Spontaneous Symmetry Breaking in a Laser: The Experimental Side

C. Green, G. B. Mindlin, E. J. D'Angelo, H. G. Solari, and J. R. Tredicce
Department of Physics, Drexel University, Philadelphia, Pennsylvania 19104
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We show that a qualitative theory of bifurcations can provide an understanding of the spontaneous symmetry breaking and the appearance of traveling waves leading to spatiotemporal complexity in the transverse patterns of the intensity of a CO_2 laser.

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The existence of spatial (in addition to temporal) complexity in a laser was studied recently both theoretically and experimentally.[1,2] However, the origin and the sequence of events leading to such situations are still in need of clarification.[3]

From a theoretical point of view, a laser can be described in terms of the well-known Maxwell-Bloch equations (see, for example, Ref. 4). In principle, one can solve these partial differential equations numerically over a wide region of parameter space, but the most important features of the solutions and their bifurcations can also be produced by the qualitative theory of differential equations (bifurcation theory). This qualitative method was used[5-7] to analyze nonlinear differential equations with an intrinsic spatial symmetry. Here we adopt this procedure to analyze the transverse intensity patterns that can emerge at the output of a laser, and we provide experimental evidence of certain features that appear in a laser constructed with a large-diameter CO_2 laser tube in which we are able to change the control parameter adiabatically.

The stable solutions of a symmetric system do not necessarily bear the same symmetries as the system including the boundary conditions, a phenomena usually referred to as spontaneous symmetry breaking. The symmetry of a solution (its isotropy group) is generally a subgroup of the group of symmetries of the system. An appropriate question is: For which subgroups of the entire symmetry group can we expect solutions? This question was addressed in Ref. 5 where the authors proved that the bifurcation sequences of a nonlinear system are determined largely by the symmetries of the system regardless of any other characteristic, including the details of the particular model.

A laser with an active medium confined within a circular tube and spherical mirrors has an obvious O(2) symmetry. The observation of spatial symmetry breaking and the appearance of associated temporal oscillations with a well-defined frequency and small amplitude suggest the existence of Hopf bifurcations implying an additional S^1 symmetry (time basis of the type $e^{i\Omega t}$). Furthermore, the evolution of the patterns, as shown in this paper, indicates the predominance of structures with a D_n (dihedral) symmetry with nonstandard actions (spatial bases of the type $e^{\pm il\theta}$ for different values of $l=n/2$) on the O(2) group. On the basis of these assumptions, the classification of the different patterns can be addressed using subgroups of a group that contains both the spatial symmetry of the problem $[O(2)]^8$ and the additional temporal symmetries added to the system when different modes are born. Generically, the nonlinearities couple those modes, creating a complicated behavior. We consider the easiest situation (the coupling of two modes) as a model for our experiment. In Table I adapted from Ref. 5, we tabulate the type of solutions for different isotropy subgroups corresponding to mode-mode interactions associated with actions l and m ($l\neq m$) over C^4. This is to say, we classify the patterns according to the spatiotemporal symmetries of waves of the form

$$E=1+(z_1e^{il\theta}+z_2e^{-il\theta})e^{i\Omega t}+(z_3e^{im\theta}+z_4e^{-im\theta})e^{i\Omega' t},$$

where E is the electric field, z_n for $n=1-4$ are functions of the radial coordinate and time, θ is the angular coordinate, and Ω and Ω' are arbitrary frequencies.

Beginning with the most symmetric state (0 in Table I) and proceeding in the direction of the states of lower symmetry (14 in Table I) the bifurcation sequence can be ordered as indicated. We say that a pattern A has higher symmetry than a pattern B if the isotropy subgroup of A includes the isotropy subgroup of B. Furthermore, a bifurcation of one pattern into another is allowed only when the dimension of the invariant subspace is increased by 2 due to the fact that a spatial symmetry breaking is associated with a Hopf bifurcation in the time domain. Accordingly, the structure of the bifurcation sequences corresponding to our case are shown in Fig. 1 where allowed symmetry-breaking transitions are indicated by an arrow.

It is worth observing that the different solutions are distinguished not only by their geometrical shape but also by the temporal behavior of each point of the pattern and the relative phase of any pair of points in space which follows directly from the functional form of the field.

To confirm the relevance of this theory in the understanding of the bifurcations taking place in lasers, we used a CO_2 laser with a tube cross section of 22 mm in diameter. An intracavity optical system consisting of two converging lenses with a variable distance between them provides an accurate control of the Fresnel number. The optical system at one end of the laser, consisting of two lenses and a plane mirror, acts as a mirror with a variable radius of curvature, as described in Ref.

