Weak-Scale Effective Supersymmetry

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The idea of supersymmetry at the weak scale should be tested without regard to the Planck-scale origin of any specific model. A class of low-energy supersymmetric theories is derived from four assumptions: minimal field content, R-parity conservation, absence of quadratic divergences, and naturalness of near flavor conservation. Current experiments are testing and constraining this wide class of supersymmetric models, and not just a specific $N=1$ supergravity model.

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Although we know electroweak symmetry breaking occurs at a scale of 250 GeV, the mechanism for such symmetry breaking is still undetermined. A supersymmetric generalization of the standard model offers a possible explanation of SU(2)×U(1) gauge symmetry breaking. Such an explanation avoids the standard problem of fundamental scalars: sensitivity to high-energy physics through quadratically divergent radiative corrections.

Of course, supersymmetry must be broken at some scale since we have not yet observed any of the partners of the observed low-energy spectrum. Because we know the scale of electroweak symmetry breaking, the scale of mass parameters for these operators is known, but the precise form for these operators is a model-dependent question. It is crucial to determine the model dependence of the experimental tests for supersymmetry. In particular, do current collider experiments test weak-scale supersymmetry, or only a specific model of supersymmetry breaking?

In general, these experiments are interpreted as constraints on the parameters of the minimal low-energy supergravity (MLES) model.[1] This makes a certain amount of sense as it may be the simplest viable model of low-energy supersymmetry. However, MLES makes very specific assumptions about physics at energies much higher than those we are testing: It requires an $N=1$ supergravity theory at the Planck scale with a superpotential of a very special and poorly motivated form.[2] Suppose that experiments exclude this model, or demonstrate that superpartner masses are such as to make the model uninteresting. We then learn only that the technical assumptions underlying the specific model are incorrect: This particular version of $N=1$ supergravity is not the correct Planck-scale theory. However, this interpretation of the experiments does not rule out the possibility of low-energy supersymmetry. Other relatively simple models of low-energy supersymmetry are not necessarily excluded. Such a model might have nothing to do with $N=1$ supergravity, or it might arise from supergravity in an unconventional way.[3,4]

Because it is weak-scale supersymmetry which we wish to test, we find it very unsatisfactory to test the idea of low-energy supersymmetry by individually testing the consequences of specific Planck-scale models. In this Letter, we show that current collider experiments constrain weak-scale supersymmetry in a much more general way. In fact, it is no harder to test a general class of low-energy supersymmetric theories satisfying certain mild constraints than to test the MLES model. Constructing a Lagrangian premised on our low-energy assumptions generalizes the MLES model in such a way that the experimental tests of supersymmetry assume more wide-ranging validity.

A class of low-energy supersymmetric models can be constructed by imposing some quite mild assumptions about the structure of the low-energy theory. The high-energy origin of the models is irrelevant to this discussion. We require that the following hold at the weak scale.

(1) The effective theory has a minimal particle content consistent with explaining the observed particles and being supersymmetric. This is by far the strongest of our assumptions, but some such assumption must be made, and it is a reasonable starting point since we wish to test the simplest theories first.

(2) The effective theory conserves R parity. If one allows all operators consistent with supersymmetry and the gauge symmetry, lepton- and baryon-number conservation would be badly violated, in contradiction to the lower bound on the proton lifetime. The simplest way to avoid such operators is to impose R parity which allows the fewest possible interactions. However, because there is no clear theoretical preference for R parity, it is also important to explore other models in which this assumption is relaxed and alternative symmetries such as baryon

number, lepton number, or Z_N are imposed.[4] We also will be assuming that R parity is not spontaneously broken.

(3) The effective theory has no quadratic divergences. The absence of quadratic divergences is a major motivation for low-energy supersymmetry, and we allow all supersymmetry-breaking operators providing they do not cause quadratic divergences in our effective theory.

To avoid quadratic divergences, we have only to prohibit all dimension-four supersymmetry-breaking interactions. To see this, notice that supersymmetry breaking is now accompanied by a mass parameter M, so that a quadratic divergence in an operator would have a coefficient proportional to $\Lambda^2 M$, where Λ is a cutoff scale. Only dimension-one operators could have such a coefficient. In theories with no scalars which are singlets under all symmetries of the theory, such an operator cannot occur.

In theories with singlet scalars, it is also necessary to limit the form of the $d=3$ supersymmetry-breaking terms: For example, trilinear scalar interactions of the form $\phi\phi\phi^*$ and gaugino mass mixing with chiral fermions $\lambda\psi$ must be excluded.[5] However, models derived from $N=1$ supergravity do not lead to such operators, but in a general model, they could be present.

