Inclusive πd Charge Exchange and Breakup

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The reactions $\pi^- d \rightarrow \pi^0 X$ and $\pi^+ d \rightarrow \pi^+ X$ are calculated within a relativistic three-body model and compared with recent data at 164.1 and 96.5 MeV, respectively. Contrary to expectations based on the impulse approximation, the differential cross sections of both reactions exhibit a strong suppression in the forward direction. It is shown that this suppression is due in the first case to the action of the Pauli principle on the two final neutrons, and in the second case to the orthogonality between the initial deuteron and final neutron-proton wave functions. The results of the full model agree well with the available differential cross sections at the two energies.

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The inclusive charge-exchange[1] and breakup[2,3] reactions $\pi^- d \rightarrow \pi^0 X$ and $\pi^+ d \rightarrow \pi^+ X$ have been measured recently and in both cases the differential cross section shows a strong suppression in the forward direction. This is a surprising result since the free pion-nucleon cross sections $\sigma_{\pi^- p \rightarrow \pi^0 n}$ and $\sigma_{\pi^+ N \rightarrow \pi^+ N}$ are all forward peaked. Thus, since the πd breakup reactions are thought to be dominated by the impulse approximation, one expects the cross sections to behave as if the pion charge-exchange and scattering processes would take place on a free nucleon and consequently to peak also in the forward direction.

In order to understand this behavior, we have performed a relativistic Faddeev calculation using the spectator-on-mass-shell approach.[4] This is the first full three-body calculation of inclusive πd breakup reactions. The reason for such a long delay is that these calculations are very time consuming, since one has to calculate the cross section for points covering the entire three-body phase space and then integrate over the variables of the two unobserved particles. In contrast, the kinematically complete breakup reactions[5-15] cover only a small portion of the available phase space and in all cases the final three particles have been restricted to lie on a plane.

In the spectator-on-mass-shell approach[4] one solves the relativistic Faddeev equations by requiring that all the spectator particles be on the mass shell and all the exchanged particles be off the mass shell, as well as by imposing a variable-mass isobar ansatz for the two-body amplitudes. This leads to integral equations that have the same degree of complexity as those of the nonrelativistic three-body problem but which are Lorentz invariant and unitary. If one labels the channels of the πNN system as d and Δ channels, then d represents the state of a NN isobar with a pion as spectator and Δ the state of a πN isobar with a nucleon as spectator. We use as input the pion-nucleon S_{11}, S_{31}, P_{11}, P_{13}, P_{31}, and P_{33} channels and the nucleon-nucleon $^3S_1$, $^3D_1$, and $^1S_0$ channels. The Δ state corresponding to a spectator nucleon and the πN pair in the P_{11} channel is treated differently in that the P_{11} amplitude is decomposed into pole and nonpole parts such that the pole part can be formally identified with a nucleon. The nonpole part, however, turns out to be negligible[4] and it is consequently neglected. Thus, this theory takes explicitly into account the coupling to the nucleon-nucleon channels so that it is also able to describe[16] the pion-absorption process $\pi d \rightarrow NN$. The integral equations for an initial πd state are of the form

$$F_{id} = (1-\delta_{id})V_{id} + \sum_{j \neq i} V_{ij}\tau_j F_{jd}, \quad i,j = \Delta, d\,, \tag{1}$$

where the transition potentials are standard Feynman diagrams corresponding to a one-particle-exchange process, for example,

$$V_{\Delta d} = g_\Delta G_0 g_d\,, \tag{2}$$

where G_0 is the propagator of the exchanged particle and g_Δ and g_d the πN and NN vertex functions defined such that the corresponding two-body amplitudes in the pair's c.m. frame are given by

$$t_{\pi N} = g_\Delta \tau_\Delta g_\Delta\,, \tag{3}$$

$$t_{NN} = g_d \tau_d g_d\,, \tag{4}$$

which also defines the isobar propagators τ_Δ and τ_d. The vertex functions of the six S- and P-wave pion-nucleon channels contain the form factor $g(p) = p^l/(\Lambda^2 + p^2)$, $l = 0,1$, where p is the magnitude of the pion-nucleon relative three-momentum and $\Lambda = 700$ MeV/c. The isobar propagators τ_Δ are constructed directly from the experimental pion-nucleon phase shifts in the physical region $s_{\pi N} > (M+\mu)^2$ and from the solution of fixed-t dispersion relations[17] and crossing symmetry[18] in the unphysical region $0 < s_{\pi N} < (M+\mu)^2$. The nucleon-nucleon form factors and isobar propagators g_d and τ_d for the $^3S_1$-$^3D_1$ and $^1S_0$ channels are obtained by applying the unitary-pole approximation to the Paris potential or so-called PEST1 approximation.[19]