TABLE I. Isotropy subgroups of $O(2)\times T^2$ (adapted from Ref. 5). The action of $S(\lambda,\mu,\nu)$, K, $Z(\theta,\phi_1,\phi_2)$, and $Z_k(\theta,\phi_1,\phi_2)$ over the vectors of the basis (z_1,z_2,z_3,z_4) is defined as

$$S(\lambda,\mu,\nu)(z_1,z_2,z_3,z_4)=(e^{i\psi(\mu+\lambda l)}z_1,e^{i\psi(\mu-\lambda l)}z_2,e^{i\psi(\nu+\lambda m)}z_3,e^{i\psi(\nu-\lambda m)}z_4)\,,$$

$$K(z_1,z_2,z_3,z_4)=(z_2,z_1,z_4,z_3)\,,$$

$$Z(\theta,\phi_1,\phi_2)=(e^{i(l\theta+\phi_1)}z_1,e^{i(\phi_1-l\theta)}z_2,e^{i(m\theta+\phi_2)}z_3,e^{i(\phi_2-m\theta)}z_4),\quad Z_k(\theta,\phi_1,\phi_2)\cong KZ(\theta,\phi_1,\phi_2)K\,.$$

	Isotropy subgroup	Basis	Comments
0	$O(2)\times T^2$	$z_1=z_2=z_3=z_4=0$	Trivial-fully symmetric
1	$S(0,0,1)\times S(1,-1,0)$	$z_2=z_3=z_4=0$	l-traveling wave (l-TW)
2	$S(0,1,0)\times S(1,0,-m)$	$z_1=z_2=z_4=0$	m-TW
3	$S(0,0,1)\times K\times Z(\pi/l,\pi,0)$	$z_1=z_2,\ z_3=z_4=0$	l-standing wave (l-SW)
4	$S(0,1,0)\times K\times Z(\pi/m,0,\pi)$	$z_1=z_2=0,\ z_3=z_4$	m-SW
5	$S(0,0,1)\times Z(\pi/l,\pi,0)$	$z_3=z_4=0$	l-TW+l-SW
6	$S(0,1,0)\times Z(\pi/m,0,\pi)$	$z_1=z_2=0$	m-TW+m-SW
7	$S(1,l,m)$	$z_1=z_3=0$	l-TW+m-TW (same direction)
8	$S(1,l,-m)$	$z_1=z_4=0$	l-TW+m-TW (opposite direction)
9	$K\times Z(\pi,l\pi,m\pi)$	$z_1=z_2,\ z_3=z_4$	l-SW+m-SW
10	$Z_k(0,\pi,0)\times Z(\pi,l\pi,m\pi)$	$z_1=-z_2,\ z_3=z_4$	l-SW+m-SW (m odd)
11	$Z_k(0,0,\pi)\times Z(\pi,l\pi,m\pi)$	$z_1=z_2,\ z_3=-z_4$	l-SW+m-SW (m even, l odd)
12	$Z(\pi/l,\pi,m\pi/l)$	$z_3=0$	Three frequencies ($l\neq1$)
13	$Z(\pi/m,l\pi/m,\pi)$	$z_1=0$	Three frequencies ($m\neq1$)
14	$Z(\pi,l\pi,m\pi)$	$z_n\neq0$	Three frequencies

2. The time-averaged intensity pattern is observed on infrared imaging plates, while the instantaneous behavior in time of two isolated points in the pattern is detected by two HgCdTe detectors.

In Fig. 2 we show sequences of patterns observed as the distance between lenses is changed. Different sequences are obtained from apparently identical initial conditions in which the pattern has a stable Gaussian profile. It is clear that even if Fig. 2 can be used as a clue to identify different paths in the symmetry-breaking process, it does not provide enough information to classify the sequence unambiguously, nor is there sufficient evidence that the theoretical process described previously is really taking place in the laser. At this point it becomes important to consider the local evolution of the intensity in time, the frequencies of oscillation, and the relative phases of different points in the pattern.

