\Theta)|^2=17°$, wavelength $=1$ μm, and emitter spacing $=5$ μm. Θ is the angle with respect to the normal to the CWL end face.

Let us discuss the connection between the present turbulence and the Wang-Winful instabilities in laser diode arrays.[7,8] If a typical value of nonlinearity, e.g., the α parameter in LD, is introduced, the out-of-phase mode (i.e., $\Delta\phi_i\to\pi$), which has been experimentally observed in most LD arrays to date, is found to be realized stably. This implies that the adequate amplitude-phase coupling stemming from the α nonlinearity suppresses the intrinsic phase turbulence shown in Fig. 2(a). The favorable relative phase depends on the sign of the α parameter, and the stable in-phase mode is realized when $\alpha<0$, while $\alpha>0$ in real LD. When the coupling coefficient increases, intrinsic phase turbulence tends to be suppressed. However, in return, the Wang-Winful instability featuring sustained relaxation oscillations resulting from the α nonlinearity becomes dominant and large-amplitude fluctuations appear in addition to phase turbulence. This nonlinearity introduces a frequency detuning into active FM-mode locking.

If such phase turbulence takes place, the far-field pattern of CWL, which is a Fourier transform of the near-field pattern, is easily expected to exhibit unstable temporal evolutions. In short, the far-field pattern moves around chaotically in time and deterministic spot dancing develops, even though the total near-field intensity is constant. Snapshots of the far-field patterns, corresponding to Fig. 2(a) at different times, are shown in

Fig. 2(b).

In the case of open CWL, eigenmodes are degenerate in lasing energy and persistent phase turbulence occurs only around dynamically unstable equilibria with $\Delta\phi_i = \pm\pi/2$. As for looped CWL, on the contrary, the equilibrium bifurcates into S supermodes with different lasing energies, where $S=(N+1)/2$ for N odd and $S=(N+2)/2$ for N even. Consequently, much more complicated behavior is expected to occur resulting from the interplay between nondegenerate supermode solutions. As expected, when the initial conditions are distributed, relative phases wander chaotically between unstable orbits localized around supermodes ("attractor ruins") as is shown in Fig. 3(a). After some transient spatiotemporal chaos (STC), the system is captured by the $l=2$ attractor ruin connected with STC and fluctuates chaotically with a frequency of $\approx q$ in region a. Such localized phase turbulence survives for a long period of time, and then the system jumps out from this local attractor ruin, without being frozen to the $l=2$ supermode solution, and begins to search for other attractors chaotically while changing relative phases abruptly by an amount $\approx \pm 2\pi m + 2\pi n/N$ $(m,n=0,1,2,\ldots)$. This suggests that the basin of attraction of $l=2$ and its equivalent $l=3$ is quite narrow. In fact, these supermode solutions are confirmed to be realized only when initial conditions are set sufficiently close to steady-state solutions. The $l=0$ supermode is also found to have an extremely narrow basin of attraction. Such an abrupt jump is referred to as *phase hopping* hereafter. A similar phenomenon called "mode-hopping" intensity noise has been observed in multimode laser diodes, where a Langevin force has been introduced into multimode rate equations for population-photon-number dynamics to explain this phenomenon.[13] In our case, a self-induced internal chaotic force results in phase hopping. In this regime (region b), several supermodes are excited and seem to compete with each other, resulting in STC. In region c, the system goes by way of the $l=1$ attractor ruin and is finally frozen to the $l=1$ supermode with an extremely long relaxation time ($\approx 10^5\tau$), although this is not depicted in the figure since its damping rate is extremely small. This implies that the basin of attraction of the $l=1$ and its equivalent $l=4$ supermodes is wide enough for the system to relax to these equilibria in this case.

