Factorization Test Using $\bar{B}^0 \to D^{*+}\pi^-$ and an Estimate of f_{D_s} Using $B \to DD_s^-$

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(Received 24 September 1990)

We describe a test of factorization comparing the decays of the $\bar{B}^0$ meson to $D^{*+}\pi^-$ with $D^{*+}l^-\bar{\nu}_l$. Assuming that factorization works and using the Isgur and Wise theory of a universal form factor, we estimate the decay constant of the D_s meson to be $276 \pm 45 \pm 44$ MeV.

PACS numbers: 14.40.Jz, 13.20.Jf, 13.25.+m

One theoretical approach to analyzing two-body hadronic decays of mesons containing heavy quarks, such as D or B mesons, is to assume that the decay amplitude can be written as a product of two currents in a similar manner to the way the decay amplitude is constructed for semileptonic decays. For example, the semileptonic decay $B \to Dl\nu$ is described by the product of two terms, a hadronic current and a leptonic current, as $\langle D|J_\mu|B\rangle \times \langle \nu|\gamma_\mu(1-\gamma_5)|l\rangle$. In the case of two-body hadronic decay the lepton term is replaced by another hadron term. For example, if the decay is $B \to D\pi^-$, the amplitude is given by $\langle D|J_\mu|B\rangle\langle 0|A_\mu|\pi^-\rangle$. This approach is called factorization and has been used by several authors for charm decays.[1,2] The quark-level diagram for B decay is shown in Fig. 1(a). This approach does not allow for final-state interactions between the two final-state particles. The approach was extended by Bauer, Stech, and Wirbel[3] to include cases where the virtual W^- does not form a final-state meson but shares its quarks as shown in Fig. 1(b). In this case the color of the quarks from the W must match the color of the other quarks. As this is not generally true this diagram is often called "color suppressed." We will be concerned here only with the non-color-suppressed processes which can occur only via the diagram in Fig. 1(a).

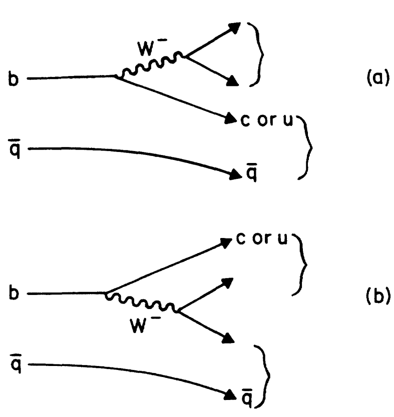

FIG. 1. Decay mechanism of the B meson: (a) Simple spectator and (b) color-suppressed spectator.

It is our aim first to test factorization in the case where a π^- is produced along with a D^{*+} meson. We will do this by comparing the reaction $\bar{B}^0 \to D^{*+}\pi^-$ with $\bar{B}^0 \to D^{*+}l^-\bar{\nu}_l$. Here the hadronic matrix element for the B-to-D^{*+} current is the same in both cases when evaluated at the momentum transfer squared (q^2) equal to m_π^2. Bjorken has long advocated this test.[4] According to Bjorken, if factorization is correct,

$$\frac{\Gamma(\bar{B}^0 \to D^{*+}\pi^-)}{d\Gamma(\bar{B}^0 \to D^{*+}l^-\bar{\nu}_l)/dq^2|_{q^2=m_\pi^2}} = 6\pi^2 f_\pi^2 |V_{ud}|^2 . \quad (1)$$

Since the values of the pion decay constant f_π and the Kobayashi-Maskawa element $|V_{ud}|$ are known, we can use available data on B decay to check if factorization works. Next, we assume that factorization is valid and add the formulation of Isgur and Wise that has one universal form-factor function governing heavy-quark transitions.[5] A check on these two assumptions is provided by comparing $\Gamma(\bar{B}^0 \to D^{*+}\pi^-)$ with $\Gamma(\bar{B}^0 \to D^+\pi^-)$. We then use published CLEO measurements of the reactions $B \to D^{*+}D_s^-$, $B \to D^0 D_s^-$, and $B \to D^+ D_s^-$ to find a value of f_{D_s}. We use the equations given by Rosner for these explicit processes to do our calculations.[6]

The decay $\bar{B}^0 \to D^{*+}l^-\bar{\nu}$ is selected through a missing-mass technique. Here the D^{*+} decays as $D^{*+} \to D^0\pi^+$ and the D^0 as $D^0 \to K^-\pi^+$ or $K^-\pi^+\pi^-\pi^+$. We use data from CLEO (Ref. 7) and ARGUS.[8] Although a detailed discussion of the analysis technique used by the two collaborations is found in their papers, we will give a brief description here.

