New Look at α Decay of Heavy Nuclei

B. Buck
Department of Theoretical Physics, Oxford University, 1 Keble Road, Oxford OX1 3NP, United Kingdom

A. C. Merchant
Instituto de Estudos Avançados, Centro Técnico Aeroespacial, 12.225 São José dos Campos, São Paulo, Brazil

S. M. Perez
Physics Department, University of Cape Town, Private Bag, Rondebosch 7700, South Africa
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Geiger-Nuttall plots of the accurate modern data on partial half-lives for α decay yield very striking linear correlations. The plots for isotopic sequences which cross the neutron magic number $N=126$ show clearly the presence of different linear relations for $N<126$ and $N>126$. We indicate that this observation and all other data on ground-state to ground-state α decays for even-even nuclei with $76 \leq Z \leq 100$ may be accounted for very well by a simple model with fixed parameters. An important ingredient in the model is the proposal that preformed α particles in the parent nuclei move in orbits with large values of a global quantum number.

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A striking feature of α radioactivity is the occurrence[1] of linear correlations involving the decay half-lives $T_{1/2}$ and the corresponding Q values:

$$\log_{10} T_{1/2} = aQ^{-1/2} + b , \tag{1}$$

which are traditionally called Geiger-Nuttall plots. In particular, this relation was found[2] to hold well for the ground-state to ground-state decays of even-even nuclei having fixed proton number Z and varying neutron number N, with $N>126$. Quite spectacular correlations result from the use of the recent accumulation of accurate decay data,[3] including some cases for which the group of isotopes has $N<126$. The isotopic sequences $^{220}_{90}$Th, $\ldots$, $^{232}_{90}$Th and $^{172}_{78}$Pt, $\ldots$, $^{190}_{78}$Pt provide good examples.

Of special interest are instances where the groups straddle the neutron magic number $N=126$, as for the sequence $^{192}_{84}$Po, $\ldots$, $^{218}_{84}$Po. Here we find that the data separate into two groups with $N<126$ and $N>126$, respectively, each group having its own linear correlation of the above form. We interpret this as evidence for a cluster structure of the system in which the α particle can move in two distinct orbits about the core. These orbits have different values of the global quantum number $G=2n+L$, where n is the number of nodes in the wave function and L is the orbital angular momentum. The value of G should be chosen so that all the nucleons in the α particle occupy states immediately above the Fermi surface of the daughter nucleus. The G should remain constant while a major shell of the daughter nucleus is being filled, and increase sharply at the shell closure. Although the combination $2n+L$ is, strictly speaking, characteristic of the oscillator potential, it is also a good guide for assigning quantum numbers associated with other nuclear potential shapes since the resulting nucleon shell structures are similarly arranged.

Table I shows the results of a statistical fit of Eq. (1) to the α-decay data[3] for the $_{78}$Pt, $_{84}$Po, and $_{90}$Th isotopes. A complete account of all the data for ground-state to ground-state decays of heavy even-even nuclei with $76 \leq Z \leq 100$ will be given elsewhere.[4] We have analyzed these correlations using a simple α-particle-core potential of the square-well + (surface-charge) Coulomb form:

$$V = -V_N + C/R \ (r<R), \quad V = C/r \ (r>R) , \tag{2}$$

where the product of charges $C=2(Z-2)e^2$. The potential depth V_N and radius R are related through a Bohr-Sommerfeld condition, i.e., for an $L=0$ α particle with global quantum number G and separation energy Q, the result is

$$R = \frac{\pi}{2}(G+1)\left[\frac{2\mu}{\hbar^2}\left(Q+V_N-\frac{C}{R}\right)\right]^{-1/2} , \tag{3}$$

with μ the reduced mass.

The novel idea is to use Eq. (3) to find R from the en-

TABLE I. The values of the slope a, intercept b, and correlation coefficient c obtained from a statistical fit of Eq. (1) to α-decay data. I is the number of isotopes used in each fit.

Element	I	a	b	c
$_{78}$Pt [a]	10	126.59 ± 0.90	-50.92 ± 0.39	0.99979
$_{84}$Po [a]	8	136.49 ± 1.38	-52.01 ± 0.59	0.99970
$_{84}$Po [b]	3	128.52 ± 0.35	-49.71 ± 0.12	0.99999
$_{90}$Th [b]	7	140.80 ± 0.44	-51.97 ± 0.18	0.99998

[a]Neutron number $N<126$.
[b]Neutron number $N>126$.