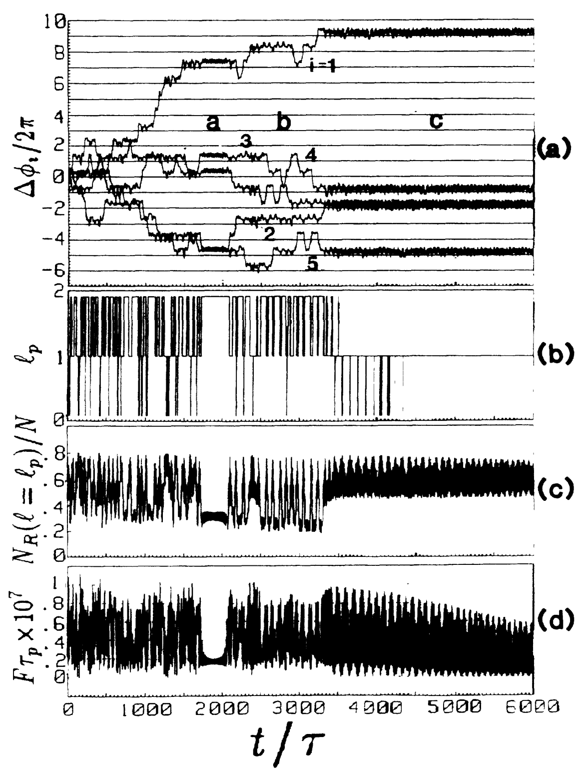

FIG. 3. Chaotic itinerancy between attractor ruins for $N=5$ looped CWL. Adopted parameter values are the same as for Fig. 2. (a) Relative phase, (b) nearest supermode number, (c) norm from nearest supermode, and (d) destructive force for phase locking.

To characterize the dynamics in the STC region, we introduce the "norm" from supermode solutions defined by

$$N_R = \sum_{i=1}^{N} |\cos(\Delta\phi_i) - \cos(2\pi l/N)|$$

and plot supermode number l_p, whose norm indicates a minimum value, and $N_R(l=l_p)/N$ in Figs. 3(b) and 3(c), respectively. These figures indicate the connectivity of ruins (i.e., $[l=0]\rightleftharpoons[l=1]\rightleftharpoons[l=2]$) and which supermode the system moves around most closely and the distance from it. There is a clear correspondence between l_p and N_R/N. In short, the $l_p=2$ supermode appears at the bottoms of norms, while $l_p=1$ appears at the tops of norms in the STC region. In particular, the system is captured to the $l=2$ ruin only when the norm is adequately small (region a), while it is captured to the $l=1$ ruin even when the norm is relatively large (region c), reflecting the difference in the basin of attraction mentioned above. In region c, the norm approaches zero very slowly, and finally the $l=1$ supermode is established. Such superslow damping to equilibria, whose time scale is much longer than other lasing time scales, occurs generally whenever the system can find the basin of attraction and could be an example of *stagnation phenomena* often observed in high-dimensional chaotic systems.[14] The relaxation time increases when pumping is increased. This result is in sharp contrast to the fact that the relaxation time to the steady state decreases with an increase in pumping in usual population-photon-number dynamics (i.e., relaxation oscillations) in which "phase" dynamics are omitted. The physical interpretation is now being investigated.

What relationships are there between population dynamics and phase turbulence, especially phase hopping?

To investigate this problem, we introduce here the quantity of "destruction force" for phase locking F. F is defined as $(1/N)\sum_{i=1}^{N}\frac{1}{2}[G(N_i)-1/\tau_p]$, which corresponds to the first term of the right-hand side of Eq. (1). This quantity expresses the contribution of population dynamics to field dynamics through Eq. (2) and equals zero when phase locking is established. From the temporal evolution of F shown in Fig. 3(d), the emitter partition noise increases when F increases. Moreover, phase hopping is found to occur at the time when F abruptly increases, although F peaks are modulated on the time scale of background emitter partition noise. This result strongly implies that population dynamics destroy phase locking between emitters.

From extensive numerical simulations, it found that the probability of the system finding the basin of attraction as in region c in Fig. 3(a) decreases as N increases, while the number of attractor ruins, among which chaotic itinerancy takes place, increases. This means that the basin of attraction becomes narrower as N increases. Needless to say, chaotic itinerancy can be eliminated and phase locking established immediately when the coupling coefficient is strong enough.[15]

In conclusion, self-induced phase turbulence, which is inherent in phase-coupled laser systems, is predicted. This phenomenon is shown to be caused by the destruction of phase locking with the introduction of population dynamics. It has been shown that there are two typical fluctuation phenomena with different time scales, i.e., the relatively fast emitter partition noise and the slow phase-hopping process leading to chaotic itinerancy. Additionally, superslow relaxation into equilibria via a chaotic sea has been predicted based on numerical simulations. It is worth noting that the predicted behaviors are produced by a self-induced internal chaotic force which is formed with a strong correlation with the past evolution of the system, yielding a more effective search for the paths through which the itinerancy is realized than by an externally applied random force.[16] The predicted chaotic itinerancy is expected to be generally observed in the transition process where local chaos joins together and develops into global chaos. Indeed, similar chaotic itinerancy has been predicted in multimode Maxwell-Bloch lasers,[16] coupled nonlinear optical element systems,[17] and coupled map lattices.[2]

