Relic Gravitational Waves and Extended Inflation

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In extended inflation, a new version of inflation where the transition from an inflationary to a radiation-dominated Universe is accomplished by bubble nucleation, bubble collisions supply a potent—and potentially detectable—source of gravitational waves. The energy density in relic gravitons from bubble collisions is expected to be about 10^{-5} of closure density. Their characteristic wavelength depends upon the reheating temperature $T_{\rm RH}$: $\lambda \sim (10^4\ {\rm cm})[(10^{14}\ {\rm GeV})/T_{\rm RH}]$. If black holes are produced by bubble collisions, they will evaporate producing shorter-wavelength gravitons.

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Inflation provides a means of understanding the smoothness and flatness of the Universe and the origin of the primeval density fluctuations necessary to trigger structure formation, as well as an elegant solution to the magnetic-monopole problem.[1,2] Testing the "inflationary paradigm" is not a simple matter. Inflation makes but three robust predictions: (i) a flat Universe, i.e., $\Omega_{\rm tot}=1.0$, where $\Omega_{\rm tot}$ is the ratio of the total energy density to the critical energy density; (ii) a nearly scale-invariant spectrum of adiabatic density perturbations; and (iii) the presence of a spectrum of relic gravitational waves—and the absence of the 0.9-K thermal background of relic gravitational waves that might otherwise be expected. The first two of these predictions can be confronted with a variety of cosmological observations and experiments, including comparison of the Hubble age with other independent age determinations, determination of the spectrum of anisotropies in the cosmic microwave background radiation (CMBR), detailed modeling of structure formation and comparison to the observed distribution of galaxies, and the search for exotic dark matter such as axions or neutralinos. (Exotic dark matter is required since primordial nucleosynthesis constrains the mass density in baryons: $\Omega_B \lesssim 0.12$.[3])

The third test is the most challenging, but also the most decisive. (Indeed, as a historical matter both flatness and the scale-invariant fluctuation spectrum were proposed before inflation.[4]) Quantum fluctuations, the underlying mechanism for the production of density perturbations, also lead to the production of relic gravitational waves: During inflation the transverse, traceless tensor components of the metric (the graviton degrees of freedom) are excited by de Sitter quantum fluctuations. Later, during the post-inflationary epoch, as a given mode reenters the horizon its dimensionless rms amplitude is about $h \simeq 2H/\sqrt{\pi}m_{\rm Pl}$, where H is the value of the Hubble parameter during inflation. Once inside the horizon (i.e., physical wavelength λ less than H^{-1}), the mode can be described as relic gravitons. The spectrum extends from about 10^5 to about 10^{28} cm, the present Hubble scale. The present energy density per octave in relic gravitons is[5] (i) $\Omega_{\rm GW}(\lambda) \simeq (4/3\pi)(H/m_{\rm Pl})^2$ for $\lambda \simeq H_0^{-1} \simeq 6000 h_{1/2}^{-1}$ Mpc: (ii) $\Omega_{\rm GW}(\lambda)$ decreases as λ^2 for scales between the present Hubble scale and about $26h_{1/2}^{-1}$ Mpc; (iii) $\Omega_{\rm GW}(\lambda) \simeq 10^{-5}(H/m_{\rm Pl})^2$ is constant on scales between $26h_{1/2}^{-1}$ Mpc and about $10^{-7}[(1\ {\rm GeV})/T_{\rm RH}]$ Mpc; and (iv) $\Omega_{\rm GW}(\lambda)$ again decreases as λ^2 down to the smallest wavelengths, about $10^{-7}[(1\ {\rm GeV})/T_{\rm RH}^{1/3}M^{2/3}]$ Mpc (see Fig. 1). In these expressions,

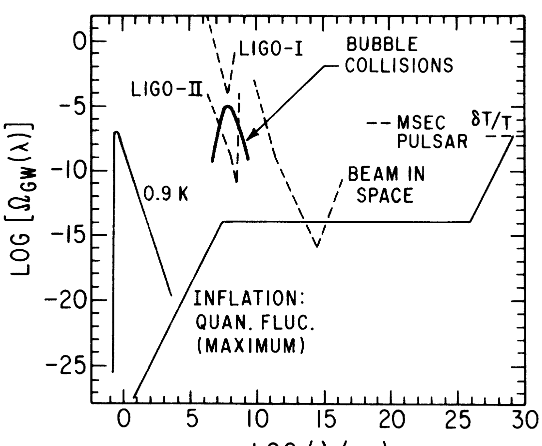

FIG. 1. Fraction of critical density in gravitational waves per octave $\Omega_{\rm GW}(\lambda)$ vs wavelength λ. Shown are the 0.9-K background expected in the standard cosmology, the radiation produced by bubble collisions in extended inflation (for $T_{\rm RH}=3\times10^{10}$ GeV), the "maximal spectrum" that can arise due to quantum fluctuations ($M \simeq 10^{16}$ GeV and $T_{\rm RH} \sim 3\times10^{10}$ GeV), the limits provided by the large-angle isotropy of the CMBR and the millsecond pulsar (Ref. 15), and the projected capabilities of some future detectors (Ref. 17).