(4) The effective theory maintains a Glashow-Iliopoulos-Maiani-type mechanism. We require that flavor violation is present only in terms proportional to the Yukawa coupling matrices λ_U, λ_D, and λ_E. These matrices break the individual chiral symmetries of the three flavors of quark doublet, up quark, down quark, lepton doublet, and right-handed lepton. Were these matrices zero, the symmetries would be exact. Which symmetry-breaking Yukawa matrix is present in any particular flavor-violating interaction is determined by the chiral-symmetry structure of the operator. This will be elaborated below in the explicit realization of the supersymmetry-breaking Lagrangian.

The MLES also has the first two assumptions. The change in viewpoint is to replace the technical assumption of $N=1$ supergravity with assumptions (3) and (4). The advantages of this generalization are as follows.

(i) Clearly, these assumptions are much milder than the strong assumptions of MLES, which is one of the many possible models satisfying our criteria.

(ii) These assumptions are probably necessary for an interesting low-energy supersymmetric theory. Requirement (3) is necessary for a solution of the gauge hierarchy problem. Requirement (4) is the minimal assumption which guarantees the theory is consistent with stringent low-energy restrictions on flavor violation.

(iii) Experimental tests of supersymmetry have a more general interpretation. If this class of models is excluded, we have learned something important: Assumptions (1) or (2) are too strong, or else weak-scale supersymmetry is ruled out.

(iv) Our class of models, which we call minimal effective supersymmetry, is no harder to test or constrain than MLES models. Experimental constraints have much wider applicability. Most additional parameters of this class of models will be irrelevant to experimental tests.

We now construct the most general Lagrangian consistent with our assumptions.

The first two assumptions imply that the supersymmetric potential is the supersymmetric generalization of the standard model. The gauge group is $SU(3)\times SU(2)\times U(1)$ and the supersymmetric interactions are described by the superpotential

$$f=U^c\lambda_U QH_2+D^c\lambda_D QH_1+E^c\lambda_E LH_1+\mu H_1 H_2 . \quad (1)$$

The most general supersymmetry-breaking operators allowed by our assumptions are the following: gaugino masses,

$$\tfrac{1}{2}m_{\tilde g}\tilde g\tilde g+\tfrac{1}{2}m_{\tilde w}\tilde w\tilde w+\tfrac{1}{2}m_{\tilde b}\tilde b\tilde b+\text{H.c.}, \quad (2.1)$$

"pure" trilinears ($\phi\phi\phi$),

$$A_U\tilde U^c\lambda_U(I+c_1\lambda_U^\dagger\lambda_U+c_2\lambda_D^\dagger\lambda_D+\cdots)\tilde Q H_2+A_D\tilde D^c\lambda_D(I+c_3\lambda_D^\dagger\lambda_D+c_4\lambda_U^\dagger\lambda_U+\cdots)\tilde Q H_1$$
$$+A_E\tilde E^c\lambda_E(I+c_5\lambda_E^\dagger\lambda_E+\cdots)\tilde L H_1+\text{H.c.}, \quad (2.2)$$

"mixed" trilinears ($\phi\phi\phi^*$),

$$A_U'\tilde U^c\lambda_U(I+c_1'\lambda_U^\dagger\lambda_U+c_2'\lambda_D^\dagger\lambda_D+\cdots)\tilde Q H_1^*+A_D'\tilde D^c\lambda_D(I+c_3'\lambda_D^\dagger\lambda_D+c_4'\lambda_U^\dagger\lambda_U+\cdots)\tilde Q H_2^*$$
$$+A_E'\tilde E^c\lambda_E(I+c_5'\lambda_E^\dagger\lambda_E+\cdots)\tilde L H_2^*+\text{H.c.}, \quad (2.3)$$

scalar $\phi^*\phi$ masses,

$$m_{H_1}^2 H_1^* H_1+m_{H_2}^2 H_2^* H_2+m_Q^2\tilde Q^*(I+c_6\lambda_U^\dagger\lambda_U+c_7\lambda_D^\dagger\lambda_D+\cdots)\tilde Q+M_{U^c}^2\tilde U^{c*}(I+c_8\lambda_U\lambda_U^\dagger+\cdots)\tilde U^c$$
$$+m_{D^c}^2\tilde D^{c*}(I+c_9\lambda_D\lambda_D^\dagger+\cdots)\tilde D^c+m_L^2\tilde L^*(I+c_{10}\lambda_E^\dagger\lambda_E+\cdots)\tilde L+m_{E^c}^2\tilde E^{c*}(I+c_{11}\lambda_E\lambda_E^\dagger+\cdots)\tilde E^c, \quad (2.4)$$

and a $\phi\phi$ Higgs-boson mass-mixing term,

$$BH_1H_2+\text{H.c.} \quad (2.5)$$

The parameters c_i are in general independent. The unit matrix I denotes terms which are flavor diagonal. The terms proportional to two powers of the Yukawa matrices are the leading-order flavor-symmetry-breaking effects. The "$+\cdots$" refers to higher-order terms in flavor symmetry breaking.

