Natural Inflation with Pseudo Nambu-Goldstone Bosons

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We show that a pseudo Nambu-Goldstone boson, with a potential of the form $V(\phi)=\Lambda^4[1 \pm \cos(\phi/f)]$, can naturally give rise to an epoch of inflation in the early Universe. Successful inflation can be achieved if $f \sim m_{\text{Pl}}$ and $\Lambda \sim m_{\text{GUT}}$. Such mass scales arise in particle-physics models with a gauge group that becomes strongly interacting at a scale $\sim\Lambda$, e.g., as can happen in superstring theories. The density fluctuation spectrum is non-scale-invariant, with extra power on large length scales.

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The inflationary-universe model was proposed[1] to solve several cosmological puzzles, notably the horizon, flatness, and monopole problems. During the inflationary epoch, the energy density of the Universe is dominated by vacuum energy, $\rho \simeq \rho_{\text{vac}}$, and the scale factor of the Universe expands exponentially: $R(t) \propto e^{Ht}$, where the Hubble parameter $H=\dot{R}/R \simeq (8\pi G\rho_{\text{vac}}/3)^{1/2}$ during inflation. If the interval of exponential expansion satisfies $\Delta t \gtrsim 60H^{-1}$, a small causally connected region of the Universe grows sufficiently to explain the observed homogeneity and isotropy of the Universe, to dilute any overdensity of magnetic monopoles, and to flatten the spatial hypersurfaces, $\Omega \equiv 8\pi G\rho/3H^2 \to 1$.

To satisfy a combination of constraints on inflationary models,[2] in particular, sufficient inflation and microwave-background anisotropy limits[3] on density fluctuations, the potential of the field responsible for inflation (the inflaton) must be very flat. For a general class of inflation models involving a single slowly rolling field (including new,[4] chaotic,[5] and double-field inflation[6]), the ratio of the height to the (width)^4 of the potential must satisfy[7]

$$\chi \equiv \Delta V/(\Delta\phi)^4 \lesssim 10^{-6}\text{-}10^{-8}\,, \tag{1}$$

where ΔV is the change in the potential $V(\phi)$ and $\Delta\phi$ is the change in the field ϕ during the slowly rolling portion of the inflationary epoch. [For extended inflation, $\chi \lesssim 10^{-15}$ (Ref. 8).] Thus, the inflaton must be extremely weakly self-coupled, with effective quartic self-coupling constant $\lambda_\phi < \mathcal{O}(\chi)$ (Ref. 7) (in realistic models, $\lambda_\phi < 10^{-12}$).

While a number of workable inflation models [satisfying Eq. (1)] have been proposed,[9] none of them is compelling from a particle-physics standpoint. In some cases, the tiny coupling λ_ϕ is simply postulated *ad hoc* at tree level, and then must be fine tuned to remain small in the presence of radiative corrections. But this merely replaces a cosmological naturalness problem with unnatural particle physics. The situation is improved in models where the smallness of λ_ϕ is protected by a symmetry, e.g., supersymmetry. In these cases,[10] λ_ϕ may arise from a small ratio of mass scales; however, the required mass hierarchy, while stable, is itself unexplained. It would be preferable if such a hierarchy, and thus inflation itself, arose dynamically in particle-physics models.

