Experimental Demonstration of Chaotic Scattering of Microwaves

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Reflection of microwaves from a cavity is measured in a frequency domain where the underlying classical chaotic scattering leaves a clear mark on the wave dynamics. We check the hypothesis that the fluctuations of the S matrix can be described in terms of parameters characterizing the chaotic classical scattering. Absorption of energy in the cavity walls is shown to significantly affect the results, and is linked to time-domain properties of the scattering in a general way. We also show that features whose origin is entirely due to wave dynamics (e.g., the enhancement of the Wigner time delay due to time-reversal symmetry) coexist with other features which characterize the underlying classical dynamics.

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In the present Letter we report on the first experimental demonstration of what can be called "chaotic wave scattering," namely, the scattering of waves from systems for which the underlying classical (ray) dynamics is chaotic. Our aim is to show that the fluctuations in the scattered wave function, which were predicted to manifest the underlying classical chaotic dynamics,[1] are indeed observed. A microwave-scattering experiment was chosen for this study since in this kind of experiment one can measure both the amplitude and the phase of the reflected wave, which is not the case in quantum-mechanical measurements. In addition, numerical simulations were performed, and some of the results are presented here. This threefold comparison between experiment, theory, and simulation serves to clarify the link between classical chaotic scattering and its wave-mechanical analog.

Classical chaotic scattering and its wave-dynamics counterpart have been extensively discussed in the literature.[2,3] We shall not attempt to summarize this subject here, but rather discuss its characteristic features, as they appear in the example we study. We consider the scattering of microwaves from an "elbow"-shaped cavity (see Fig. 1) which is fed by a waveguide. A detailed semiclassical analysis of this system will be published elsewhere.[4] From the classical (geometrical optics) point of view, this is nothing but an opened Sinai billiard. It is also a variant of the n-disk-scattering problem[2,5] since the elbow can be considered as a symmetry-reduced four-disk system. The elbow is different, however, in confining the asymptotic dynamics to the waveguide, in contrast to the n-disk problem, where the particles can scatter in any direction. Of prime importance is the classical probability to stay a given time t inside the cavity. For problems of the elbow type it can be shown[6] that this probability can be written as $P(t)=\gamma\exp(-\gamma t)$ (see Fig. 2), where γ is intimately related to the parameters which specify the classical chaotic dynamics. It is important to emphasize that in the system considered here there are relatively few trajectories which stay a short time in the elbow: On average, a trajectory goes through many collisions with the walls before it scatters out (see Fig. 1). For this reason one can neglect the contributions of short paths (or their wave counterparts) in the present analysis.

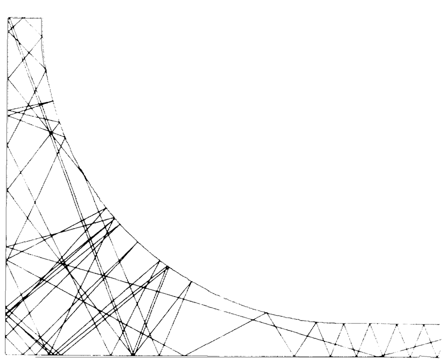

FIG. 1. The elbow system and a typical classical trajectory.

In contrast with classical dynamics, waves can travel in the waveguide only in a discrete and finite set of modes, which correspond to quantized values of the transverse momentum $k_t = n\pi/d$, and $1 \le n \le kd/\pi = L$, where k is the wave number and d the width of the waveguide. The scattering problem is completely determined in terms of the scattering matrix S, which is an $L\times L$ unitary and symmetric matrix. The symmetry is due to the fact that our problem is invariant under time reversal. The probability to scatter from mode n to

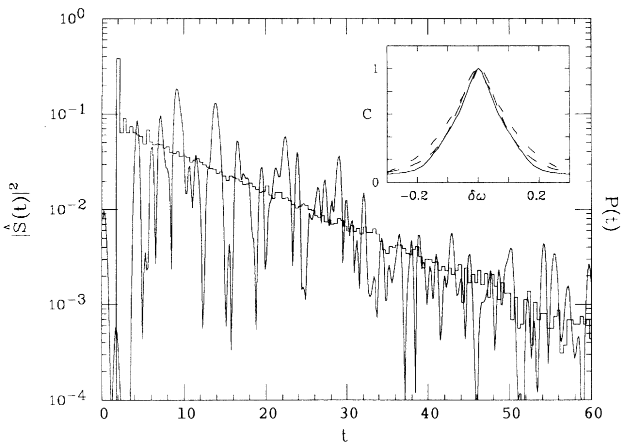

FIG. 2. The time spectrum $|\hat{S}(t)|^2$ and the histogram of the classical staying time distribution. Inset: Measured (solid line), theoretical (dashed line), and simulated (dash-dotted line) absolute value squared of the autocorrelation function of S with respect to ω.

mode m is given by $|S_{nm}|^2$. The extreme case, which is studied here, is when $L=1$. In this case the propagation in the waveguide is fully wave mechanical. However, the scattering of the wave in the cavity can still be described in semiclassical terms, as the size of the cavity is large compared to the wavelength.

