Pseudospin Symmetry and Quantized Alignment in Nuclei

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Nine superdeformed bands in several nuclei of the mass-190 region all have transitions of the same or equivalent energies to within an average of about 1 keV—almost identical. Furthermore, it is found that transitions of the same energy do not always connect levels of the same spin, indicating noncollective angular momentum alignments that are all very nearly integers (0, $1\hbar$, or $2\hbar$). Possible explanations involving pseudospin are discussed.

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Only occasionally in any area of science is something really unexpected found, but that seems to be the case in nuclear-structure physics just now. Rotational bands, or regions of bands, in *different* superdeformed nuclei have been found that have identical (or equivalent—i.e., identical averages of) transition energies to within an average of about 1 keV—much more similar than expected. Since the transition energy (these are $E2$ transitions with $\Delta I=2$) is very nearly twice the rotational frequency (dE/dI), this means that the rotational frequencies of the two bands are very similar and also implies that the (dynamical) moments of inertia are almost equal. This surprising similarity was announced at the recent Copenhagen Workshop and Symposium on the Nucleus at High Spins. It was already noticed for two superdeformed bands in the mass-190 region, but thought to be accidental, and later was announced[1] for two pairs of bands in the mass-150 region.

There have been three approaches to understanding these nearly identical transition energies. In the mass-150 region, Nazarewicz *et al.*[2] have concentrated on the properties of an $\Omega=\frac{1}{2}$ band, $[301]\frac{1}{2}$, pointing out that when the decoupling parameter is exactly ± 1 the levels in such a band coincide with those of the core, and proposing that a doubly closed shell at ^{152}Dy ($N=86$, $Z=66$) renders the superdeformed core sufficiently stable so that the addition or subtraction of a particle has a very small effect on its properties. They pointed out that pseudospin symmetry would lead to a decoupling parameter close to ± 1. Ragnarsson[3] took a direct calculational approach, working with a pure harmonic-oscillator model, since the required accuracy cannot be attained with more realistic cranked shell models. He found that in such a model there are, indeed, orbitals (including the above-mentioned $\Omega=\frac{1}{2}$ orbital) where the addition or subtraction of a particle has about as small an effect on the transition energies as is observed experimentally. In the mass-190 region the spins of the superdeformed band members can be reasonably reliably determined, and Stephens *et al.*[4] have used the near-integer aligned spin of one band relative to another (1.00 ± 0.04) as evidence for (quantized) pseudospin alignment and therefore approximate pseudospin symmetry. They have proposed that the lack of any other change in the rotational properties upon adding or subtracting nucleons may be due to compensating deformation and alignment effects, such as occur in harmonic-oscillator models that have SU(3) symmetry.

These approaches have a reasonable overlap. It is easy to show that the aligned spin i is related to the decoupling parameter a in an $\Omega=\frac{1}{2}$ band: $2i=(-1)^{I-1/2}a$. Thus decoupling parameters of ± 1 correspond to alignments of $\frac{1}{2}$, the expected value for the alignment of the pseudo (intrinsic) spin, where the $\pm$ sign is determined by the signature (related to the spin) as indicated in the above expression. Such alignment[5] arises due to the mixing of states having the intrinsic spin coupled parallel to the orbital angular momentum with states having antiparallel coupling (e.g., $f_{7/2}$ and $h_{9/2}$). The mixed states ("pseudo" $g_{7/2,9/2}$ in this example) have a weak residual coupling of the "pseudo" intrinsic spin (pseudospin) to the "pseudo" orbital angular momentum so that the pseudospins ($\frac{1}{2}\hbar$) can be more easily aligned by the Coriolis force; however, this is a general property of the normal-parity states, and not limited only to $\Omega=\frac{1}{2}$ bands. If the appropriate pairs of states were degenerate, the mixing would be complete (the pseudospin symmetry exact) and, while precise degeneracy does not occur, such pairs of states do tend to lie close together in energy due to the systematic compensation of spin-orbit energy shifts with those due to anharmonicity (flattening) in realistic oscillator potentials.