Nambu-Goldstone bosons are ubiquitous in particle-physics models: They arise whenever a global symmetry is spontaneously broken. If there is additional explicit symmetry breaking, these particles become pseudo Nambu-Goldstone bosons (PNGBs). In models with a large global-symmetry-breaking scale f, PNGBs are very weakly interacting, since their couplings are suppressed by inverse powers of f. For example, in "invisible" axion models[11,12] with Peccei-Quinn scale $f_{\text{PQ}} \sim 10^{15}$ GeV, the axion self-coupling is $\lambda_a \sim (\Lambda_{\text{QCD}}/f_{\text{PQ}})^4 \sim 10^{-64}$. [This simply reflects the hierarchy between the QCD and grand-unified-theory (GUT) scales, which arises from the slow logarithmic running of α_{QCD}.] Because of the nonlinearly realized global symmetry, the potential for PNGBs is exactly flat at tree level. The symmetry may be explicitly broken by loop corrections, as in schizon[13] and axion[11] models. In the case of axions, for example, the PNGB mass arises from nonperturbative gauge-field configurations (instantons) through the chiral anomaly. When the associated gauge group becomes strong at a mass scale Λ, instanton effects give rise to a periodic potential of height $\sim\Lambda^4$ for the PNGB field.[14] Since the nonlinearly realized symmetry is restored as $\Lambda \to 0$, the flatness of the PNGB potential is natural in the sense of 't Hooft.[15]

The resulting PNGB potential is generally of the form

$$V(\phi)=\Lambda^4[1 \pm \cos(N\phi/f)]\,. \tag{2}$$

We will take the positive sign in Eq. (2) (this choice has no effect on our results) and, unless otherwise noted, assume $N=1$, so the potential, of height $2\Lambda^4$, has a unique minimum at $\phi=\pi f$ (we assume the periodicity of ϕ is $2\pi f$). We show below that, for appropriately chosen values of the mass scales, namely, $f \sim m_{\text{Pl}}$ and $\Lambda \sim m_{\text{GUT}} \sim 10^{15}$ GeV, the PNGB field ϕ can drive inflation. [This is consistent with Eq. (1), since $\chi \sim (\Lambda/f)^4 \sim 10^{-13}$.] These mass scales can arise naturally in particle-physics models. For example, in the hidden sector of superstring theories, if a non-Abelian group remains unbroken, the running gauge coupling can become strong at the GUT scale;[16] then the role of the PNGB inflaton might be played, e.g., by the model-independent axion.[17]

For temperatures $T \lesssim f$, the global symmetry is spontaneously broken, and the field ϕ describes the phase degree of freedom around the bottom of a "Mexican hat." Since ϕ thermally decouples at a temperature $T \sim f^2/m_{\rm Pl} \sim f$, we assume it is initially laid down at random between 0 and $2\pi f$ in different causally connected regions. Within each Hubble volume, the evolution of the field is described by

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0\,, \tag{3}$$

where Γ is the decay width of the inflaton. In the temperature range $\Lambda \lesssim T \lesssim f$, the potential $V(\phi)$ is dynamically irrelevant, because the forcing term $V'(\phi)$ is negligible compared to the Hubble damping term. (In addition, for axion models, $\Lambda \to 0$ as $T/\Lambda \to \infty$ due to the high-temperature suppression of instantons.[14]) Thus, in this temperature range, aside from the smoothing of spatial gradients in ϕ, the field does not evolve. Finally, at $T \lesssim \Lambda$, in regions of the Universe with ϕ initially near the top of the potential, the field starts to roll slowly down the hill toward the minimum. In those regions, the energy density of the Universe is quickly dominated by the vacuum contribution $[V(\phi) \simeq 2\Lambda^4 \gtrsim \rho_{\rm rad} \sim T^4]$, and the Universe expands exponentially. Since the initial conditions for ϕ are random, our model is closest in spirit to chaotic inflation.[5] Note that PNGBs may also provide a flat potential for double-field inflation.[6]

To successfully solve the cosmological puzzles of the standard cosmology, an inflationary model must satisfy a variety of constraints.