The semiclassical theory[1] predicts the following form for the frequency autocorrelation function:

$$C_{ij}(\delta\omega)=\langle S_{ij}^*(\omega+\tfrac{1}{2}\delta\omega)S_{ij}(\omega-\tfrac{1}{2}\delta\omega)\rangle \propto \frac{1}{1-i\,\delta\omega/\gamma}. \tag{1}$$

Thus, the classical parameter γ, defined above, appears as a frequency correlation length in the wave-mechanical description.

The relation of the wave-mechanical behavior to the underlying classical dynamics is best observed in the time domain where the representation of the S matrix is simply the Fourier transform of $S(\omega)$, $\hat{S}(t)$. One can show that $|\hat{S}(t)|^2$ should approach the classical expression $\exp(-\gamma t)$, once the time t is sufficiently long so that the density of the classical trajectories which live in the elbow for this time is large enough.

A second important time-domain quantity is the Wigner time delay,[7,8] defined as

$$T(\omega)=-iS^*(\omega)\frac{\partial S(\omega)}{\partial\omega}=\frac{\partial\Phi(\omega)}{\partial\omega}, \tag{2}$$

where $S=\exp[i\Phi(\omega)]$. For elastic scattering, $T(\omega)$ coincides with the classical delay time in the semiclassical limit. $T(\omega)$ is a wave-mechanical function whose value depends on the proximity and distribution of the poles of S. However, its average value can be calculated semiclassically[4] to be

$$\langle T(\omega)\rangle=(1+1/L)\gamma^{-1}. \tag{3}$$

The term $1/L$ in (3) is caused by coherent interference of time-reversed paths, in systems with time-reversal symmetry.

In the limit $L\gg 1$, (3) reduces to $\langle T\rangle=\gamma^{-1}$, which is just the mean staying time of a classical particle in the scattering area. However, in our case $L=1$, and $\langle T\rangle=2\gamma^{-1}$, twice the classical value. This wave-mechanical enhancement of the staying time is directly observable in experiments such as the one described here.

The experimental setup in which we tested the above predictions consists of an elbow-shaped cavity made of polished brass, whose straight wall is $\sim$50 cm long. The openings of the cavity have a rectangular cross section which is 47 mm wide and 22 mm high. One of the ports is closed by a brass plate, while the other is connected via a linear taper to a standard waveguide-to-coax adapter, which is in turn connected to an automatic network analyzer (model HP 8510B). The network analyzer can completely separate the transmitted from the reflected wave, thus enabling precise measurement of the reflection coefficient over a wide range of frequencies.

In the present experiment the frequency range was 6–8 GHz, scanned in 2.5-MHz steps (giving a total of 801 data points). In this frequency range either one or two modes can propagate in the waveguide. However, since the waveguide is connected to the coax via a single symmetrically placed pin, the antenna can only transmit and receive one mode, while the second mode is reflected back into the cavity. Thus, the S matrix is inherently 1×1 over the whole frequency range. Since the system is essentially invariant with respect to scaling of the length and frequency, we will normalize the length of the cavity and the speed of light to 1. In these units the value of γ, evaluated *classically*, is 0.1.

Figure 2 shows the power spectrum of the measured complex reflection coefficient. The small peak at $t=0$ is the result of reflection at the coax-waveguide interface, and its small size ensures that the matching was good. Superimposed is the classical staying-time distribution $P(t)$ for this system. We can see that both the time spectrum and the classical staying-time probability indeed decrease exponentially on average, with the same slope γ. This compares favorably with the semiclassical prediction.

The autocorrelation (1) was calculated from the data, and its absolute value squared is shown in the inset of Fig. 2, together with the semiclassical result from (1) and the results of a numerical simulation. One can see that there is good agreement, and, in particular, the correlation length of $\gamma=0.1$ is accurately reproduced.

Since the S matrix is 1×1, one would expect the amplitude of the reflection coefficient to be identically 1. However, this is not so, as can be seen from the inset in

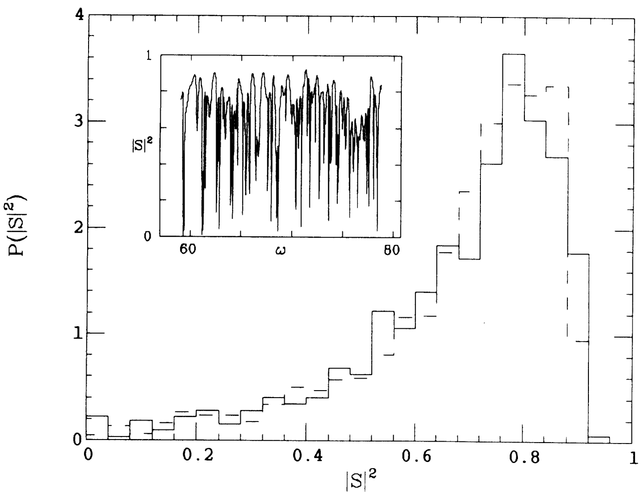

FIG. 3. Distribution of the measured (solid line) and simulated (dashed line) value of $|S|^2$. Inset: Measured reflected power $|S|^2$ as a function of frequency.