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[1]G. Nicolis and I. Prigogine, *Self-Organization in Nonequilibrium Systems* (Wiley, New York, 1977); *Chemical Oscillations, Waves, and Turbulence,* edited by H. Haken (Springer-Verlag, New York, 1984).

[2]*Formation, Dynamics, and Statistics of Patterns,* edited by K. Kawasaki *et al.* (World Scientific, Singapore, 1989).

[3]J. E. Ripper and T. L. Paoli, Appl. Phys. Lett. **17**, 371 (1979).

[4]K. Otsuka, IEEE J. Quantum Electron. **13**, 895 (1977).

[5]K. Otsuka, Electron. Lett. **19**, 723 (1983).

[6]For a recent review, see D. Botez and D. E. Ackley, IEEE Circuits Devices Mag. **2**, 8 (1986).

[7]S. S. Wang and H. G. Winful, Appl. Phys. Lett. **52**, 1774 (1988).

[8]H. G. Winful and S. S. Wang, Appl. Phys. Lett. **53**, 1894 (1988).

[9]S. E. Harris and O. P. McDuff, IEEE J. Quantum Electron. **1**, 245 (1965).

[10]D. J. Kuizenga and A. E. Siegman, IEEE J. Quantum Electron. **6**, 694 (1979).

[11]Assume that $G(N_i)=G(N_{\rm th})=1/\tau_p$; i.e., the population inversion is uniform across all the emitters. The steady state of the system then bifurcates into N equilibria, which have different lasing frequencies. The amplitude and frequency of the lth equilibrium are $E_i^l=\sin[i\pi l/(N+1)]$ and $\Omega^l=\omega+2q\times\cos[\pi l/(N+1)]$, where $l=1,2,\ldots,N$. However, these array eigenmodes [that is, supermodes; see J. K. Butler, D. E. Ackley, and D. Botez, Appl. Phys. Lett. **44**, 293 (1984)] are not realized if population dynamics [Eq. (2)] are introduced, because the N_i distribution becomes nonuniform, reflecting the nonuniform field distribution.

[12]In some coupled two-laser systems (i.e., $N=2$), chaotic relaxation oscillations have been observed when the perturbation frequency is close to the natural frequency of relaxation oscillations. This phenomenon can generally be understood in terms of the response of coupled two-relaxation oscillators. (See, for example, G. L. Lippi *et al.*, Opt. Commun. **53**, 129 (1985); T. Mukai and K. Otsuka, Phys. Rev. Lett. **55**, 1171 (1985); K. Otsuka and T. Mukai, in *Optical Chaos,* edited by J. Chrostowski and N. B. Abraham, SPIE Proceedings No. 667 (Society of Photo-Optical Instrumentation Engineers, Bellingham, WA, 1986), p. 122; A. V. Bondarenko *et al.*, Zh. Eksp. Teor. Fiz. **95**, 807 (1989) [Sov. Phys. JETP **68**, 461 (1989)).

[13]M. Ohtsu, Y. Teramachi, Y. Otsuka, and A. Osaki, IEEE J. Quantum Electron. **22**, 535 (1986).

[14]Y. Aizawa, Prog. Theor. Phys. **81**, 249 (1989).

[15]This self-induced phase turbulence is also eliminated if *diffusive* coupling is introduced instead of *phase* coupling. In such a case, j in Eq. (1) can be omitted and the locking becomes AM locking, where the equilibria are defined as eigenvalues of the corresponding quantum-mechanical harmonic oscillator [H. Haken and M. Pauthier, IEEE J. Quantum Electron. **4**, 454 (1968)].

[16]K. Ikeda, K. Otsuka, and K. Matsumoto, Prog. Theor. Phys. Suppl. (to be published).

[17]K. Otsuka and K. Ikeda, Phys. Rev. A **39**, 5209 (1989); (unpublished).