Initially, we distinguish two possible paths in the process of breaking the initial radial symmetry of the pattern shown in Fig. 2(a): (1) the generation of two intensity peaks [Fig. 2(b)] with the simultaneous appearance of temporal oscillations with a frequency of the order of 2 MHz (see Fig. 3) for every point in the pattern except

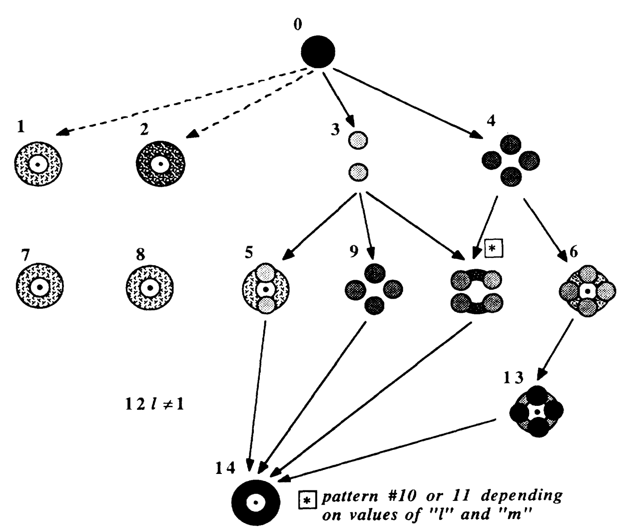

FIG. 1. Bifurcations of $O(2)\times T^2$. The patterns shown correspond to the average intensity for the case $l=1$, $m=2$. The dashed lines indicate transitions not observed in our laser while the solid lines indicate those that have been observed. These patterns must be compared to those shown in Fig. 2.

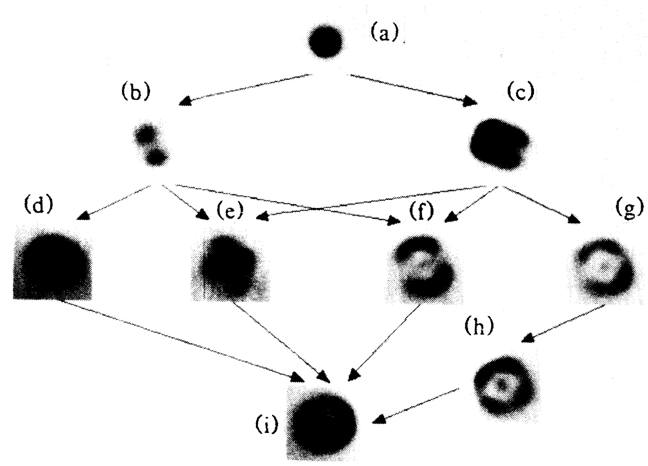

FIG. 2. Spatial patterns of the average laser intensity as observed in the thermal plates. From (a) to (i) we show the bifurcations producing the spontaneous symmetry breaking of a Gaussian pattern in our laser.

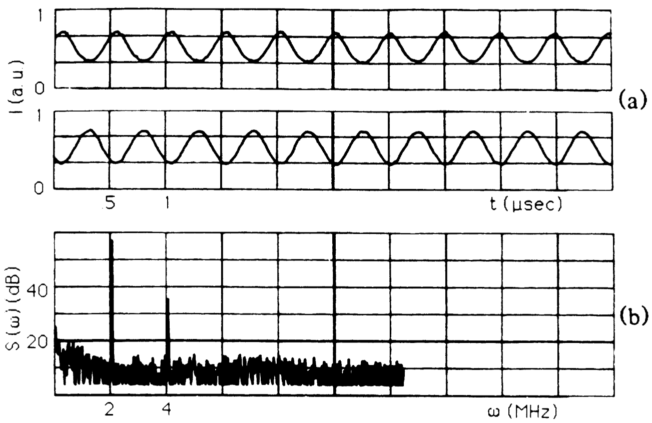

FIG. 3. (a) Intensity as a function of time measured at the two peaks of Fig. 2(e); (b) the power spectrum detected at one of the peaks.

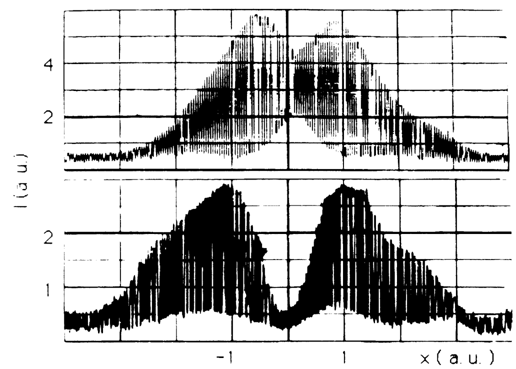

FIG. 4. Cross sections of pattern 2(d) at two different positions. The lower trace is a cut through the center of the beam and perpendicular to the position of the two intensity peaks. The upper trace is a cut parallel to the previous one passing through the intensity peak.

for those lying on a line passing through the center of the beam and perpendicular to the line joining the two peaks; (2) the generation of four peaks [Fig. 2(c)] in the averaged intensity with the appearance of temporal oscillations at every point of the pattern except for those lying on two lines perpendicular to each other and separating the peaks. In both cases, the lines with no oscillations are the boundaries through which there is a discontinuous change in phase by π rad.

