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Experimental Control of Chaos

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We have achieved control of chaos in a physical system using the method of Ott, Grebogi, and Yorke [Phys. Rev. Lett. **64**, 1196 (1990)]. The method requires only small time-dependent perturbations of a single-system parameter and does not require that one have model equations for the dynamics. We demonstrate the power of the method by controlling a *chaotic* system around unstable periodic orbits of order 1 and 2, switching between them at will.

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In a recent Letter, Ott, Grebogi, and Yorke[1] (OGY) demonstrated that one can convert the motion of a chaotic dynamical system to periodic motion by controlling the system about one of the many unstable periodic orbits embedded in the chaotic attractor, through only small time-dependent perturbations in an accessible system parameter. They demonstrated their method numerically by controlling the Hénon map.

Far from being a numerical curiosity that requires experimentally unattainable precision, we believe this method can be widely implemented in a variety of systems including chemical, biological, optical, electronic, and mechanical systems. In this Letter we report the control of chaos in a physical system, a parametrically driven magnetoelastic ribbon, using the method of OGY.

Theoretical background.— The method is based on the observation that unstable periodic orbits are dense in a typical chaotic attractor. Their method assumes only the following four points. First, the dynamics of the system can be represented as arising from an n-dimensional nonlinear map (e.g., by a surface of section or time one return map), the iterates given by $\xi_{n+1} = \mathbf{f}(\xi_n, p)$, where p is some accessible system parameter. Second, there is a specific periodic orbit of the map which lies in the attractor and around which one wishes to stabilize the dynamics. Third, there is maximum perturbation δp_* in the parameter p by which it is acceptable to vary p from the nominal value p_0. Finally, one assumes that the position of the periodic orbit is a function of p, but that the local dynamics about it do not vary much with the allowed small changes in p. Note that while the dynamics is assumed to arise from a map, one needs no model for the global dynamics. These assumptions would seem to allow for the control of any chaotic system for which a faithful Poincaré section can be constructed. The construction of a map from and the location of periodic orbits in[2] experimental data are straightforward processes.

To control chaotic dynamics one only needs to learn the *local* dynamics around the desired periodic orbit by observing iterates of the map near the desired orbit and fitting them to a local linear approximation of the map $\mathbf{f}$.[3] From this, one can find the stable and unstable eigenvalues as well as the local stable and unstable manifolds (given by the eigenvectors). Next, by changing p slightly and observing how the desired orbit changes position, one can estimate the partial derivatives of the orbit location with respect to p.

To control the chaos, one attempts to confine the iterates of the map to a small neighborhood of the desired orbit. When an iterate falls near the desired orbit, we change p from its nominal value p_0 by δp, thereby changing the location of the orbit and its stable manifold, such that the *next* iterate will be forced back toward the stable manifold of the *original* orbit for $p = p_0$. [Figure 1 illustrates this method for the case of a saddle fixed point located at $\xi_F(p_0)$.] That the method of OGY

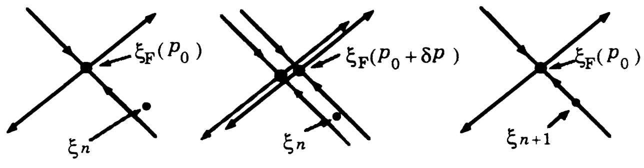

FIG. 1. Schematic of the OGY control algorithm for a saddle fixed point: (a) The nth iterate ξ_n falls near the fixed point $\xi_F(p_0)$. (b) Turn on the perturbation of p to move the fixed point. (c) The next iterate is forced onto the stable manifold of $\xi_F(p_0)$. Turn off the perturbation.

rests on attempting to force the dynamics to stay in the neighborhood of an unstable periodic orbit in the attractor makes it quite different from other previously published methods from removing chaos.[4]

