Two-Component Approach to the "Proton Spin" Puzzle in Generalized Skyrme Models

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We demonstrate that a cancellation mechanism between quark and gluon contributions to the axial-vector singlet form factor is operative in a generalized Skyrme model. The calculation of the nonelectromagnetic part of the neutron-proton mass difference plays a crucial role in this analysis.

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In this Letter we will show that the cancellation mechanism between quark and gluon components proposed in the QCD parton approach[1] to the "proton spin" puzzle[2-4] can be to some extent explicitly verified in the framework of generalized Skyrme-type models. To set notation, let the form factors of the ath quark axial-vector current be defined by

$$\frac{(p_0 p_0')^{1/2}}{m}\langle p(\mathbf{p}')|i\bar{q}_a\gamma_\mu\gamma_5 q_a|p(\mathbf{p})\rangle = i\bar{u}(\mathbf{p}')\left[\gamma_\mu\gamma_5 H_a(q^2)+\frac{iq_\mu}{2m}\gamma_5\tilde{H}_a(q^2)\right]u(\mathbf{p}), \quad (1)$$

where m is the nucleon mass and $q_\mu = p_\mu - p_\mu'$. The U(1) axial-vector current J_μ^5 is defined by $J_\mu^5 = i\sum_a \bar{q}_a\gamma_\mu\gamma_5 q_a$ and its corresponding form factor of interest is $H(q^2) \equiv H_1+H_2+H_3$. The European Muon Collaboration (EMC) experiment[2] "measured"[3] the linear combination $4H_1(0)+H_2(0)+H_3(0)$ while neutron β decay yields $1.25 = H_1(0)-H_2(0)$. Knowing a third linear combination $R \equiv H_1(0)+H_2(0)-2H_3(0)$ enables one to disentangle all three $H_a(0)$. Relating R to the hyperon β decays using SU(3) (Ref. 5) and the Cabibbo theory resulted in the unexpected conclusion[3] that $H(0)$ is close to zero. On the other hand, the naive quark model suggests that $H(0)$ should be unity, since it appears to be given by twice the quark-spin contribution to the total angular momentum of the proton. It is considered[1] that the solution to this apparent paradox is related to the gluonic anomaly G in the divergence of J_μ^5:

$$\partial_\mu J_\mu^5 = 2i\sum_a m_a\bar{q}_a\gamma_5 q_a + G\,, \quad (2)$$

m_a being the mass of quark a. Similarly, in the QCD parton approach the matrix element of J_μ^5 is separated into a quark piece Δq and a remaining gluonic piece as follows:

$$H(0) = \Delta q - 3\Delta\Gamma\,. \quad (3)$$

It is expected that $\Delta q \sim 1$ and that the first term should be canceled by the gluonic contribution $-3\Delta\Gamma$. It is, however, not easy to verify this feature directly from QCD, and serious difficulties encountered by attempts to explain the EMC data along the lines of Ref. 1 are discussed in Ref. 4. Interesting attempts to overcome these difficulties were made by many workers[6] using (2). They used essentially pole saturation techniques and attempted to identify the first term on the right-hand side of (2) with the first term on the right-hand side of (3), etc. However, it was found[6] that peculiar results emerged reflecting the large isospin violations of the light-quark masses appearing in the pseudoscalar density terms $m_a\bar{q}_a\gamma_5 q_a$. More recently, Shore and Veneziano,[7] using Ward identities for the relevant two- and three-point functions defined according to a special prescription, were able to derive a sensible two-component decomposition along the lines of the Goldberger-Treiman-type relations discussed in Ref. 6. Using pole saturation their result is

$$H(0) = (\sqrt{3}F_\pi/2m)(g_{\eta' NN} - \sqrt{3}F_\pi m_{\eta'}^2 g_{GNN})\,, \quad (4)$$

where corresponding terms in (3) and (4) are to be identified. g_{GNN} is an effective coupling constant[8] for the composite anomaly field G to the nucleon while $g_{\eta' NN}$ is the Yukawa coupling constant for the unmixed [SU(3) singlet] η' field. Still, it is hard to verify the postulated cancellation mechanism since g_{GNN} is completely unknown at present while $g_{\eta' NN}$ is rather difficult[9] to reliably extract from experiment.