$\Omega_{GW}(\lambda) \equiv (\lambda\, d\rho_{GW}/d\lambda)/\rho_{crit}$ is the fraction of critical density contributed per octave, the present Hubble parameter $H_0 = 50 h_{1/2}$ km sec^{-1} Mpc^{-1}, T_{RH} is the reheat temperature, and M^4 is the vacuum energy during inflation.

The gravitational waves just entering the horizon today ($\lambda \sim 10^{28}$ cm) lead to a quadrupole anisotropy in the temperature of the CMBR of magnitude comparable to their dimensionless amplitude: $\delta T/T \sim H/m_{Pl}$. The observed isotropy of the CMBR on large angles,[6] $\delta T/T \lesssim 3\times 10^{-5}$, constrains H/m_{Pl} to be less than about 3×10^{-5}. In turn, this constrains the entire spectrum of relic gravitational waves. In particular, the long plateau region is constrained to contribute at most 10^{-14} of the critical density, which makes prospects for detection bleak. [Note, the dimensionless rms wave amplitude on the scale λ, h_λ, and the energy density per octave are related, $h_\lambda \simeq \lambda (G\rho_\lambda)^{1/2}$; short-wavelength fluctuations correspond to smaller absolute metric distortions, for a fixed energy density.]

Our main purpose in this Letter is to point out that in models of extended inflation[7] there is an additional and probably much more important source of gravitational waves, whose fractional contribution to the critical density is expected to be about 10^{-5}. The origin of these gravitational waves traces to the fundamental difference between slow-rollover inflation and extended inflation: the mechanism for terminating the inflationary phase. Whereas in slow-rollover inflation the transition to a radiation-dominated phase is basically smooth, proceeding through the decay of the inflaton field,[8] in extended inflation the transition occurs through bubble nucleation and percolation. Bubble collisions result in significant production of gravitational waves. It is also possible —and even highly plausible—that mini black holes are produced at the collision sites.[9] These holes evaporate through the Hawking process and produce comparable amounts of gravitational radiation, but at shorter wavelengths.

The reason that bubble nucleation can successfully lead to reheating in extended inflation is the fact that the nucleation rate per Hubble volume per Hubble time ($\varepsilon \equiv \Gamma/H^4$) varies during inflation: At early times the Universe is hung up in the false vacuum (as in old inflation) because ε is very small; at late times ε becomes greater than some critical value[10] (which is of "order unity," but could be as small as 10^{-6}) and rapid bubble nucleation and percolation occur, returning the Universe to a radiation-dominated phase. The rate at which ε changes determines the spectrum of bubble sizes. To prevent too many large bubbles, which would ultimately result in large-temperature anisotropies in the CMBR, the transition must happen relatively fast.[10] Precisely how ε evolves is very model dependent; we will simply assume that there is a characteristic bubble size $\bar{\lambda}$. Given a specific model, one can easily compute the spectrum of bubble sizes.

Reheating through bubble collisions is an inherently violent and nonspherical process, and so one expects copious production of gravitational waves. To estimate it, we characterize the size of the bubbles when they collide relative to the Hubble radius by f: $\bar{\lambda} \equiv fH^{-1}$; f is expected to be of order unity, but depends upon the details of bubble percolation. Since the growth of bubbles is inherently relativistic, we assume that the time scale associated with bubble collisions is also $\bar{\lambda}$. The emission of gravitational waves during the collision of a few bubbles is characterized by a luminosity given by $\mathcal{L}_{GW} \sim G(d^3Q/dt^3)^2$, where Q is the quadrupole moment of the energy distribution and G is the gravitational constant. It then follows that the energy liberated in gravity waves is

$$\Delta E_{GW} \sim \bar{\lambda}\mathcal{L}_{GW} \sim GM_B^2/\bar{\lambda}\,, \tag{1}$$

where $M_B \simeq \bar{\lambda}^3 M^4$ is the mass-energy of a typical bubble. (As before, M^4 is the false-vacuum energy.) From Eq. (1) we estimate that the fraction of the false-vacuum energy that goes into gravitational waves is $\varepsilon \sim \Delta E_{GW}/M_B \sim f^2$; since $f \sim 1$, there is every reason to expect that after reheating a significant fraction of the energy density in the Universe is present in the form of gravitational waves of wavelength $\bar{\lambda}$.[11]