Experimental setup and results.—The experimental system consisted of a gravitationally buckled, amorphous magnetoelastic ribbon. The ribbon material belongs to a new class of amorphous magnetostrictive materials[5] that have been found to exhibit very large reversible changes of Young's modulus $E(H)$ with the application of small magnetic fields.[6,7] The ribbon was clamped at the base to yield a free vertical length greater than the Euler buckling length, thus giving an initially buckled configuration. The ribbon was placed within three mutually orthogonal pairs of Helmholtz coils, which allowed us to compensate for the Earth's magnetic field and to apply an approximately uniform vertical magnetic field along the ribbon. The Young's modulus of the ribbon was varied by applying a vertical magnetic field having the form $H = H_{dc} + H_{ac}\cos(2\pi f t)$. To lowest order, the ribbon was not driven by magnetic forces, but was forced by gravity as $E(H)$ was varied. The magnetic-field amplitudes were typically set in the range 0.1–2.5 Oe. A sensor measured the curvature of the ribbon near its base. Other details of the experiment can be found in Refs. 6 and 7.

The data were time-series voltages $V(t)$ acquired from the output of the sensor. Voltages were sampled at the drive period of the ac field (at times $t_n = n/f$) by triggering a voltmeter off the ac signal.

By considering the sampled voltages as arising from iterates of a map, $\mathbf{X}_n = V(t_n)$, we are able to directly apply the control theory outlined above. We selected H_{dc} to be the parameter to be varied to achieve control (i.e., $p = H_{dc}$). First, we chose a parameter region (H_{ac}, H_{dc}, and f) such that the ribbon was oscillating chaotically. In order to simplify the comparison with the theory, the parameter region chosen was one in which the dynamics of the iterates near the orbits of interest clearly appears to be two dimensional (i.e., the two-dimensional return map, $\mathbf{X}_{n+1}$ vs $\mathbf{X}_n$, is always single valued in the neighborhood of the orbits of interest). The first 2350 iterations (in gray) in Fig. 2(a) are of the uncontrolled time-series data for $H_{ac} = 2.050$ Oe, $H_{dc} = 0.112$ Oe ($= p_0$), and $f = 0.85$ Hz (from 1 to 2350 iterations). In Fig. 2(b), the return map for the uncontrolled system is shown in gray. We estimate the dynamical noise in our system, i.e., the deviation of the motion of the ribbon away from deterministic chaos, to be ± 0.005 V, since any structure on the attractor below this scale is blurred out.

We found the approximate location X_F of an unstable period-1 orbit of the map (i.e., a fixed point) by noting that any fixed point of the dynamics must lie along the $\mathbf{X}_{n+1} = \mathbf{X}_n$ line in the plot of the return map. To stabilize this fixed point we next examined the data series and found all pairs of iterates *both* of which fell within 0.05 V of the approximate fixed point. To these pairs of iterates we fit the approximate local *linear* map $\mathbf{M}$,

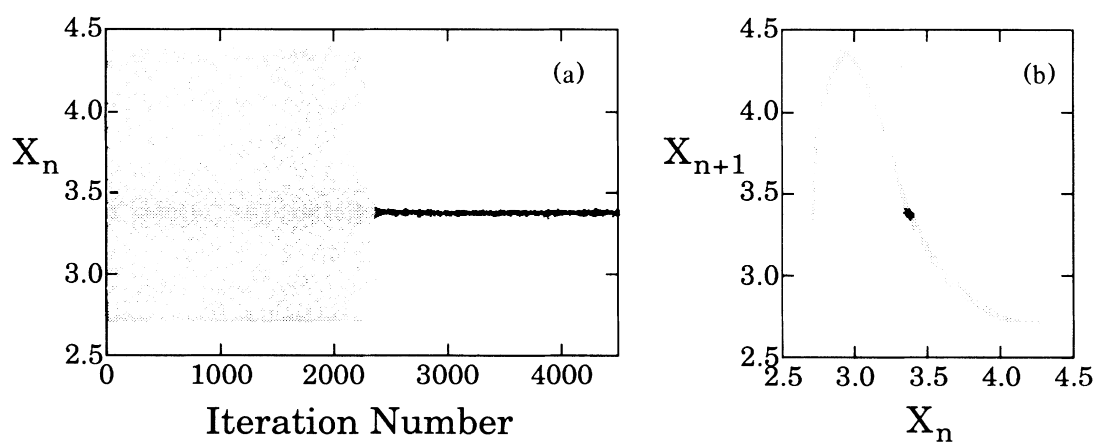

FIG. 2. (a) Time series of $\mathbf{X}_n = V(t_n)$ for H_{dc}(nominal) $= 0.112$ Oe, $H_{ac} = 2.050$ Oe, and $f = 0.85$ Hz. Control was initiated after iteration 2350. (b) The first return map ($\mathbf{X}_{n+1}$ vs $\mathbf{X}_n$) for the controlled system (in black) is superimposed on the map for the uncontrolled system (in gray). The large density of points of low values of $\mathbf{X}_n$ is due to the saturation of the sensor for large excursions of the ribbon away from the sensor.