Here we attempt to evaluate the two terms in (4) using a generalized SU(3) Skyrme model. The Skyrme approach to computing $H(0)$ was pioneered in Ref. 3 wherein it was shown that the simplest Skyrme model of pseudoscalars predicts the desired result $H(0)=0$. However, that simplest Skyrme model is widely recognized to be too crude to give a full description of hadronic properties. For example, in order to explain the neutron-proton mass difference in the Skyrme framework it is necessary[10] to enlarge the model to take account of "short-distance" effects, such as inclusion of vector mesons or explicit quark degrees of freedom. Of course, in order to treat (4) in the framework of such a generalized Skyrme model it is necessary to incorporate into the effective Lagrangian the physics which underlies the two-component decomposition in (4). This may be done by modifying an effective Lagrangian recently proposed by the present authors[11] which gives rise to (4) in a rather transparent way. That Lagrangian treated the baryons as explicit fields; now the goal is to use an analogous mechanism in a Lagrangian of mesons in which the baryons appear as solitons. For our purpose the key part of the Lagrangian is a piece[12] contrived some years ago to solve the η' mass problem in a manner consistent with the anomaly equation (2). With the standard chiral nonet field U and a pseudoscalar "ghost" field G this piece is

$$\mathcal{L}_1 = -\frac{F_\pi^2}{8}\mathrm{Tr}(\partial_\mu U\,\partial_\mu U^\dagger) + \frac{1}{6F_\pi^2 m_{\eta'}^2}G^2 + \frac{i}{12}(\ln\det U - \ln\det U^\dagger) + \frac{F_\pi}{2}\sum_{a=1}^{3} A_a(U_{aa}+U_{aa}^\dagger - 2)\,, \tag{5}$$

where $F_\pi \approx 132$ MeV, $m_{\eta'}$ is a *bare* η' mass, and the A_a are quantities proportional to the quark masses m_a. The unmixed η' field may be extracted from U by writing

$$U = e^{i\chi}\tilde{U}, \quad \det\tilde{U} = 1, \quad \eta' = \tfrac{1}{2}\sqrt{3}F_\pi \chi\,. \tag{6}$$

It is crucial that there is no kinetic term for G so it gets eliminated by its equation of motion. The U(1) axial-vector current determined from (5) is simply $J_\mu^5 = \sqrt{3}F_\pi \partial_\mu \eta'$. It is then very easy to understand the result of Ref. 3, which also incorporated the piece (5), that $H(0)=0$. We just note[13,14] that J_μ^5 is a pure gradient which can only contribute to the induced $\tilde{H}(q^2)$ form factor in (1). Now let us consider more complicated Skyrme models which take short-distance effects into account. Then it will be found that J_μ^5 has the structure

$$J_\mu^5 = \sqrt{3}F_\pi \partial_\mu \eta' + s\tilde{J}_\mu^5\,, \tag{7}$$

where s is a dimensionless constant which we introduce for later convenience and $\tilde{J}_\mu^5$ is *not* a pure gradient. Hence (7) leads in general to a nonzero value for $H(0)$. The lesson learned in Ref. 11 for incorporating the "two-component" description into the effective Lagrangian is that we should consider an additional coupling of the ghost field G to "matter" of the form

$$\mathcal{L}_2 = (t/3F_\pi^2 m_{\eta'}^2)\partial_\mu G\,\tilde{J}_\mu^5\,, \tag{8}$$

where t is a dimensionless coupling constant proportional to g_{GNN} in (4). In the models of interest $\tilde{J}_\mu^5$ is independent of η' and is also a local $\mathrm{U}(1)_A$ invariant. Since G does not carry any chiral quantum numbers, $\mathcal{L}_2$ is $\mathrm{U}(1)_A$ invariant and so will preserve the crucial anomaly equation (2). Furthermore, $\mathcal{L}_2$ does not contribute to the current J_μ^5 so (7) will continue to hold in the extended theory. Elimination of G by its equation of motion,

$$G = \sqrt{3}F_\pi m_{\eta'}^2 \eta' + t\,\partial_\mu \tilde{J}_\mu^5 \tag{9}$$

(note that G is no longer simply proportional to η' in the present case), leads to

$$\mathcal{L}_1 + \mathcal{L}_2 = \frac{-F_\pi^2}{8}\mathrm{Tr}(\partial_\mu\tilde{U}\,\partial_\mu\tilde{U}^\dagger) - \tfrac{1}{2}(\partial_\mu\eta')^2 - \tfrac{1}{2}m_{\eta'}^2\eta'^2 + \frac{F_\pi}{2}\sum_a A_a(U_{aa}+U_{aa}^\dagger-2) - \frac{t}{\sqrt{3}F_\pi}\eta'\,\partial_\mu\tilde{J}_\mu^5 - \frac{1}{6}\frac{t^2}{F_\pi^2 m_{\eta'}^2}(\partial_\mu\tilde{J}_\mu^5)^2\,. \tag{10}$$

The last two terms are the new ones. The next to last gives an additional coupling of the η' to other matter fields which is responsible for the existence of two "components" in (4). The last term gives an extra contribution to scattering processes as discussed in Ref. 11 for the Lagrangian containing explicit baryon fields.