The two configurations described above may bifurcate into four easily distinguished patterns with two or four intensity peaks [Figs. 2(d)–2(g)].

The pattern of Fig. 2(d) shows oscillations whose relative amplitude varies depending on the position. The points lying on the line that originally separated the phases in Fig. 2(b) show now an almost 100% modulation depth except for the center of the beam where the intensity is zero (Fig. 4). On a line parallel to the previous one and passing through one of the peaks we observe oscillations with a dc basis. This behavior is a clear indication of the existence of a traveling wave around the center of the beam and therefore is pattern 5 in the classification of Fig. 1.

Another observed state is represented by four peaks of the averaged intensity [Fig. 2(e)]. It is easily distinguished from the previous four-peak configuration by its temporal behavior at the maxima. Two of them, at opposite positions in space, oscillate quasiperiodically with one of the frequencies in phase and the other one out of phase (see Fig. 5), while the other two maxima show a single frequency in phase (pattern 9 of Fig. 1).

A different pattern observed in this second stage of the bifurcation sequence also consists of four maxima. All peaks oscillate with two frequencies but points lying on three different lines crossing each other at the center show oscillations at a single frequency. The amplitude of the oscillations vanishes at the center of the beam. This state corresponds to pattern 11 in Fig. 1.

Finally, the last possible state consists of four peaks with a ring. This pattern presents evidence of a traveling wave similar to the ones described previously. A change in the control parameter from this state does not seem to affect the average spatial structure [Fig. 2(h)] but the local time behavior becomes complex, suggesting the existence of a new Hopf bifurcation. We associate this situation with pattern 13 of Fig. 1.

A further decrease of the distance between the lenses yields an almost radially symmetric pattern on the average [Fig. 2(i)], but it shows a complicated behavior in time (chaotic for some spatial points) (Fig. 6).

This experimental bifurcation sequence provides evidence for the cascade shown in Fig. 1 in which only some paths are actually selected by the system. In fact, patterns 1 and 2 of Fig. 1 involving only rotating waves are not observed in our experiments. In contrast, standing waves with $l=1$ or $m=2$ are observed with an apparent preference for the $l=1$ standing wave. Fifteen out of

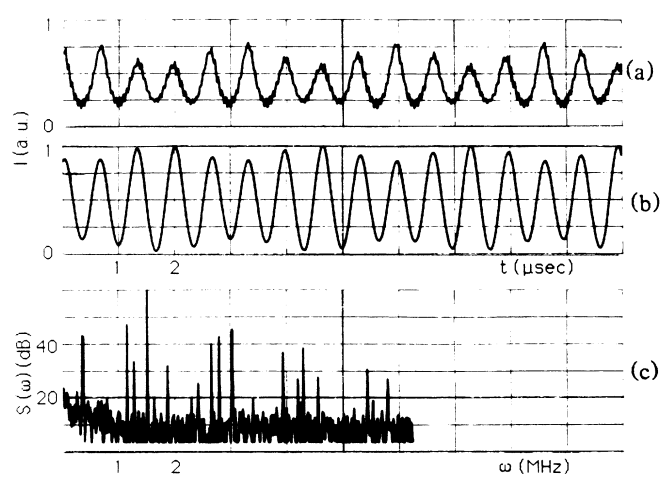

FIG. 5. (a),(b) Intensity as a function of time measured at two opposite peaks of Fig. 2(e); (c) the power spectrum corresponding to the signal detected at one peak. Note that the low frequency is out of phase while the high frequency is in phase.

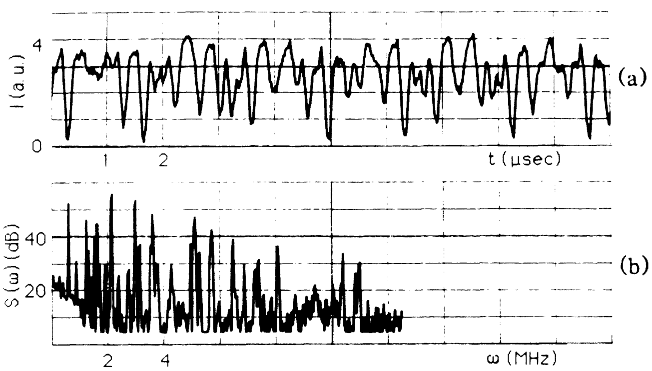

FIG. 6. (a) Intensity as a function of time for a spatial point corresponding to Fig. 2(h); (b) the power spectrum detected at that point. Note the large dips in the values of the intensity and its irregular behavior.