where

$$\xi_{n+1}-\xi_F=\mathbf{M}(\xi_n-\xi_F),\quad \xi_{n+1}=\begin{pmatrix}X_{n+2}\\X_{n+1}\end{pmatrix},$$

$$\xi_n=\begin{pmatrix}X_{n+1}\\X_n\end{pmatrix},\quad \xi_F=\begin{pmatrix}X_F\\X_F\end{pmatrix}.$$

Knowing $\mathbf{M}$, we could extract the stable and unstable eigenvalues (λ_s,λ_u) and eigenvectors $(\mathbf{e}_s,\mathbf{e}_u)$. We actually only needed λ_u and the unstable *contravariant* eigenvector[1] $\mathbf{f}_u$, given by $\mathbf{f}_u\cdot\mathbf{e}_u=1$ and $\mathbf{f}_u\cdot\mathbf{e}_s=0$.

Next, we changed H_{dc} slightly ($H_{dc}=0.120$ Oe) and collected another set of data. We again found the precise location of the fixed point and calculated $\mathbf{g}=\partial\xi_F/\partial p\approx\delta\xi_F/\delta H_{dc}$.

To control the oscillations of the ribbon, we set $p=p_0$; when $|\xi_n-\xi_F|<\delta\xi_*$, we attempted control. Here, $\delta\xi_*\approx[(\lambda_u-1)/\lambda_u]\delta p_*(\mathbf{g}\cdot\mathbf{f}_u)$ is the maximum distance from the stable manifold of ξ_F for which one can achieve control for a given δp_*. As long as the iterate was within $\delta\xi_*$ of ξ_F, we perturbed p from p_0 by $\delta p=C(\xi_n-\xi_F)\cdot\mathbf{f}_u$, where Ref. 1 gives $C=[\lambda_u/(\lambda_u-1)]/\mathbf{g}\cdot\mathbf{f}_u$. Since noise and errors in determining ξ_F, $\mathbf{f}_u$, $\mathbf{g}$, and λ_u, as well as any inaccuracies due to the linear approximation, prevented us from getting the next iterate exactly on the stable manifold, a new δp was calculated for each iterate. Note that both $\delta\xi_*$ and C can be computed at the start of the run, and that the calculations at each iterate are very simple. We could apply the changes to the applied magnetic field and change the Young's modulus of the ribbon in under 1 ms. Thus, our change in p was effectively instantaneous in relation to the 1.2-s period of the ac drive.

At the values of H_{ac}, H_{dc}, and f mentioned above, we calculated $X_F=3.398\pm0.002$, $\delta X_F/\delta H_{dc}=-337\pm50$, $\mathbf{f}_u=\binom{-0.2}{1.2}\pm\binom{0.2}{0.1}$, and $\lambda_u=-1.2\pm0.2$. These numbers are typical of our data in that the fixed point can be determined with a great deal of accuracy, but the computed values of the eigenvalues and eigenvectors are sensitive to the noise on the attractor. Fortunately, the control is quite insensitive to variations in λ_u and $\mathbf{f}_u$ (e.g., using $\lambda_u=-1.4$ yielded results similar to those using $\lambda_u=-1.2$).

We have been able to control the oscillations of our ribbon for over 200000 iterates (>64 h), with a maximum allowed perturbation of 0.01 Oe. Figure 2(a) (after 2350 iterations) shows the controlled time series (in black) and Fig. 2(b) the return map (superimposed, in black, on the attractor for the uncontrolled system). The control was to ±0.015 V of the desired fixed point, about triple the dynamic noise present on the uncontrolled attractor.