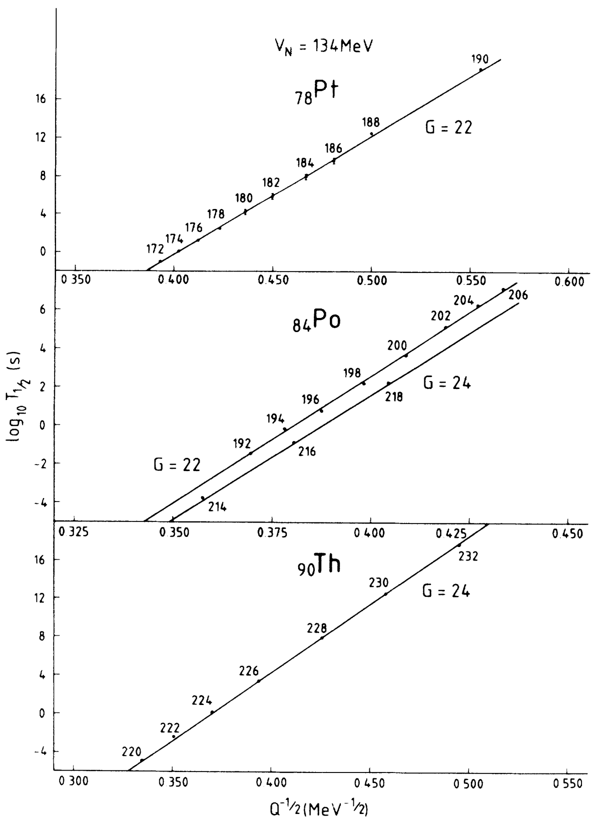

FIG. 1. Model fits to α-decay half-lives $T_{1/2}$ from Eq. (7) with parameter values from Eq. (8). Experimental data are from Ref. 3, and references therein.

suing quadratic equation for each decay. The potential depth V_N and the two values G_1 and G_2 of the global quantum number (corresponding to $N<126$ and $N>126$, respectively) remain fixed throughout. Thus R is simply related to Q and C.

The radius R is a crucial ingredient in the expression for the decay width Γ, given in semiclassical approximation[5] by

$$\Gamma=\frac{P\hbar^2K}{2\mu R}\exp\left(-2\int_R^{C/Q}k(r)dr\right), \qquad (4)$$

where P is the formation probability, and K and $k(r)$ are the wave numbers in the internal and barrier regions, respectively,

$$K=\left[\frac{2\mu}{\hbar^2}\left(Q+V_N-\frac{C}{R}\right)\right]^{1/2}, \qquad (5)$$

$$k(r)=\left[\frac{2\mu}{\hbar^2}\left(\frac{C}{r}-Q\right)\right]^{1/2}. \qquad (6)$$

The decay half-life is thus given by

$$T_{1/2}=\frac{\hbar\ln 2}{\Gamma}=P^{-1}2\ln 2\left(\frac{\mu R}{\hbar K}\right)\exp\left(2\int_R^{C/Q}k(r)dr\right). \qquad (7)$$

For each decay the separation energy Q and the product of charges C are known. Thus, once the formation probability P, the potential depth V_N, and the global quantum number G have been set, the corresponding radius R and half-life $T_{1/2}$ can be determined. There are strongly correlated ambiguities in the values of P, V_N, and G, and, in particular, the formation probability P is, in itself, poorly determined. We thus limit the number of parameters at our disposal by setting $P=1$ throughout, in common with similar analyses.[1] A value $G\sim 22$ is suggested by the Wildermuth condition for this mass region,[6] and $\Delta G=(G_2-G_1)=2$ is the smallest change compatible with angular momentum and parity conservation for the 0^+ to 0^+ decays considered here. Using the parameter values

$$P=1,\quad V_N=134\text{ MeV},\quad G_1=22,\quad G_2=24, \qquad (8)$$

all the partial half-lives for the ground-state to ground-state α decays of even-even nuclei with $76\le Z\le 100$ are fitted to within a factor of ~ 2. Typical model fits for the $_{78}$Pt, $_{84}$Po, and $_{90}$Th isotopes are shown in Fig. 1.

Most of the decay data fall into two groups with either $N\le 126$ and $Z\le 82$, or with $N>126$ and $Z>82$. Our model fits require $G_1=22$ for the former case, and $G_2=24$ for the latter. This does not determine whether the discontinuity in G is a consequence of the neutron shell closure at $N=126$, or of the proton shell closure at $Z=82$, or of a combination of these. The remaining data (with $Z>82$ and $N\le 126$) provide some evidence that the proton shell closure by itself is relatively unimportant. No effect of this closure is seen in Fig. 1 for those isotopes of $_{84}$Po which have $N\le 126$, and the decay data are very well fitted with $G=22$. A small effect of this closure could, however, explain the results of Table II for the isotopes of $_{86}$Rn and $_{88}$Ra with $N\le 126$. For these cases the model with $G=22$ consistently overestimates the experimental values of $\log_{10}T_{1/2}$ by $\Delta(\log_{10}T_{1/2})\sim 0.3$. Remarkably, out of a total of 89 decays considered in this analysis, only 10 have $|\Delta(\log_{10}T_{1/2})|\ge 0.25$, and 7 of these are to be found in Table II.