Such pseudospin symmetry would appear to be more realistic for nuclei than SU(3) symmetry, which is exact only for a complete, isolated shell in a deformed (but otherwise pure) harmonic-oscillator potential. In nuclei, the largest deviations from the pure harmonic oscillator are the spin-orbit interaction ($\mathbf{l}\cdot\mathbf{s}$) and the anharmonicity of the potential (e.g., an l^2 term). The main effect of these is to separate the intruder orbital (largest l with parallel s) from the rest of the shell. Thus a better approximation for heavier nuclei is to exclude the intruder and consider the remainder of the shell as a pseudo $(N-1)$ shell, but even this pseudo SU(3) symmetry is rather strongly broken by the $\mathbf{l}\cdot\mathbf{s}$ and l^2 (or equivalent) terms. The pseudospin symmetry is the part of these higher symmetries that most nearly survives these anharmonicities, but there could be other aspects of (pseudo) SU(3) symmetry present in these superdeformed bands. Ragnarsson's calculation, based on the pure harmonic-oscillator model, will have nearly the full SU(3) symmetry (which includes the degeneracies characteristic of pseudospin symmetry), and is thus related to the other approaches discussed.

This Letter will consider the many new bands found recently in the mass-190 region [^{190}Hg,[6] ^{191}Hg,[7,8] ^{192}Hg,[9,10] ^{193}Hg,[11] ^{194}Hg,[12,13] ^{194}Tl,[14] ^{194}Pb,[15,16] and ^{196}Pb (Ref. 15)]. Many of these superdeformed bands are related to the band in ^{192}Hg and the properties of this family seem to be highly relevant to the above ideas. It is not surprising that some bands in this region are not members of the ^{192}Hg family. A number of different orbitals will be available to the nucleons and certainly some of these will not have the special properties required to produce the very similar transition energies. Indeed, it is surprising that out of the fourteen known bands in this region (including excited bands and counting each signature as a separate band), nine *do* belong to the ^{192}Hg family. This is a much larger family than any found so far in the mass-150 region.

The superdeformed band in ^{192}Hg seems to be central to these nine bands, and we will use it as the reference band. This nucleus may represent a double-closed shell for superdeformed shapes, or its central position may be only apparent. While the zero-alignment position does depend on which of these bands is taken as reference, the main conclusions do not. To look for systematic effects, we will compare each transition in all the remaining eight bands with the transitions in ^{192}Hg. Rather than use the method of Ref. 4, which requires numerical fits to the reference band, we will use a simple linear interpolation method, which is quite good for these nuclei, and add the measured spin difference when known: $i=\Delta I+\Delta i$, where i is the alignment, ΔI is the difference between the angular momenta (e.g., between the initial spins) associated with the transitions, and Δi is the "incremental alignment" associated with the difference in transition energies. The incremental alignment is obtained by subtracting the transition energy in question from the closest transition energy in the ^{192}Hg band (ΔE_γ) and dividing the result by the energy difference between the closest two transitions in ^{192}Hg (ΔE_γ^0). In this linear interpolation procedure, all values will fall between ± 0.5 and the nearly identical transition energies mentioned initially give values very close to zero. Since successive transitions in ^{192}Hg differ from each other by two spin units, 2 times the above ratio gives the incremental alignment ($\Delta i = 2\Delta E_\gamma/\Delta E_\gamma^0$).

The incremental alignment has the important advantage that it does not require knowledge of the spins of the states. This quantity is plotted against rotational frequency for all eight bands in Fig. 1. We have also included the excited superdeformed bands, ^{153}Dy* (Ref. 17) and ^{151}Tb*,[1] referred to the band in ^{152}Dy.[18] (For the other known pair in the mass-150 region, namely, ^{151}Tb and ^{150}Gd*, it is not clear which to use as reference.) It is apparent that the data cluster around the lines drawn at ± 1 (these are identical), ± 0.5, and 0, although almost all the values for bands in the mass-190 region decrease below frequencies of about 0.2 MeV, and the values for one ^{191}Hg* band decrease at the highest frequencies. Since aligned angular momentum is not expected to be quantized, this is a very puzzling result.