The breakup (normal and charge-exchange) amplitudes are given by

$$A_{\rm br}=\sum_{i=1}^{2}(g_{\Delta_i}+g_{d_3}\tau_{d_3}V_{d_3\Delta_i})\tau_{\Delta_i}F_{\Delta_i d}\,, \tag{5}$$

where we have taken the nucleons as particles 1 and 2 and the pion as particle 3, and have labeled the different contributions to the amplitude according to the spectator particle. The second term inside the parentheses is of course the nucleon-nucleon final-state interaction (FSI). Up to this point in the calculation there is no difference between the normal breakup and the charge-exchange reaction. It is only through the isospin part of the vertex functions, g_{Δ_1}, g_{Δ_2}, and g_{d_3}, that one projects out the appropriate amplitude for either of the two processes. The impulse approximation is obtained as a special case of Eq. (5) by replacing $F_{\Delta_i d}$ by $V_{\Delta_i d}$ and dropping the FSI term; that is

$$A_{\rm br}^{\rm IA}=\sum_{i=1}^{2}g_{\Delta_i}\tau_{\Delta_i}V_{\Delta_i d}\,. \tag{6}$$

We show in Fig. 1 the results for the charge-exchange reaction at 164.1 MeV measured by Moinester *et al.*[1] The dotted line corresponds to the naive impulse approximation in which one assumes that only the proton contributes to the charge-exchange process with the neutron acting only as spectator. This curve was obtained by including only one of the two terms in Eq. (6) and ignoring the factor $\frac{1}{2}$ in the cross section due to the presence of two identical particles in the final state. Indeed, this cross section is very close to the free $\pi^- p\to\pi^0 n$ cross section. The dashed line is the result of the impulse approximation when one includes in Eq. (6) the contribution of both terms (as well as the factor $\frac{1}{2}$ in the cross section due to having two identical particles in the final state). As one sees this already gives rise to the suppression in the forward direction. It is easy to show that the term $i=2$ in Eq. (6) is precisely the exchange diagram of the term $i=1$ required by the Pauli principle in the case of a final state with two neutrons. This result holds of course to all orders and is automatically included in the expression for the exact result (5), since the FSI term contains only Pauli-allowed states. This clearly shows that although intuitively one may think that in the impulse approximation only one nucleon contributes to the charge-exchange process, Eq. (6) and Fig. 1 show that the contribution of the second nucleon is essential in order to explain the behavior of the cross section in the forward direction. The solid line is the result of the full calculation and it shows that the main effect of the multiple-scattering terms is to lower the cross section everywhere by approximately 20%. The full calculation in the backward direction underestimates the data by about 1 standard deviation. We have no explanation for the origin of this discrepancy.

We show in Fig. 2 the corresponding results for the normal breakup reaction at 96.5 MeV measured by Khandaker *et al.*[2,3] The dashed line is the result of the impulse approximation (both nucleons included) and it

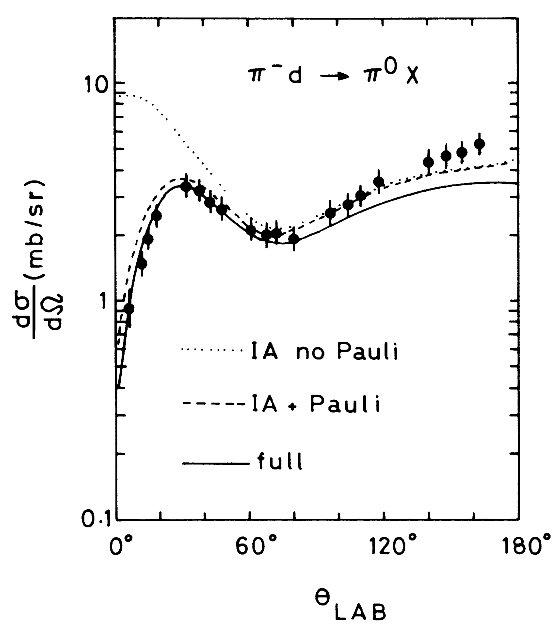

FIG. 1. Differential cross section of the reaction $\pi^- d\to\pi^0 X$ at 164.1 MeV. The solid line is the result of the exact calculation and the dashed (dotted) line is the result of the impulse approximation taking into account (not taking into account) the Pauli principle. The experimental points are from Ref. 1.

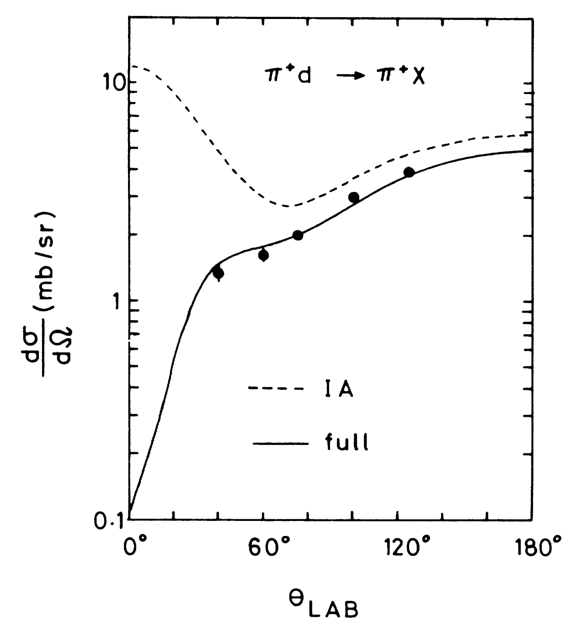

FIG. 2. Differential cross section of the inelastic reaction $\pi^+ d\to\pi^+ X$ at 96.5 MeV. The solid line is the result of the exact calculation and the dashed line is the result of the impulse approximation. The experimental points are from Refs. 2 and 3.