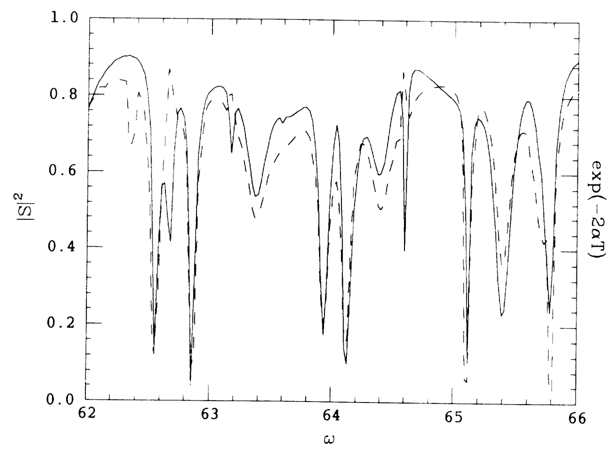

FIG. 4. Measured reflected power $|S|^2$ (solid line) and the prediction of (4) $|S|^2 = \exp(-2\alpha T)$ (dashed line) as a function of frequency.

Fig. 3, where the reflected power $|S|^2$ as a function of frequency is shown. One observes an average value of 0.7, with sharp deviations. This behavior is due to absorption of energy because of the finite conductivity of the cavity walls. This effect can be modeled by adding a small imaginary term α to the frequency. In Fig. 3 we compare the experimentally observed distribution of $|S|^2$ (solid line) with that of a numerical simulation (dashed line), taking $\alpha = 0.013$. The simulation reproduces all the features of the measured distribution within the experimental accuracy. Note that the absorption taken is quite small (a factor of e^{-1} for every 38 m), but still has dramatic consequences for the distribution of $|S|^2$. The smallness of the ratio α/γ (=0.13) explains why $|\hat{S}(t)|^2$ and $C(\delta\omega)$ are only marginally affected by absorption.

We can understand the dependence of $|S(\omega)|^2$ on ω by making use of the analyticity of $S(\omega)$ [and of $\Phi(\omega)$] in a strip around the real axis.[9] Because of unitarity, $\Phi(\omega)$ is real on the real axis, and

$$S(\omega + i\alpha) \approx S(\omega) e^{-\alpha \partial\Phi/\partial\omega} = S(\omega) e^{-\alpha T(\omega)}. \tag{4}$$

Thus, the inclusion of absorption affects S as if the loss mechanism acts during the time $T(\omega)$. However, as was shown before, T is not the classical time delay, and is subject to wave-mechanical corrections, most notably the factor-of-2 enhancement due to time-reversal symmetry. Only in the many-mode limit is this enhancement negligible, and we regain the classical behavior.

The analysis presented above was tested by comparing the measured reflected power $|S|^2$ with $\exp[-2\alpha T(\omega)]$, where $T(\omega)$ was obtained from the *measured* phase $\Phi(\omega)$ by numerical differentiation according to (2). The results are shown in Fig. 4. Only a limited frequency range is shown so that details can be seen. One can see that there is great similarity between the two quantities. Differences arise near sharp dips because the numerical differentiation is not accurate there. However, the similarity extends over the whole frequency range, and the correlation between the two quantities is ~0.8. An additional result is the average time delay, which was calculated from the experimental data to be ~19, in good agreement with the predicted value $2\gamma^{-1} \approx 20$.

This work demonstrates the utility of microwave experiments in the study of quantum chaotic scattering. In a way, it complements the work on spectral fluctuations of eigenfrequencies of chaotic billiard systems described in Ref. 10. Our results show that fluctuations of the S matrix for chaotic wave scattering can be described in terms of semiclassical arguments, based on the chaotic nature of the underlying classical dynamics. The effect of absorption in chaotic scattering was also studied, and the total energy loss was shown to be linked to the Wigner time delay. A predicted wave-mechanical enhancement of the classical time delay as a result of time-reversal symmetry was also confirmed here. The semiclassical considerations leading to the results demonstrated in this Letter can be extended to the fluctuating behavior of the S matrix under a change of geometry, magnetic field, or other parameters.[4,11] Such effects are relevant to experiments on ballistic transport of charge carriers in mesoscopic systems. Recent experiments[12,13] and theoretical work[14-16] on the variation of the magnetoconductance of miniature junctions have shown that the gross features observed are due to classical effects. The fluctuations described here occur on a finer scale (5–30 mT), and further experiments at this field resolution are called for. When this work was in its final stages of preparation we received a paper[11] where numerical simulations indeed show these fluctuations and their relation to the underlying chaotic scattering. A prediction not tested here was the connection between the statistics of the S matrix and random-matrix theory.[17] A study of this link necessitates measuring S matrices of dimensions larger than 1, and work to this end is in progress now.

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