The solid and dashed lines in Fig. 1 have a special significance. Alignments are projections and therefore add algebraically (rather than vectorially). In an exact pseudospin scheme, each odd nucleon in a natural-parity orbital will contribute an alignment of $\pm\frac{1}{2}$, so that, insofar as only natural-parity orbitals are involved, any odd-mass nucleus would have half-integer ($\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots$) alignment and even-mass nuclei would have integer alignments. We will call these "natural" alignments. (Unnatural alignments can occur, even in an exact pseudospin scheme, by occupying both natural-parity and in-

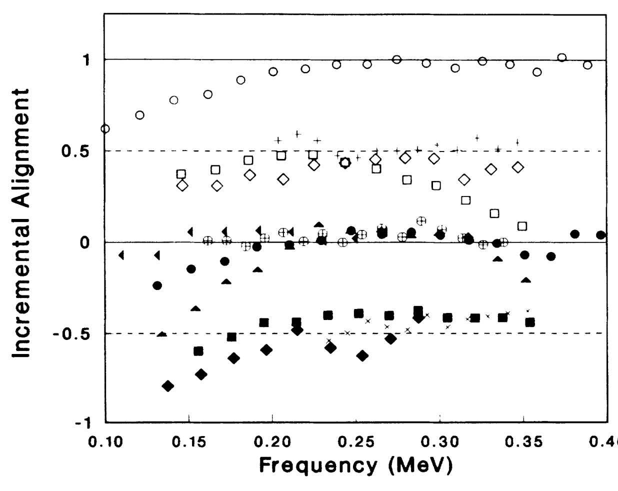

FIG. 1. Incremental aligned angular momentum (see text) as a function of rotational frequency for eight superdeformed bands [nucleus (signature)] relative to the band in ^{192}Hg: □, ^{191}Hg(+); ■, ^{191}Hg(−); ◇, ^{193}Hg(+); ◆, ^{193}Hg(−); ●, ^{194}Hg(1); ○, ^{194}Hg(0); ◀, ^{194}Tl; and ▲, ^{194}Pb. Incremental alignments for three bands in the mass-150 region relative to the band in ^{152}Dy are included: ⊕, ^{151}Tb; +, ^{153}Dy(+); and × ^{153}Dy(−). For the mass-150 region the abscissa scale should be doubled.

truder states. Intruder states have no pseudospin partner states, and thus the intrinsic spins remain strongly coupled to the orbital angular momentum. Low-Ω intruder states usually produce large changes in the moment of inertia that destroy the close similarities that we are exploring, but higher-Ω intruder states can have very small effects on the moments of inertia, as well as nearly zero alignment.) It is easy to see that the solid lines in Fig. 1 always correspond to natural alignments and the dashed lines to unnatural alignments, provided the reference nuclei, ^{192}Hg and ^{152}Dy, have natural alignment. [For example, in an odd-mass nucleus with an even-even reference nucleus, ΔI is half integer so that when i is natural (also half integer) Δi must be integer (solid lines); a similar argument holds for even-mass nuclei.] It turns out that all the odd-mass nuclei in Fig. 1 lie on dashed lines except for ^{151}Tb*, and all the even-mass nuclei lie on solid lines, so that all the alignments, except that for ^{151}Tb*, must be very close to integers. (The situation for the other case in the mass-150 region is that both nuclei have either natural or unnatural alignment, the former being consistent with previous assignments involving the above-mentioned $\Omega = \frac{1}{2}$ orbital.) This result is completely independent of any spin determinations, but can be extended in the mass-190 region where the spins can be rather reliably determined.

We can use the spins to get the total alignments relative to ^{192}Hg, as discussed above. The results are shown in Fig. 2. It is impressive that after a common initial rise the alignments all reach integer values. The initial rise represents the onset of the alignment with rotational frequency and is correspondingly larger for the ^{194}Tl case where the alignment is twice as large and absent for the ^{194}Pb case where the alignment is zero. These differences in the rise are not due to the spin assignments, as they are already clearly visible in Fig. 1. This increase in alignment must be a Coriolis effect, but it is not clear whether it is due to a decrease in the pairing correlations or to a direct Coriolis alignment or some combination of these. In any case the integer alignments are established only above frequencies of about 0.2 MeV. At higher frequencies, the decrease in alignment for the $+\frac{1}{2}$ signature band of ^{191}Hg is the only obvious deviation from integer alignments in these bands, and it has been associated[8] with the [642]$\frac{3}{2}$ orbital, which is a member of a pseudospin doublet but also has some signature splitting.