The total energy density in radiation released by bubble collisions is

$$\rho_R \equiv (g_* \pi^2/30) T_{RH}^4 \simeq M^4\,, \tag{2}$$

where $T_{RH} \simeq (30/\pi^2 g_*)^{1/2} M$ is the reheat temperature and g_* counts the total number of ultrarelativistic degrees of freedom. Using the fact that $\varepsilon\rho_R = \rho_{GW} \sim G^{-1}(h_{\bar{\lambda}}/\bar{\lambda})^2$, it follows that $h_{\bar{\lambda}} \sim \varepsilon$. Assuming for the moment that the gravitational constant does indeed remain constant, then as the Universe expands $h_{\bar{\lambda}}$ evolves as R^{-1} and $\bar{\lambda}$ increases as R (R is the cosmic scale factor). Further, if we assume that the expansion is adiabatic after reheating, then the entropy per comoving volume, which is proportional to $g_*(T)R^3T^3$, remains constant. It is a simple matter to relate the value of the scale factor today to that at reheating:

$$R_0/R_{RH} = [g_*(T_{RH})/g_*(3\text{ K})]^{1/3}[T_{RH}/(3\text{ K})]\,.$$

From this it follows that the present amplitude h_λ and wavelength λ of the bubble-produced gravitational waves are

$$h_\lambda \sim 10^{-26}[(10^{14}\text{ GeV})/T_{RH}]\varepsilon\,, \tag{3a}$$

$$\lambda \sim (10^4\text{ cm})[(10^{14}\text{ GeV})/T_{RH}]\varepsilon^{1/2}\,, \tag{3b}$$

where we have taken $g_*(T_{RH})$ to be 300 and $g_*(3\text{ K}) \simeq 3.4$ and $\lambda = [T_{RH}/(3\text{ K})]\bar{\lambda}$.

In a similar manner one can use the constancy of the entropy per comoving volume and the fact that ρ_{GW} evolves as R^{-4} to find the ratio of the energy density in gravitational waves to that in photons at any epoch:

$$\frac{\rho_{GW}}{\rho_\gamma}=\varepsilon\left[\frac{g_*(T_{RH})}{2}\right]\left[\frac{g_*(T)}{g_*(T_{RH})}\right]^{4/3}. \qquad (4)$$

Relic gravitational waves contribute energy density just like any relativistic species; based upon primordial nucleosynthesis we know that any additional relativistic species can contribute no more to the energy density than photons,[3] which implies that $\varepsilon \lesssim 0.5$ [using $g_*(\text{MeV})=10.75$ and $g_*(T_{RH})=300$]. Since the fraction of critical density contributed by photons is $\Omega_\gamma \simeq 10^{-4} h_{1/2}^{-2}$, it follows that $\Omega_{GW} \simeq 4\times 10^{-5} h_{1/2}^{-2}\varepsilon$.

Since the metric perturbations at the locus of bubble collisions are of order unity, we may expect the production of large numbers of black holes. Since the bubble walls have energy of order $M_B \sim M^4/H^3 \simeq m_{Pl}^3/M^2$ when they collide, and the problem is basically geometrical, we would expect mini black holes of this mass to be formed. These black holes will have a Hawking temperature $T_H \sim m_{Pl}^2/M_B \sim M^2/m_{Pl}$, and will evaporate in a time $\tau \sim M_B^3/m_{Pl}^4 \sim m_{Pl}^5/M^6$. If the fraction of the false-vacuum energy that is converted into small black holes is greater than about $(M/m_{Pl})^2$, the energy density of small black holes will come to dominate the energy density of the Universe before they evaporate. In this case, the radiation black holes release when they evaporate will overwhelm the radiation released during reheating by bubble collisions: The entropy (and ultimately the baryon asymmetry) of the Universe is produced by black-hole evaporations. The temperature of the Universe after the mini black holes evaporate and the particles radiated thermalize should be about $T_a \sim M^3/m_{Pl}^2$. (Of course, to ensure that the Universe is radiation-dominated during nucleosynthesis T_a must be greater than about 1 MeV, which restricts M to be greater than about 10^{11} GeV. At somewhat higher values of M there may be effects on the electroweak and quark-hadron phase transitions.)