TABLE II. Deviations $\Delta(\log_{10}T_{1/2})=\log_{10}(T_{1/2}^{\text{calc}}/T_{1/2}^{\text{expt}})$ between the calculated and experimental half-lives $T_{1/2}$ for α decay of $_{86}$Rn and $_{88}$Ra isotopes with neutron number $N<126$.

Nucleus	N	$\Delta(\log_{10}T_{1/2})$
$_{86}$Rn	114	0.39 ± 0.01
$_{86}$Rn	116	0.34 ± 0.03
$_{86}$Rn	118	0.25 ± 0.03
$_{86}$Rn	120	0.25 ± 0.02
$_{86}$Rn	122	0.09 ± 0.05
$_{88}$Ra	118	0.39 ± 0.22
$_{88}$Ra	120	0.28 ± 0.12
$_{88}$Ra	122	0.26 ± 0.02

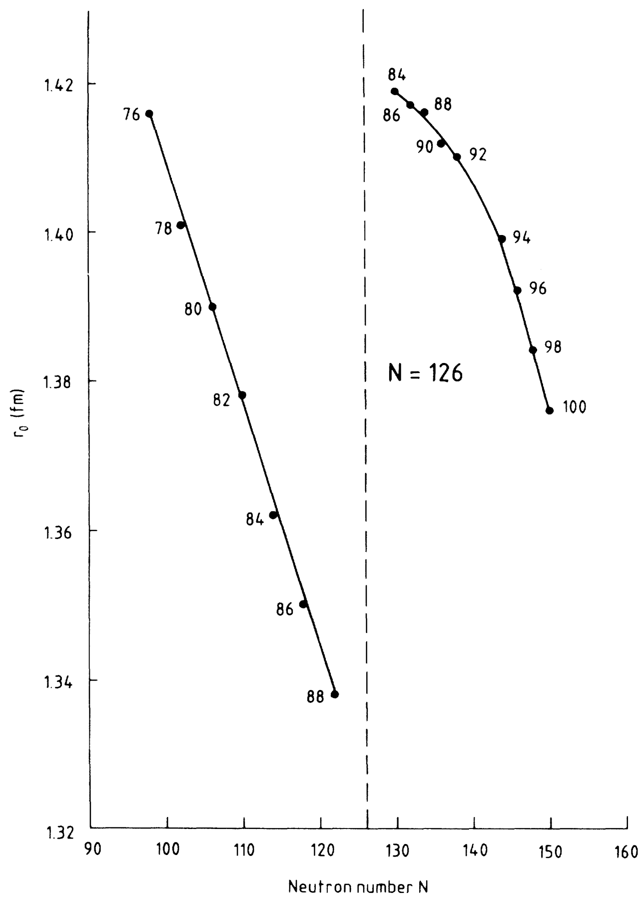

FIG. 2. Reduced radius $r_0 = R/A^{1/3}$ for some typical even-even nuclei with R from Eq. (3) and parameter values from Eq. (8). Proton numbers Z are indicated in the figure, and the lines serve to guide the eye.

This excellent agreement between the theoretical and experimental half-lives implies that, for $P=1$, the effective square-well radius R is singularly well determined by Eq. (3), with parameter values from Eq. (8). In Fig. 2 we show the behavior of the corresponding reduced radius $r_0 = R/A^{1/3}$ for various elements as a function of neutron number N. Evidently r_0 is far from constant, decreasing with increasing mass number A for both $N<126$ and $N>126$, and having a discontinuity at $N=126$. Of course, similar features are also found in other analyses[1,7] in which the individual values of R are fitted *directly* to the individual values of $T_{1/2}$ for each decay. In those analyses the assumption of a nodeless ($G=0$) wave function for the α cluster results in varying (and unphysically small) values of V_N.[7] In contrast, our model provides the underlying principles which, through Eqs. (3) and (8), generate the required features of R in a natural way.

In conclusion, we have shown that an α-particle–core potential of square-well form, with fixed depth, and radius given by the Bohr-Sommerfeld condition, reproduces all the α-decay half-lives for heavy even-even nuclei to within a factor of ~ 2. We find that the α particle has to occupy orbits with different global quantum numbers for $N<126$ and $N>126$. Excellent agreement with the data can also be obtained in a similar, but less transparent, analysis[4] based on a more realistic α-particle–core potential of the diffuse-well type. We interpret these results as strong evidence of cluster structure in these nuclei.

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