The Yukawa couplings $\lambda_{U,D,E}$ break the chiral symmetries via operators with specific transformation properties. Once these transformation properties are specified via Eq. (1), they are respected in all of the terms in Eqs. (2). This excludes terms such as $\tilde{Q}^*\lambda_U\lambda_U^+\tilde{Q}$.

Such flavor-violating terms must be present to guarantee renormalizability of the theory. For example, radiative corrections to the squark mass matrix will generate divergences which can be absorbed in $m_Q, c_6, c_7, \ldots$.

The precise values of these coefficients are sensitive to the flavor structure at higher energy scales. If they vanish at some unification scale M_U, they will be generated by renormalization-group scaling of A, A', m^2 to the weak scale and would be of order $(1/16\pi^2)\ln(M_U/M_W)$. Higher-order terms would be similarly calculated from higher loop diagrams. However, one would have to assume one had completely specified the theory between the weak and unification scales in order to precisely calculate the values of these parameters. Moreover, since there must be flavor violation in the supersymmetric potential (in order to generate known masses and mixings), it is likely that even at the unification scale M_U, there is flavor violation present in the soft-supersymmetry-breaking terms generated by renormalization-group scaling above M_U.

The constants c_i and c_i' would also arise if the particles and superpartners were composite states. If the preonic theory had a global $SU(3)^5$ flavor symmetry acting on the left-handed quark doublets, left-handed lepton doublets, right-handed up quark, right-handed down quark, and right-handed lepton, with flavor violation proportional to the matrices $\lambda_{U,D,E}$, such a flavor structure would also arise.[6] We have written the most general renormalizable interactions with c_i and c_i' strong-interaction coefficients determined by the underlying theory. In such a theory, higher-dimension nonrenormalizable interactions could also be important.[6]

In general, unless we have completely specified the high-energy theory and can thereby derive the low-energy Lagrangian, it is most useful to allow for a general form for the low-energy Lagrangian, with restrictions imposed by low-energy phenomenology. In this way, our Lagrangian is not necessarily sufficiently general, for there might not have existed independent chiral symmetries on each of the left- and right-handed quark and lepton fields (e.g., many grand unified theories). With a particular model, one can deduce the global symmetry group allowed by the interactions, and calculate the flavor violations induced by scaling (see Ref. 7, for example). It is moreover possible that flavor violation is not precisely proportional to Yukawa parameters. Nevertheless, in a viable theory, our description of flavor violation should approximately describe flavor-changing effects at low energies. With explicit models, deviations from our prescription can be explicitly calculated.

Of course, the MLES model is a special case of our effective low-energy supersymmetric model. It is derived by assuming supergravity-induced flavor-symmetric soft-supersymmetry-breaking parameters at the Planck scale, that the scaling between the Planck and unification scales can be neglected, and the existence of a desert between the weak and unification scales. With these assumptions, one can derive the values of the soft-supersymmetry-breaking parameters at the weak scale by renormalization-group scaling and applying the assumed boundary conditions at M_U.

One thereby derives

$$A'_{U,D,E}=0 \tag{3.1}$$

together with the boundary conditions at M_U,

$$\begin{aligned} &m_{\tilde{g}}=m_{\tilde{w}}=m_{\tilde{b}}\,, \quad A_U=A_D=A_E=A\,,\\ &m_Q^2=m_{U^c}^2=m_{D^c}^2=m_L^2=m_{E^c}^2=m_{H_1}^2=m_{H_2}^2\,,\\ &c_1=c_2=\cdots=c_{11}=0\,, \end{aligned} \tag{3.2}$$

and at low energies, for example,

$$\begin{aligned} &c_2=c_4=\frac{3}{16\pi^2}\ln\left(\frac{M_U}{M_W}\right),\\ &c_6=c_7=\frac{3+A^2}{8\pi^2}\ln\left(\frac{M_U}{M_W}\right). \end{aligned} \tag{4}$$

Clearly, these relations are uncertain because of the many assumptions about the high-energy theory.

We now discuss the experimental implications of the effective low-energy supersymmetric model and compare them to those of the MLES model. We will see that it is no more difficult to constrain the former, despite the apparent proliferation of parameters. We will see that most searches and tests are in fact identical for the two cases.