We have also controlled the motion about a period-2 oscillation (again for over 50000 iterates and with $\delta p_*=0.01$ Oe). The same procedure outlined above is followed except using $\xi_n=\binom{X_{2n+2}}{X_{2n}}$. The control was adjusted only at every other data iterate, about the periodic point at $X_F=3.926\pm0.004$. As a demonstration of the versatility of the method, Fig. 3 shows time-series data while the system was switched between no control and control about the fixed point or the period-2 orbit, again with the same values of H_{dc}(nominal), H_{ac}, and f as for Fig. 2.

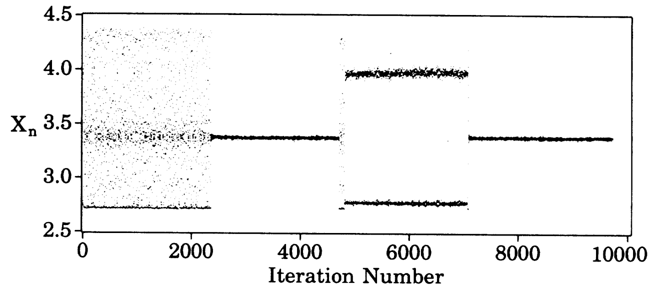

FIG. 3. Time-series data as the system is switched from no control to control about the fixed point (at $n=2360$), to control about the period-2 orbit (at $n=4800$), and back to control around the fixed point (at $n=7100$).

In conclusion, we have demonstrated the first control of chaos in a physical system, using the method of Ott, Grebogi, and Yorke. Some advantages of this method are the following: (1) No model for the dynamics is required; (2) the computations required at each iterate are minimal; (3) the required changes in the parameter can be quite small; (4) different periodic orbits can be stabilized for the *same* system in the *same* parameter range; (5) control can be achieved even with imprecise measurements of the eigenvalues and eigenvectors; and (6) this method is *not* restricted to periodically driven mechanical systems, but extends to any system whose dynamics can be characterized by a nonlinear map.

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contrast this method with the resonant control method of Hübler and co-workers that has been applied to nonlinear pendula and other oscillators with nonlinear potentials. Their method differs from the method described above in at least three important respects. (1) One must have or construct model equations for the dynamics. (2) One must be able to modify the *driving force* of these equations, and these modifications can be rather large. The method of OGY requires no model equations and the perturbations could be to *any* accessible system parameter. (3) Rather than apply corrections as the dynamics wanders from a given unstable orbit, the resonant control method seeks to modify the underlying dynamical system such that the goal dynamics become *stable* solutions of the system (and thus uses no feedback).

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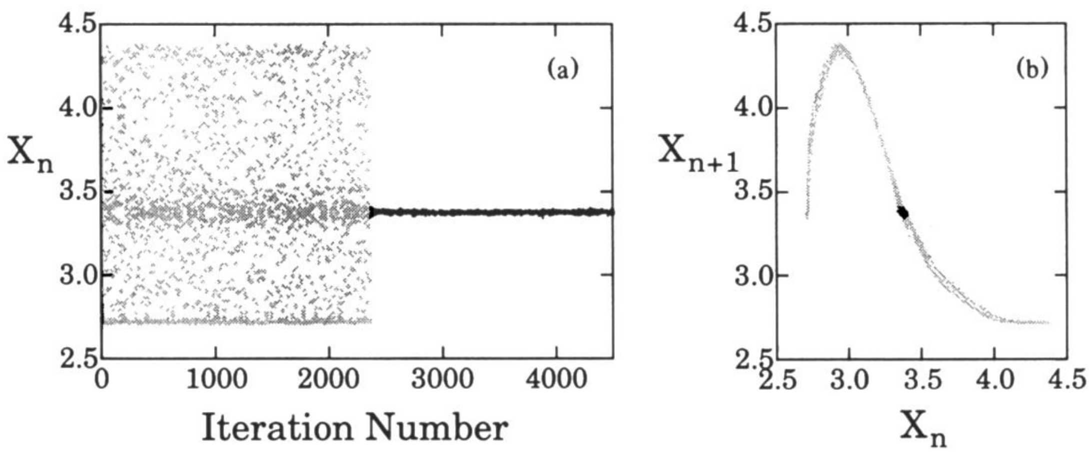

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