Because B mesons are produced nearly at rest at the $\Upsilon(4S)$ resonance, one can use the fact that the energy of the B, E_B, equals the beam energy E_{beam}, and approximate the B momentum p_B, which is 325 MeV/c, as zero, to write the missing-mass squared as

$$M^2 = [E_{\text{beam}} - (E_{D^*} + E_l)]^2 - (\mathbf{p}_{D^*} + \mathbf{p}_l)^2 .$$

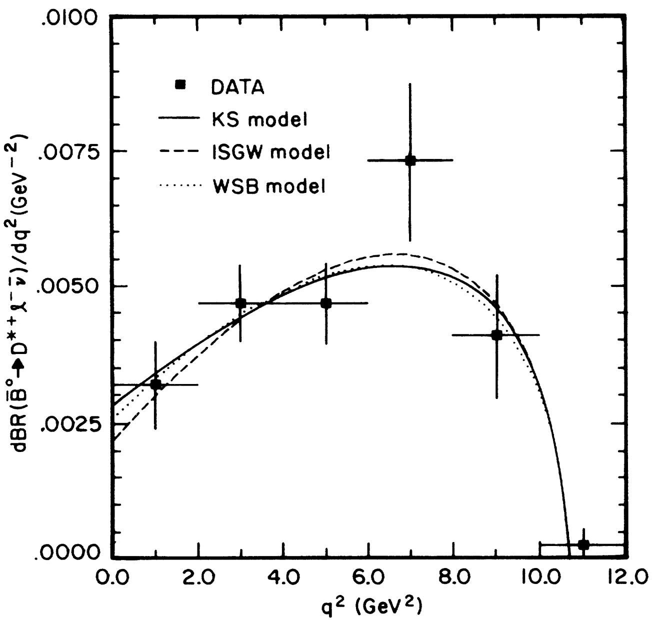

FIG. 2. The q^2 distribution for the decay $\bar{B}^0 \to D^{*+} l^- \bar{\nu}$ in units of (branching ratio)$\times$GeV^{-2}. This is a weighted average of CLEO and ARGUS data (see text). The curves are fits of various models.

Signal events have M^2 consistent with zero.

The same approximation is used to find q^2 from the measurement of the momentum of the D^{*+} meson from the following formula:

$$q^2 = (E_{\text{beam}} - E_{D^{*+}})^2 - (\mathbf{p}_B - \mathbf{p}_{D^{*+}})^2$$
$$= (E_{\text{beam}} - E_{D^{*+}})^2 - \mathbf{p}_{D^{*+}}^2 . \qquad (2)$$

The q^2 distribution is obtained by selecting events with $|M^2| < 1$ GeV^2 which are consistent with only a missing $\bar{\nu}$. In the CLEO analysis the background due to $B \to DX$ and $D \to K l^- \bar{\nu}$ is minimized by selecting $p_l > 1.4$ GeV/c. The resulting q^2 distribution is then corrected for loss in efficiency using the model of Isgur, Scora, Grinstein, and Wise (ISGW).[9] The analysis performed by the ARGUS Collaboration is similar, and differs mainly in that they use electrons with p_l as low as 0.4 GeV/c.[8]

The q^2 distributions from the two experiments are then normalized to the averaged value of $B(\bar{B}^0 \to D^{*+} l^- \bar{\nu}) = (4.8 \pm 0.4 \pm 0.7)\%$.[10] We take the weighted average of the two distributions and display the resulting distribution in Fig. 2. Both data sets have been corrected for experimental acceptance in lepton momentum (worse for CLEO) and in q^2. The smearing in q^2 introduced by the approximation of setting the B momentum to zero in Eq. (2) dominates the experimental resolution. The smearing is particularly severe near $q^2 = 0$.