To obtain (4) from our generalized Skyrme model let us consider the equation of motion for the η' field. After recognizing that the existence of the second term in the axial-vector current (7) means that the part of the Lagrangian we have not written contains η' only in the term $(-s/\sqrt{3}F_\pi)\partial_\mu\eta'\tilde{J}_\mu^5$, we easily find

$$(-\Box + m_{\eta'}^2)\eta' = [(s-t)/\sqrt{3}F_\pi]\partial_\mu\tilde{J}_\mu^5\,, \tag{11}$$

where the quark-mass terms (i.e., leading to η-η' mixing) have been neglected for simplicity. Now take matrix elements of both sides between proton states at zero momentum transfer. On the left-hand side we get $ig_{\eta'NN}\bar{u}\gamma_5 u$ by definition. Then, using (7), (1), and the Dirac equation on the right-hand side, we conclude that

$$g_{\eta'NN} = \frac{s-t}{s}\frac{2mH(0)}{\sqrt{3}F_\pi}\,. \tag{12}$$

Rewriting (12) in the desired form (4) leads us to identify

$$g_{GNN} = \frac{t}{t-s}\frac{g_{\eta'NN}}{\sqrt{3}F_\pi m_{\eta'}^2}\,. \tag{13}$$

This is as far as we can go without specifying the details of the "short-range" part of the total Lagrangian.

Let us first consider the short-range physics to be described by vector mesons. Chiral Lagrangians containing both vectors and pseudoscalars have been studied for many years but it is only relatively lately that the terms proportional to the Levi-Civita symbol $\epsilon_{\mu\nu\alpha\beta}$, which are the important ones for our purpose, have been fully discussed. The particular Lagrangian[15] we shall use was found to give improved results, compared to the Skyrme model of only pseudoscalars, for baryon static properties,[16] form factors,[17] scattering[18] as well as the n-p mass difference.[10] We spare the reader complicated details but just note that $\tilde{J}_\mu^5$ in (7) is [see Eq. (4.5) of Ref. 14]

$$\tilde{J}_\mu^5 = \epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left[\frac{i}{2\sqrt{2}g}\left(\frac{\gamma_1}{3}+\frac{\gamma_2}{2}\right)\tilde{P}_\nu\tilde{P}_\alpha(2g\rho - iv)_\beta - \frac{\gamma_2}{\sqrt{2}}F_{\nu\alpha}(\rho)(2g\rho - iv)_\beta - \frac{i}{2\sqrt{2}g}\left(\gamma_3 + \frac{\gamma_2}{2}\right)(2g\rho - iv)_\nu(2g\rho - iv)_\alpha(2g\rho - iv)_\beta\right], \tag{14}$$

where ρ is the vector-meson nonet, $(\tilde{P}, v) = \tilde{U}^{-1/2}d\tilde{U}^{1/2} \pm d\tilde{U}^{1/2}\tilde{U}^{-1/2}$, g is a coupling constant, and $\gamma_1, \gamma_2, \gamma_3$ are numerical constants obtained[16,17] by fitting the model to experiment. The quantity s introduced in (7) corresponds to a small modification of the Lagrangian[15-17] for the interactions of the η'. In that Lagrangian, in analogy with Okubo's original[19] statement of the Okubo-Zweig-Iizuka rule, all fields appear as nonets and only one trace appears in each term. But that procedure is somewhat debatable for the η'. Hence we modify a typical term by the replacement

$$\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}(\partial_\mu U^{1/2}Z_{\nu\alpha\beta}) \to \epsilon_{\mu\nu\alpha\beta}\left[\exp\left(\frac{2i\eta'}{\sqrt{3}F_\pi}\right)\mathrm{Tr}(\partial_\mu \tilde{U}^{1/2}Z_{\nu\alpha\beta}) + \frac{is}{\sqrt{3}F_\pi}\partial_\mu\eta'\mathrm{Tr}(U^{1/2}Z_{\nu\alpha\beta})\right] \tag{15}$$

so that $s=1$ is the original situation with nonet symmetry. [For simplicity, in the particular term in (15), $Z_{\nu\alpha\beta}$ is assumed to contain only vector meson fields.] This has the effect that previous calculations[14,20] of $H(0)$ in this model should now be interpreted as

$$H(0) = (0.30 \pm 0.03)s\,, \tag{16}$$

where the "theoretical uncertainty" discussed in Ref. 14 is included. In presenting this estimate we have made the approximation that the "back reaction" of the new last two terms in (10) on the collective soliton properties and profiles of fields other than the η' is negligible. This seems justified from previous calculations[10,17] showing (at the two-flavor level) that the η field had only a tiny effect on the *other* field profiles, which are necessary to evaluate $H(0)$, for example.