The Higgs-boson mass relations for the MLES model[8] can be derived from the three unknown parameters of the Higgs-boson potential: $m_{H_1}^2$, $m_{H_2}^2$, and B. In minimal effective supersymmetry, there are these same three parameters, so the Higgs-boson mass relations will be the usual ones.

In both models, the absence of Z decays to charginos and neutralinos constrains the parameters μ, B, $m_{\tilde{w}}$, $m_{\tilde{b}}$, and $\tan\beta=v_2/v_1$, where v_i is $\langle H_i\rangle$. In either case, a unification condition could relate the Majorana $\tilde{w}$ and $\tilde{b}$ masses.

The hadron-collider searches for squarks and gluinos decaying with a missing energy signature are usually taken to depend on two parameters, the gluino mass $m_{\tilde{g}}$ and the squark mass $m_{\tilde{q}}$ which is assumed to be the mass for five degenerate flavors of left- and right-handed squarks. In low-energy effective supersymmetry, the only difference is that the latter assumption is relaxed: $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{c}_L$, $\tilde{s}_L$, and $\tilde{b}_L$ will be degenerate with mass m_Q, $\tilde{u}_R$ and $\tilde{c}_R$ will be degenerate with mass m_{U^c}, and $\tilde{d}_R$, $\tilde{s}_R$, and $\tilde{b}_R$ are degenerate with mass m_{D^c}. The top squarks may be heavier or lighter than the up and charm squarks.

One might expect flavor violations to be very different in MLES and the effective supersymmetric model. However, there is in fact very little difference. Most of the parameters in (2.2), (2.3), and (2.4) do not violate flavor because they are diagonal in the mass eigenstate basis. In effective supersymmetry all flavor violation is described by the usual Kobayashi-Maskawa matrix and the parameters c_2, c_4, c_2', c_4', c_6, and c_7. In fact, at leading order, there are only four relevant flavor-violating parameters, $\bar{A}_U = A_U v_2 c_2 + A_U' v_1 c_2'$, $\bar{A}_D = A_D v_1 c_4 + A_D' v_2 c_4'$, c_6, and c_7. In fact, probably only c_6 and c_7 could be experimentally observed. This is because the two other parameters are suppressed by an extra power of a quark mass. Although λ_t might be large, flavor violation proportional to λ_t^2 will always exceed that proportional to $\lambda_t^2 \lambda_b$. The only measurable difference between the effective supersymmetric model and MLES is that flavor violations in the up and down systems are described by two independent parameters, rather than one [as seen in Eq. (4)].

A better way to distinguish the models is the mass ratios $m_Q^2 : m_{U^c}^2 : m_{D^c}^2 : m_L^2 : m_{E^c}^2$. Because each type of fermion could in principle have a different mass parameter, these masses can be independent. Of course, even in MLES the relation between these parameters can only be determined with full knowledge of the high-energy theory (e.g., a unification condition and the desert hypothesis). It is impossible to really test whether the low-energy theory is derived from $N=1$ supergravity.

A potential advantage of minimal effective supersymmetry is that the parameter μ is no longer required in the supersymmetric potential because our general soft-symmetry-breaking potential permits a B term which is in principle unrelated to the parameter μ. In the MLES model, it is necessary to take the parameter μ to be of the order of the weak scale because the B parameter is proportional to μ, and nonzero B is required for suitable electroweak symmetry breaking. It is hard to understand why this parameter is roughly the same size as terms in the supersymmetry-breaking potential. The effective supersymmetric model allows μ to be zero. The neutralino and chargino mass matrices would then be much more predictive. One chargino would be lighter than the W and one neutralino would be massless at tree level, with interesting phenomenological consequences.[9]

The minimal low-energy supergravity theory has two CP-violating phases in addition to the phase δ of the standard model, which can be taken to be the phases of A and B. Even in models with a mechanism for setting the strong CP-violating phase $\bar{\theta}$ to zero, radiative corrections to the neutron electric dipole moment constrain these phases to be less than 10^{-3}–10^{-2}.[10] It is a failing of the model that a mechanism for explaining these small phases has not yet been found. The effective supersymmetric model also has this problem. There are now phases in $A_{U,D,E}$, $A_{U,D,E}'$, and B, for example, which are similarly constrained. In a subsequent paper,[9] we show that the addition of a single chiral superfield allows us to construct an R-invariant supersymmetric model in which the number of parameters of the low-energy theory is significantly reduced. In particular, $m_{\tilde{g}}$, $m_{\tilde{w}}$, $m_{\tilde{b}}$, $A_{U,D,E}$, $A_{U,D,E}'$, and μ all vanish. This R-invariant low-energy effective supersymmetric model suppresses radiative corrections to the neutron electric dipole moment.

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