Although we have measurements over the entire q^2 range, the binning is sufficiently coarse to require a fit to the data to find the values of $dB(\bar{B}^0 \to D^{*+} l^- \bar{\nu})/dq^2$ at $q^2 = m_\pi^2$. We use the predicted shapes from three different models of semileptonic B decay, those of ISGW,[9] Körner and Schuler (KS),[11] and Wirbel, Stech, and Bauer (WSB).[12] The results are sensitive to the model used. The value for dB/dq^2 at $q^2 = m_\pi^2$ is $(2.2 \pm 0.2 \pm 0.4) \times 10^{-3}$ GeV^{-2} using the ISGW model, $(2.8 \pm 0.2 \pm 0.5) \times 10^{-3}$ GeV^{-2} using the KS model, and $(2.6 \pm 0.2 \pm 0.5) \times 10^{-3}$ GeV^{-2} using the WSB model. Note that the systematic errors will cancel in the ratio given by the left-hand side of Eq. (1) because they are dominated by the errors on the $D^{*+} \to \pi^+ D^0$ branching ratio as well as the individual D^0 branching ratios. There are three independent measurements of the $\bar{B}^0 \to D^{*+} \pi^-$ branching ratio.[13-15] These are listed in Table I. The branching ratios have all been normalized assuming that the $\Upsilon(4S)$ decays 50% into $B^0 \bar{B}^0$. This is the same assumption used in deriving the $\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l$ branching ratio. Again, any error will cancel in the ratio. The average for $B(\bar{B}^0 \to D^{*+} \pi^-)$ is $(0.30 \pm 0.06)\%$.

Using these data and a value for $|V_{ud}|$ of 0.9744,[16] we find a value of $f_\pi^2 = 0.024 \pm 0.006$ GeV^2 using the ISGW fit, 0.019 ± 0.005 GeV^2 using KS, and 0.020 ± 0.005 GeV^2 using WSB. All of these values are consistent with the value found from π^- decay of 0.017 GeV^2.[16] Thus, the factorization hypothesis is tested at the 25% level.

Isgur and Wise have shown that in the limit of infinitely massive quarks all semileptonic heavy-quark decays can be expressed in terms of one universal form factor.[5] Adding the factorization assumption to the Isgur-Wise model, relations among exclusive two-body heavy-meson decays and semileptonic decays can be derived. These have so far been limited to the non-color-suppressed decays of Fig. 1(a). One prediction from Rosner[6] relates the ratio of widths for a B to decay into a D or a D^* and a pseudoscalar meson P^- as

$$\frac{\Gamma(\bar{B}^0 \to D^{*+} P^-)}{\Gamma(\bar{B}^0 \to D^+ P^-)} = \left(\frac{1+\sqrt{\zeta}}{1-\sqrt{\zeta}} \right)^2 \frac{\lambda(1,\zeta,y)}{[(1+\sqrt{\zeta})^2 - y]^2} , \qquad (3)$$

TABLE I. Exclusive two-body B branching ratios (%).

Mode	CLEO (Ref. 13)	CLEO (Ref. 14)	ARGUS (Ref. 15)	Average
$D^{*+}\pi^-$	$0.33 \pm 0.09 \pm 0.06$	$0.27 \pm 0.13 \pm 0.08$	$0.28 \pm 0.09 \pm 0.06$	$0.30 \pm 0.06 \pm 0.06$
$D^{+}\pi^-$	$0.27 \pm 0.08 \pm 0.05$	$0.51 \pm 0.27 \pm 0.14$	$0.48 \pm 0.11 \pm 0.11$	$0.35 \pm 0.06 \pm 0.08$

where $\zeta = m_D^2/m_B^2$, $y = q^2/m_B^2 = m_P^2/m_B^2$, and $\lambda(1,\zeta,y) = 1+\zeta^2+y^2-2\zeta-2y-2\zeta y$.

For the case of P^- being a π^- we have

$$B(\bar{B}^0 \to D^{*+}\pi^-)/B(\bar{B}^0 \to D^+\pi^-) = 1\,. \qquad (4)$$

Measurement of the latter mode are also shown in Table I. The ratio of branching ratios defined in Eq. (4) has an average value of $0.9 \pm 0.2 \pm 0.1$. The systematic error arises from the error on $B(D^{*+} \to \pi^+ D^0)$, since it does not cancel in the ratio. This ratio is consistent with unity.