(1) Slow-rolling regime.— The field is said to be slowly rolling (SR) when its motion is overdamped, i.e., $\ddot{\phi} \ll 3H\dot{\phi}$ (N.B., $\Gamma \ll H$), and two conditions are met:[2]

$$|V''(\phi)| \lesssim 9H^2, \text{ i.e., } \left(\frac{2|\cos(\phi/f)|}{1+\cos(\phi/f)}\right)^{1/2} \lesssim \frac{\sqrt{48\pi} f}{m_{\rm Pl}} \tag{4}$$

and

$$\left|\frac{V'(\phi) m_{\rm Pl}}{V(\phi)}\right| \lesssim \sqrt{48\pi}, \text{ i.e., } \frac{\sin(\phi/f)}{1+\cos(\phi/f)} \lesssim \frac{\sqrt{48\pi} f}{m_{\rm Pl}}\,. \tag{5}$$

From Eqs. (4) and (5) the existence of a broad SR regime requires $f \gtrsim m_{\rm Pl}/\sqrt{48\pi}$ (required below for other reasons). The SR regime ends when ϕ reaches a value ϕ_2, at which one of the inequalities (4) or (5) is violated. For example, for $f = m_{\rm Pl}$, $\phi_2/f = 2.98$ (near the minimum of the potential), while for $f = m_{\rm Pl}/\sqrt{24\pi}$, $\phi_2/f = 1.9$. Clearly, as f grows, ϕ_2/f approaches π. (Here and below, we assume inflation begins at a field value $0 < \phi_1/f < \pi$; since the potential is symmetric about its minimum, we can just as easily consider the case $\pi < \phi_1/f < 2\pi$.)

(2) Sufficient inflation.— We demand that the scale factor of the Universe inflates by at least 60 e-foldings during the SR regime,

$$\begin{aligned} N_e(\phi_1,\phi_2,f) &\equiv \ln(R_2/R_1) \\ &= \int_{t_1}^{t_2} H\,dt = -\frac{8\pi}{m_{\rm Pl}^2}\int_{\phi_1}^{\phi_2}\frac{V(\phi)}{V'(\phi)}d\phi \\ &= \frac{16\pi f^2}{m_{\rm Pl}^2}\ln\left[\frac{\sin(\phi_2/2f)}{\sin(\phi_1/2f)}\right] \geq 60\,. \end{aligned} \tag{6}$$

Using Eqs. (4) and (5) to determine ϕ_2 as a function of f, the constraint (6) determines the maximum value ($\phi_1^{\rm max}$) of ϕ_1 consistent with sufficient inflation. The fraction of the Universe with $\phi_1 \in [0,\phi_1^{\rm max}]$ will inflate sufficiently. If we assume that ϕ_1 is randomly distributed between 0 and πf from one horizon volume to another, the probability of being in such a region is $\phi_1^{\rm max}/\pi f$. For example, for $f = 3m_{\rm Pl}$, $m_{\rm Pl}$, $m_{\rm Pl}/2$, and $m_{\rm Pl}/\sqrt{24\pi}$, the probability is 0.7, 0.2, 3×10^{-3}, and 3×10^{-41}. The fraction of the Universe that inflates sufficiently drops precipitously with decreasing f, but is large for f near $m_{\rm Pl}$.

(3) Density fluctuations.— Inflationary models generate density fluctuations[18] with amplitude at horizon crossing $\delta\rho/\rho \simeq 0.1H^2/\dot{\phi}$, where the right-hand side is evaluated when the fluctuation crossed outside the horizon during inflation. Fluctuations on observable scales are produced 60–50 e-foldings before the end of inflation. The largest-amplitude perturbations are produced 60 e-foldings before the end of inflation,

$$\frac{\delta\rho}{\rho} \simeq \frac{0.3\Lambda^2 f}{m_{\rm Pl}^3}\left(\frac{8\pi}{3}\right)^{3/2}\frac{[1+\cos(\phi_1^{\rm max}/f)]^{3/2}}{\sin(\phi_1^{\rm max}/f)}\,. \tag{7}$$

Constraints on the anisotropy of the microwave background[3] require $\delta\rho/\rho \leq 5\times10^{-5}$, i.e.,

$$\Lambda \leq \begin{cases} 2\times10^{16}\ \text{GeV for } f = m_{\rm Pl}\,, & \text{(8a)} \\ 3\times10^{15}\ \text{GeV for } f = m_{\rm Pl}/2\,. & \text{(8b)} \end{cases}$$

Thus, to generate the fluctuations responsible for large-scale structure, Λ should be comparable to the GUT scale, and the inflaton mass $m_\phi = \Lambda^2/f \sim 10^{11}$-$(2\times10^{12})$ GeV.