Graviton production in the evaporation process will be about 10% of that of photons,[12] from which it follows that ρ_{GW}/ρ_γ evolves as

$$\rho_{GW}/\rho_\gamma = 0.1[g_*(T)/g_*(T_a)]^{4/3}; \qquad (5a)$$

this implies that today $\Omega_{GW} \simeq 10^{-6} h_{1/2}^{-2}$, comparable to that produced by bubble collisions. (In this case the gravity waves produced by bubble collisions will be greatly diluted by the entropy produced by mini-black-hole evaporations.) Unlike the other particles radiated as the mini holes evaporate, the gravitons will not thermalize and will have a distribution characterized by the temperature T_H. (However, they do not correspond to a blackbody distribution at this temperature, because their number density is too small by a factor of $T_a^3/T_H^3 \sim M^3/m_{Pl}^3$.) The present wavelength of these gravity waves is very different than those produced by bubble collisions:

$$\lambda \sim T_H^{-1}\left(\frac{T_a}{3\text{ K}}\right) \simeq (10^{-6}\text{ cm})\left(\frac{T_{RH}}{10^{14}\text{ GeV}}\right). \qquad (5b)$$

In the case that the mini black holes contribute only a small fraction of the energy density of the Universe when they evaporate [fraction of false-vacuum energy converted into black holes less than about $(M/m_{Pl})^2$], the gravitational waves radiated are subdominant to those produced by bubble collisions.

Many models of extended inflation are based upon Brans-Dicke-like theories of gravity.[13] In such theories, the "gravitational constant" varies because it is set by the value of a scalar field that evolves with time. If the value of the gravitational constant today is different than that at the epoch of reheating, we must reexamine our previous estimates for graviton production. To begin, write the gravitational part of the action as

$$S=-\int d^4x \frac{\mathcal{R}}{16\pi G}, \qquad (6)$$

where $\mathcal{R}$ is the curvature scalar and G is the effective gravitational constant. When $\mathcal{R}$ is linearized to extract the graviton degrees of freedom, and the metric is specialized to the Robertson-Walker form, the graviton part of the action becomes

$$S=\int d\eta\, d^3x \frac{R^2}{16\pi G}[(\partial_\eta h)^2-(\partial_i h)^2], \qquad (7)$$

where η, defined by $d\eta = dt/R$, is conformal time, and for simplicity the indices on the metric perturbation $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ have been suppressed. The graviton wave equation is

$$\frac{\partial}{\partial\eta}\left(\frac{R^2}{G}\frac{\partial}{\partial\eta}h\right)=\frac{\partial}{\partial x}\left(\frac{R^2}{G}\frac{\partial}{\partial x}h\right). \qquad (8)$$

Provided that the variations in R^2/G are slow, one may find approximate solutions by the method of geometrical optics. They take the form $h \sim a(\eta)\exp[i(kx-\omega t)]$, where in the zeroth approximation k and ω are constant and at next order $a^2 \propto G/R^2$. The comoving energy density ($E \propto R^3 \rho_{GW}$)

$$E=\frac{\delta(\sqrt{g}\mathcal{L})}{\delta(\partial h/\partial t)}\frac{\partial h}{\partial t}-\mathcal{L}$$

is now easily evaluated (note that $\mathcal{L}=0$ at lowest order). One finds that $E \propto 1/R$, as for ordinary radiation, with no G dependence. Since our original estimate of the fraction of the false-vacuum energy that goes into gravitational waves did not depend upon G, our previous results for Ω_{GW} and h_λ are unaffected. (The variation of G *does* affect the amplitude of the gravitons produced as quantum fluctuations, as discussed in Ref. 14.)

Finally, we comment on the detectability of these relic gravitational waves. While the fraction of critical density they contribute is expected to be of order 10^{-5}, their characteristic wavelength depends upon the reheat temperature and whether they were produced by bubble collisions, $\lambda \sim (10^4\ \text{cm})[(10^{14}\,\text{GeV})/T_{\text{RH}}]$, or black-hole evaporations, $\lambda \sim (10^{-6}\ \text{cm})[T_{\text{RH}}/(10^{14}\ \text{GeV})]$. Unfortunately, the "best detector" at present, the millisecond pulsar,[15] is sensitive to much longer wavelengths, $\lambda \sim 10^{19}$ cm. In the future, the most promising detectors appear to be the proposed laser-interferometric gravitational-wave observatory (LIGO) or a beam in space.[16]

To summarize, in extended inflation one expects an additional source of gravitational waves of a characteristic wavelength determined by the reheating temperature and whether the dominant source is bubble collisions or black-hole evaporations, which contribute about 10^{-5} of critical density. The prospects for their detection depend upon their wavelength and therefore the reheat temperature. If detected, their characteristic wavelength would provide a measure of the reheat temperature and a way of distinguishing between inflation and other cosmological phase transitions, e.g., the electroweak or QCD phase transitions.[17] By way of contrast, the gravity waves that arise as de Sitter space quantum fluctuations have a spectrum that extends from about 10^5 to about 10^{28} cm and a much smaller amplitude. Consideration of the gravitational radiation from the "popping" of vacuum bubbles at the end of extended inflation provides a new observational handle on this spectacular moment in the history of the Universe.

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