Now we see that if $H(0)$ is specified s is given, so it remains only to determine t. From (11) we note that the *source* of the η' field is proportional to $s-t$, in contrast to the current J_μ^5 in (7) which only involved s. We must find a quantity which depends on the source of the η' field. Such a quantity is[10] the nonelectromagnetic part of the neutron-proton mass difference. To understand why this should be the case consider the two-flavor limit (which turns out to give the dominant contribution). From the last term in (5), this should arise as a term proportional to $\mathrm{Tr}[\tau_3(U+U^\dagger)]$. We may parametrize $U=e^{\sqrt{2}i\eta/F_\pi}(\cos\theta + i\hat{\mathbf{n}}\cdot\boldsymbol{\tau}\sin\theta)$; substituting in this expression shows that the n-p mass difference is proportional to the η field. Now from (11) one sees that η will only get excited [i.e., acquire a nonzero value by "cranking" with the ansatz $\eta = \frac{1}{2}\eta(r)\boldsymbol{\Omega}\cdot\hat{\mathbf{x}}$, where $\boldsymbol{\Omega}$ is the collective angular velocity of the soliton] when the source $[(s-t)/\sqrt{3}F_\pi]\,\partial_\mu\tilde{J}_\mu^5$ exists. In the two-flavor model with only pseudoscalars this source does not exist and neither does the n-p mass difference. Evidently, the calculation[10] with vectors included can be used to estimate $s-t$. There, denoting the nonelectromagnetic part of the neutron-proton mass difference as Δ, one finds

$$\begin{aligned}2\pm 0.3\ \mathrm{MeV} &= \Delta = \Delta(\mathrm{vect}) + \Delta(\eta) + \Delta(\mathrm{SU}(3)) \\ &\approx [0.12 + 1.11(s-t) + 0.40]\ \mathrm{MeV}\,,\end{aligned} \tag{17}$$

where[21] $\Delta(\mathrm{vect})$ is the piece explicitly due to the vector-meson fields, $\Delta(\eta)$ is the main piece due to the η meson, and $\Delta(\mathrm{SU}(3))$ is an estimate of the additional effect due to incorporation of the third flavor. Notice that a factor $s-t$ multiplies the old value[10] of $\Delta(\eta)$, in accordance with the discussion above. From (17) we finally obtain the estimate

$$s-t \approx 1.30 \pm 0.50\,, \tag{18}$$

where an additional 10% theoretical uncertainty in the Skyrme model calculation of Δ has been included.

Now let us put things together. The experimental uncertainty in $H(0)$ is non-negligible. Taking R to be[5] about 0.30 suggests that roughly $-0.2 \le H(0) \le 0.3$ as one may see from Fig. 1 of Ref. 14; this range is not too sensitive to R. It is amusing to notice that, regardless of the value of $H(0)$, (12), (16), and (18) predict $g_{\eta' NN} \approx 3.2$. If the first term of (4) is interpreted[7] as the portion of the proton's total angular momentum due to quark spins, this result predicts it to be $(40\pm 20)\%$. For allowed values of $H(0)$, the predicted central values of s, t, and g_{GNN} [from (13)] are displayed in Table I. It is important to notice that the central value of g_{GNN} is always positive in Table I. This shows, from (4), that the "gluonic piece" always reduces the magnitude of the "quark-spin" piece. This is in qualitative agreement with the expectations expressed in Ref. 1. Of course, the generalized Skyrme model gives, in any event, a smaller quark-spin contribution to $H(0)$ than does the naive quark model. We also see from Table I that smaller values of $H(0)$ are associated with more severe deviations from the nonet ansatz, $s=1$.

It is possible to carry out a similar analysis for any generalized chiral model. Another popular way to represent short-distance effects is to include explicit quark degrees of freedom. There are a rather large number of models available. For simplicity, we may look at the "chiral quark model" in which complexities associated with bag boundary conditions are not present. There, in the notation of Sec. V of Ref. 14 (which contains additional references), we have for the short-range piece in (7),

$$\tilde{J}_\mu^5 = i(1+\tilde{g})\sum_a \bar{q}_a\gamma_\mu\gamma_5 q_a\,, \tag{19}$$

where the q_a are now the "chiral quarks" and $\tilde{g}$ is proportional to an extra derivative-type Yukawa interaction. The treatment, including introduction[22] of terms associated with s and t, proceeds using the same Eqs. (5)-(13) above.

The set of generalized Skyrme models is of course a rather large one so the specific results will be model dependent. It is somewhat encouraging that, as noted in Ref. 14, the vector-meson and chiral quark models give similar results for (16).

TABLE I. Predicted parameters for allowed values of $H(0)$.

$H(0)$	s	t	g_{GNN} (GeV^{-3})
−0.2	−0.67	−1.97	38.9
−0.1	−0.33	−1.63	32.3
0	0	−1.30	25.7
+0.1	0.33	−0.97	19.2
+0.2	0.67	−0.63	12.5
+0.3	1	−0.30	5.9

A more detailed discussion of this approach in the vector-meson and chiral quark models will be given elsewhere.

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