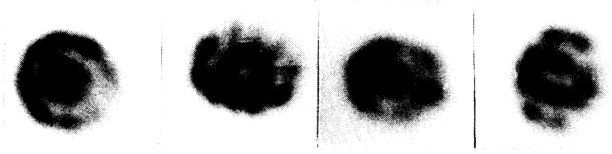

FIG. 7. Complex average intensity patterns observed by increasing the Fresnel number of the cavity. Note the appearance of different number of peaks in the internal and external regions of the pattern which indicates the coexistence of different values of "l" and "m."

twenty consecutive sweeps across the bifurcation point led to the pattern of Fig. 2(b) indicating that its basin of attraction is larger than the one corresponding to the pattern of Fig. 2(c).

From the pattern shown in Fig. 2(i), upon decreasing the distance between lenses, we find new structures involving the appearance of more rings and up to 24 intensity peaks. These bifurcation sequences have different values of l and m in different rings. Some typical patterns are shown in Fig. 7. A detailed analysis of these patterns is beyond the scope of this Letter.

In conclusion, we have demonstrated that the spatiotemporal behavior of lasers can be described qualitatively by group theory, that the structures observed are created from the symmetry breaking of the $O(2)\times T^2$ group, and that this spontaneous symmetry breaking is at the origin of a gradual complexity in space and time by separating regions in which the dynamical behavior is essentially different.[9] After the appearance of two or three frequencies the temporal behavior of the local intensity becomes usually chaotic and almost any degree of spatial symmetry tends to disappear.

After several bifurcations the cross correlation function between two points in the pattern decreases as the distance between points increases. A decreasing cross correlation function is usually an indicator of turbulent behavior.[10] In such situations a more careful study of the particular equations of the laser is necessary if one is to make any predictions about the behavior of the system.

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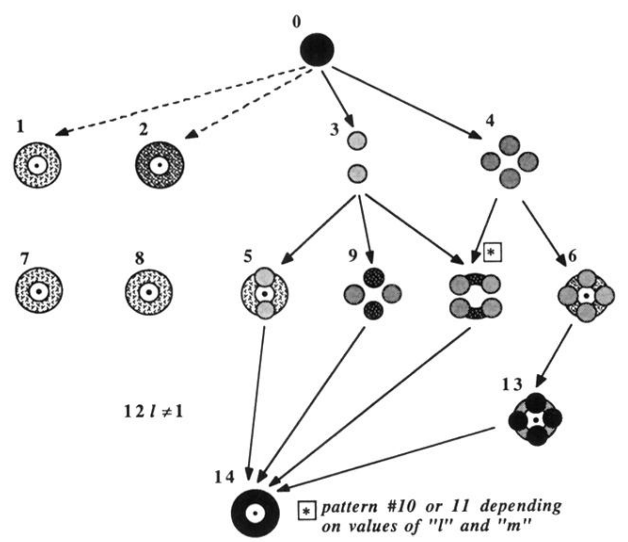

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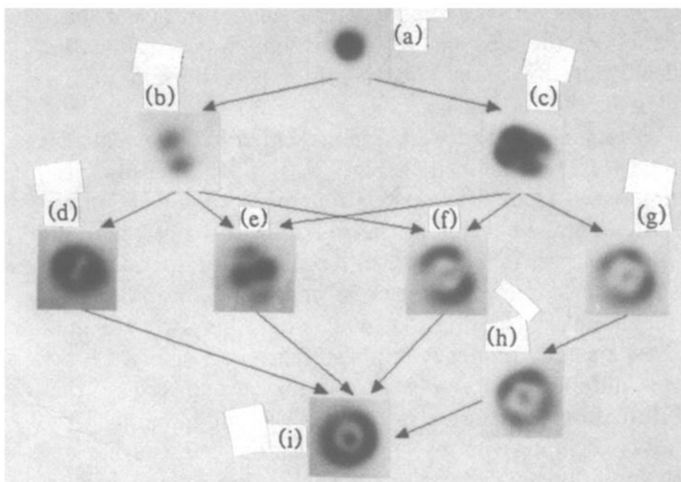

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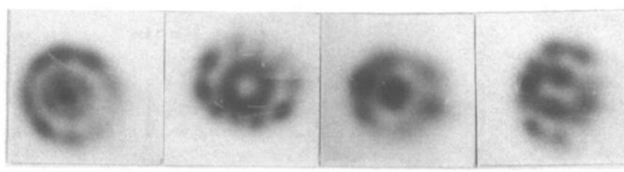

FIG. 7. Complex average intensity patterns observed by increasing the Fresnel number of the cavity. Note the appearance of different number of peaks in the internal and external regions of the pattern which indicates the coexistence of different values of "l" and "m."