We now turn to an estimate of f_{D_s}. In order to make this estimate we use a different formula, similar to Eq. (1), but one that takes into account explicitly the kinematics for a B decay into two charmed mesons.[6] We have

$$\frac{\Gamma(\bar{B}^0 \to D^{*+}D_s^-)}{d\Gamma(\bar{B}^0 \to D^{*+}l^-\bar{\nu}_l)/dq^2|_{q^2=m_{D_s}^2}} = \delta(\zeta,y)6\pi^2 f_{D_s}^2 |V_{cs}|^2\,, \qquad (5)$$

where

$$\delta(\zeta,y) = \frac{\lambda(1,\zeta,y)(1+\sqrt{\zeta})^2}{[(1+\sqrt{\zeta})^2-y][4y(1+\zeta-y)+(1-\zeta)^2-y(1-\sqrt{\zeta})^2]}\,. \qquad (6)$$

Evaluating these terms at $q^2 = m_{D_s}^2$, we find that $\delta(\zeta,y)$ is 0.41.

$B(\bar{B}^0 \to D^{*+}D_s^-)$ has been measured by CLEO (Ref. 17) and is listed in Table II. We have rescaled the published branching ratio using the newly found $B(D_s^+ \to \phi\pi^+)$ value of $(3.1 \pm 1.0)\%$, which is the average of CLEO (Ref. 18) and ARGUS (Ref. 19) results. From the fits to the q^2 distribution we find dB/dq^2 at $q^2 = m_{D_s}^2$ is $(4.9 \pm 0.4 \pm 0.7) \times 10^{-3}$ GeV^{-2}. All three models give the same result. From Eq. (5) we find $f_{D_s}^2 = 0.14 \pm 0.08 \pm 0.05$ GeV^2, where we take $|V_{cs}|$ equal to $|V_{ud}|$, assuming unitarity of the Kobayashi-Maskawa matrix. The systematic error arises solely from the uncertainty in $B(D_s^+ \to \phi\pi^+)$. Since the measurement of $B(\bar{B}^0 \to D^{*+}D_s^-)$ is based on only three events, the statistical error is too large to find a meaningful result. We can improve the result by using in addition the measurements of $B(\bar{B}^0 \to D^+D_s^-)$ and $B(B^- \to D^0D_s^-)$ (see Table II) using the prediction that a universal form factor governs both the $\bar{B}^0 \to D^{*+}D_s^-$ and $\bar{B}^0 \to D^+D_s^-$ decays. We have already shown that current (meager) data are consistent with the prediction of Eq. (3), at least in the case of $D^{*+}\pi^-/D^+\pi^-$. Here we average the measurement of the neutral- and charged-B decays to improve statistics, assuming that the neutral-to-charged B fraction on the $\Upsilon(4S)$ is not very different from 1 and that the charged- and neutral-B lifetimes are equal. There is evidence for both of these assertions.[20] The weighted average of the $B \to DD_s^-$ branching fractions is $(1.0 \pm 0.4 \pm 0.3)\%$. We must use a new kinematic correction factor which is calculated by dividing $\delta(\zeta,y)$ by the right-hand side of Eq. (3). For our case this factor is 0.56. This results in a value of $f_{D_s}^2 = 0.068 \pm 0.027 \pm 0.021$ GeV^2.

Using both measurements we find $f_{D_s}^2 = 0.076 \pm 0.025 \pm 0.023$ GeV^2. This gives a value of $f_{D_s} = 276 \pm 45 \pm 44$ MeV.

TABLE II. Exclusive B-to-double-charm branching ratios (%).

$\bar{B}^0 \to D^{*+}D_s^-$	$1.6 \pm 0.9 \pm 0.5$
$\bar{B}^0 \to D^+D_s^-$	$0.77 \pm 0.45 \pm 0.25$
$B^- \to D^0D_s^-$	$1.9 \pm 0.8 \pm 0.6$

Rosner has used the same exclusive branching ratios and a different approach to find a value of 259 ± 74 MeV.[6] Both of these values are consistent with purely theoretical predictions, which vary widely.

In conclusion, we show that factorization works near $q^2=0$ at the $\pm 25\%$ level. We find that the ratio of widths for $(\bar{B}^0 \to D^{*+}\pi^-)/(B^0 \to D^+\pi^-)$ is consistent with unity as expected in the Isgur-Wise approach, if factorization is valid. We estimate a value for $f_{D_s} = 276 \pm 45 \pm 44$ MeV assuming factorization. These conclusions can be vastly improved when larger samples of B-decay data become available.

We thank J. D. Bjorken for stimulating our interest in testing factorization and thank J. L. Rosner for several interesting and useful discussions. We are grateful to the ARGUS and CLEO Collaborations for their extensive efforts in providing the results used in this paper. This work was funded by the National Science Foundation and the U.S. Department of Energy.

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