In this model, the fluctuations deviate from a scale-invariant spectrum. For $f \lesssim 3m_{\rm Pl}/4$, the amplitude grows with mass scale M as[2] $\delta\rho/\rho \sim M^{m_{\rm Pl}^2/48\pi f^2}$. Thus, the primordial power spectrum (at fixed time) is a power law, $|\delta_k|^2 \sim k^n$, with spectral index $n \simeq 1 - m_{\rm Pl}^2/8\pi f^2$. The extra power on large scales (compared to the scale-invariant $n=1$ spectrum) may have important implications for large-scale structure.[19]

In new inflation models, the density perturbation constraint usually requires a very large number of e-foldings of the scale factor. Here, many regions of the Universe will inflate less than 60 e-foldings and generate acceptable density fluctuations. Thus, this model might be easily embedded in double-inflation scenarios that also seek to produce extra power on large scales.[20]

(4) Quantum fluctuations.—The semiclassical treatment of the scalar field requires the initial value of ϕ to exceed its quantum fluctuations, i.e., $\phi_1 \gtrsim \Delta\phi = H/2\pi$. For example, this requires that $\phi_1/f > 10^{-7}$ for $f = m_{\rm Pl}$. Since $\phi_1^{\max} \gg H/2\pi$ over the parameter range of interest, this constraint does not place significant restrictions on the model.

(5) Reheating.—At the end of the SR regime, the field ϕ oscillates about the minimum of the potential, and gives rise to particle and entropy production. The decay of ϕ into fermions and gauge bosons reheats the Universe to a temperature[2]

$$T_{\rm RH} = \left(\frac{45}{4\pi^3 g_*}\right)^{1/4} \min[[H(\phi_2)m_{\rm Pl}]^{1/2}, (\Gamma m_{\rm Pl})^{1/2}], \quad (9)$$

where g_* is the number of relativistic degrees of freedom. On dimensional grounds, the decay rate is $\Gamma \simeq g^2 m_\phi^3/f^2 = g^2\Lambda^6/f^5$, where g is an effective coupling constant. [For example, in the original axion model,[12] $g \propto \alpha_{\rm EM}$ for two-photon decay, and $g^2 \propto (m_\psi/m_\phi)^2$ for decays to light fermions ψ.] For $f = m_{\rm Pl}$ and $g_* = 10^3$, we find $T_{\rm RH} = \min[6\times 10^{14}$ GeV, $10^8 g$ GeV]. Since we generally expect $g \lesssim 1$, the reheat temperature will be $T_{\rm RH} \lesssim 10^8$ GeV, too low for conventional GUT baryogenesis, but high enough if baryogenesis takes place at the electroweak scale. Alternatively, the baryon asymmetry can be produced directly during reheating through baryon-violating decays of ϕ or its decay products. The resulting baryon-to-entropy ratio is $n_B/s \simeq \epsilon T_{\rm RH}/m_\phi \sim \epsilon g\Lambda/f \sim 10^{-4}\epsilon g$, where ϵ is the CP-violating parameter;[9] provided $\epsilon g \gtrsim 10^{-6}$, the observed asymmetry can be generated.