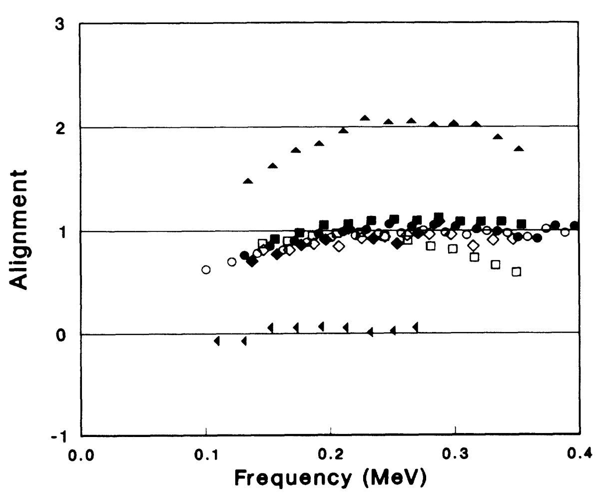

FIG. 2. Total aligned angular momentum as a function of rotational frequency for the eight bands in the mass-190 region relative to ^{192}Hg. The symbols represent the same nuclei as in Fig. 1.

In general, the results in Figs. 1 or 2 support the occurrence of pseudospin alignment. The fact that many bands are found to have very nearly integer alignments (and one or two very nearly half-integer alignments) strongly suggests that alignment is quantized in a way that could occur naturally if that alignment were composed of fully aligned pseudo (intrinsic) spins (i.e., units of $\frac{1}{2}\hbar$). Such a quantization of aligned spin is not expected in any other reasonably simple scheme we have thought of. Futhermore, the details of the mass-150 region can be understood reasonably easily in this picture, though no spins are known. The natural half-integer alignment in ^{151}Tb* is that expected if the proton hole (in the ^{152}Dy core) is in a pseudospin orbital (e.g., the $\Omega = \frac{1}{2}$ band discussed[2]) and the integer alignment of the bands in ^{153}Dy*, though unnatural, is consistent with the value of zero expected for the high-Ω intruder orbit [514]$\frac{9}{2}$.

However, the details of the mass-190 region are very puzzling. Figure 2 shows that all the alignments currently known in this region are integer. The ^{193}Hg bands, with one neutron outside the ^{192}Hg core, have alignment 1, not $\frac{1}{2}$ as might be expected, and the same is true for the ^{191}Hg* bands with one neutron hole. Even if our spin determinations are wrong for some unknown reason, Fig. 1 already shows these bands have unnatural (i.e., integer) alignments. The spin assignments only indicate that the alignment in these bands is 1 rather than some other integer. Only if the mass assignments are wrong would this conclusion be changed, and that seems unlikely. It is, of course, possible to generate such alignments in an exact pseudospin picture (two particles in aligned orbitals together with one in an alignment-zero intruder orbital), but these are not the simplest assignments. The data in Fig. 2 seem to suggest that the alignment quantum is often 1 rather than $\frac{1}{2}$ and this is an interesting result that we would like to consider briefly.

An alignment of 1 can arise by putting two particles in a doubly degenerate aligned-pseudospin orbital. In a system having reasonably good pseudospin, there would be many such orbitals, and mixing all the resulting alignment-1 states might generate a low-lying collective state having alignment 1 (e.g., ^{194}Hg* and possibly even

^{153}Dy*). Coupling a single alignment-0 (high-Ω intruder) particle to this state would give alignment-1 bands (^{191}Hg* and ^{193}Hg); whereas, coupling an alignment-$\frac{1}{2}$ particle would give the expected half-integer-alignment bands (^{151}Tb*). This would be a completely new type of collective excitation in nuclei, but requires estimates of the mixing to find out if it is feasible.

To summarize, it appears that for many, but not all, of these superdeformed rotational bands the alignments tend to be quantized in units of $\frac{1}{2}\hbar$ or $1\hbar$. Nothing except pseudospin alignment has been thought of to account for this quantization. Further, it appears that in the mass-190 region the aligned quantum may often be $1\hbar$ rather than the expected $\frac{1}{2}\hbar$, suggesting pairs of pseudospin-aligned particles might be involved. This raises the interesting question of whether low-lying collective states having alignment 1 would occur in a nucleus with rather good pseudospin symmetry. This is an interesting possibility, but even if correct, it does not answer all the questions about these superdeformed bands. We still need to understand why the pseudospin symmetry is so good, and why other effects, like deformation changes, orbital alignment changes, pairing changes, and/or size and shape changes, do not affect the energy levels and mask the pseudospin effects that seem to be there. In our previous Letter[4] we suggested that this aspect may involve systematic cancellations that occur in the contributions to the moment of inertia for systems having (remnants of) SU(3) symmetry. This still seems likely to us.

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