(6) Spatial gradients and topological defects.—Above, we assume that the Universe is vacuum dominated $[\rho \simeq V(\phi)]$ when ϕ begins to roll down the hill. Otherwise, the onset of inflation could be delayed or even prevented altogether. Several sources of energy density could be problematic: spatial ϕ gradients, global cosmic strings, and domain walls. For a spatial fluctuation with amplitude $\delta\phi$ and wavelength L, the gradient energy density is $(\nabla\phi)^2 \simeq (\delta\phi/L)^2$. Requiring this to be less than the potential $V(\phi)$ at the onset of inflation leads to the constraint $LH \gtrsim 3(\delta\phi/f)f/m_{\rm Pl}$, i.e., large-amplitude gradients $(\delta\phi \sim f)$ must have wavelengths longer than the Hubble length, $L \gtrsim H^{-1}$, at $T \sim \Lambda$.[21] Gradients on subhorizon scales $(L \lesssim H^{-1})$ are expected to be smoothed out by the beginning of inflation: Since the potential is inoperative for $T \gtrsim \Lambda$, these gradients are damped (redshifted away) by the Hubble expansion.[22] Thus, the gradient energy density at $T \sim \Lambda$ is at most comparable to the potential and quickly becomes subdominant;[23] the net effect is to delay only slightly the onset of inflation. This conclusion follows as long as there also exists a long-wavelength mode $(L \gg H^{-1})$ with amplitude $\delta\phi \sim f$; in this case, there will be regions with $\phi_1 < \phi_1^{\max}$ which inflate.[23] Since ϕ is initially Poisson distributed, we expect roughly equal power on all scales at $T \sim f$, i.e., $\delta\phi_L \sim f$ independent of L (at least for $L \gtrsim m_{\rm Pl}^{-1}$); consequently, gradients should be innocuous, and the probability for inflation will be given by the estimates in Sec. (2) above.

To be conservative, however, we can assume that we must be in a region of the Universe that is homogeneous over at least ~ 1 horizon volume at the onset of inflation. We model the Universe as a tetrahedral lattice with vertices separated by a Hubble length and assume the field is uncorrelated from one vertex point to another. Requiring each point of a tetrahedron to have $0 \leq \phi_1 \leq \phi_1^{\max}$ (or the equivalent at other maxima of the potential), we find that the fraction of the Universe that is homogeneous and inflates is $P = 2N(\phi_1^{\max}/2\pi f N)^4$, where N is the number of distinct minima of the potential. For $f = m_{\rm Pl}$, $\phi_1^{\max}/f = 0.6$ and $P = 2\times 10^{-4} N^{-3}$; for $f = m_{\rm Pl}/2$, $\phi_1^{\max} \simeq 10^{-2}$ and $P = 10^{-11} N^{-3}$. From this argument, the scale f must be very near $m_{\rm Pl}$ to avoid fine tuning the initial conditions. On the other hand, if the previous paragraph is correct, the constraints from gradients are not so severe; ultimately, the issue should be settled by numerical simulations.

Initial gradients in ϕ may also lead to global cosmic strings, which form in the symmetry breaking at $T \sim f$, and to domain walls, which may form at $T \sim \Lambda$.[24] The energy density in strings, which correspond to configurations in which ϕ/f winds around 2π and have dimensionless mass per unit length $G\mu \sim (f/m_{\rm Pl})^2 \sim 1$, is comparable to the gradient density estimated above, assuming of order one string per horizon. The initial energy density in domain walls, $\rho_{\rm DW} \simeq \sigma H$, where $\sigma \simeq f\Lambda^2$ is the wall surface energy per unit area, will also be of the same order of magnitude. Since their energy densities redshift away, topological defects do not prevent the Universe from inflating, but, like gradients, delay briefly the onset of inflation. Once inflation takes place, our observable Universe lies deep inside a single domain with $\phi = \pi f$, so both strings and domain walls are inflated away.

In conclusion, a pseudo Nambu-Goldstone boson, with a potential [Eq. (2)] that arises naturally from particle-physics models, can lead to successful inflation if the global symmetry-breaking scale $f \simeq m_{\rm Pl}$ and $\Lambda \simeq m_{\rm GUT}$.

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