peaks in the forward direction like the free $\pi^+N \rightarrow \pi^+N$ cross sections. The solid line which is the result of the full calculation describes the data well and it shows a dramatic fall off in the forward direction of 2 orders of magnitude with respect to the impulse approximation (that is with respect to the free πN cross sections). We should point out that to a very good approximation the result of the solid line in Fig. 2 can also be obtained if one uses just the impulse approximation with FSI included; that is replacing $F_{\Delta_i d}$ in Eq. (5) by $V_{\Delta_i d}$. The strong suppression of the cross section in the forward direction obtained in Fig. 2 is a consequence of the orthogonality between the initial deuteron and final neutron-proton wave functions as we will show next.

If one identifies in the Born term of Eq. (1) $G_0 g_d \equiv \psi_d$ as the deuteron wave function, the Born term can be written using Eq. (2) as

$$V_{\Delta_i d} = g_{\Delta_i} \psi_d , \tag{7}$$

so that one can define an operator T_i such that

$$T_i \psi_d = g_{\Delta_i} \tau_{\Delta_i} F_{\Delta_i d} , \tag{8}$$

where, in particular, making the approximation $F_{\Delta_i d} \simeq V_{\Delta_i d}$, one finds using Eqs. (7) and (8) that $T_i \simeq g_{\Delta_i} \tau_{\Delta_i} g_{\Delta_i} = t_{\pi N}^{(i)}$. Thus, using Eqs. (4) and (8), the breakup amplitude (5) can be rewritten as

$$A_{\mathrm{br}} = (1 + t_{NN} G_0) \sum_{i=1}^{2} T_i \psi_d = \chi_{NN}^{(-)} \sum_{i=1}^{2} T_i \psi_d . \tag{9}$$

Calling $\mathbf{k}_3'$ the initial pion momentum in the c.m. frame and $\mathbf{k}_3$ and $\mathbf{p}_3$ the final pion momentum and the nucleon-nucleon relative momentum, Eq. (9) can be written as

$$\begin{aligned} \langle \mathbf{k}_3 \mathbf{p}_3 | A_{\mathrm{br}} | \mathbf{k}_3' \rangle &= \langle \mathbf{k}_3 \chi_{\mathbf{p}_3}^{(-)} | \sum_{i=1}^{2} T_i | \mathbf{k}_3' \psi_d \rangle \simeq \langle \mathbf{k}_3 \chi_{\mathbf{p}_3}^{(-)} | \sum_{i=1}^{2} t_{\pi N}^{(i)} | \mathbf{k}_3' \psi_d \rangle \\ &\simeq \sum_{i=1}^{2} \langle \mathbf{p}_i | t_{\pi N}^{(i)} | \mathbf{p}_i' \rangle \int d\mathbf{r}\, e^{i(\mathbf{k}_3 - \mathbf{k}_3') \cdot \mathbf{r}} \chi_{\mathbf{p}_3}^{(-)*}(\mathbf{r}) \psi_d(\mathbf{r}) \rightarrow 0 \text{ if } \mathbf{k}_3 \rightarrow \mathbf{k}_3' . \end{aligned} \tag{10}$$

In the first step of Eq. (10) we have used the approximation $T_i \simeq t_{\pi N}^{(i)}$, in the second step the factorization approximation, and in the last step the orthogonality between the deuteron and the continuum neutron-proton wave functions which requires $\mathbf{k}_3 \simeq \mathbf{k}_3'$. The first approximation $T_i \simeq t_{\pi N}^{(i)}$ has been found numerically to be a very good approximation as mentioned before. The second or factorization approximation is apparently valid to a very good accuracy due to the sharpness with which the deuteron wave function in momentum space peaks at $p = 0$. Finally, the orthogonality of the two wave functions applies only at the point $\mathbf{k}_3 = \mathbf{k}_3'$ which is never reached in the breakup process; however, due to the smallness of the deuteron's binding energy one comes very close to it. Also, in the forward direction the cross section is dominated by the contributions with pion momenta $|\mathbf{k}_3| \simeq |\mathbf{k}_3'|$. We should also mention that the argument of orthogonality applies only for the $^3S_1$-$^3D_1$ channel; however, the effect of the $^1S_0$ channel (which is included in the calculation) is known to be negligible.[12] The result of Fig. 2 clearly shows that all of these approximations are very well satisfied. It would be interesting if the measurements of Khandaker *et al.*[2,3] were extended to angles smaller than 40° in order to check more accurately the prediction of Fig. 2.

Finally, we compare in Fig. 3 the results of the full model (solid lines) and of the impulse approximation

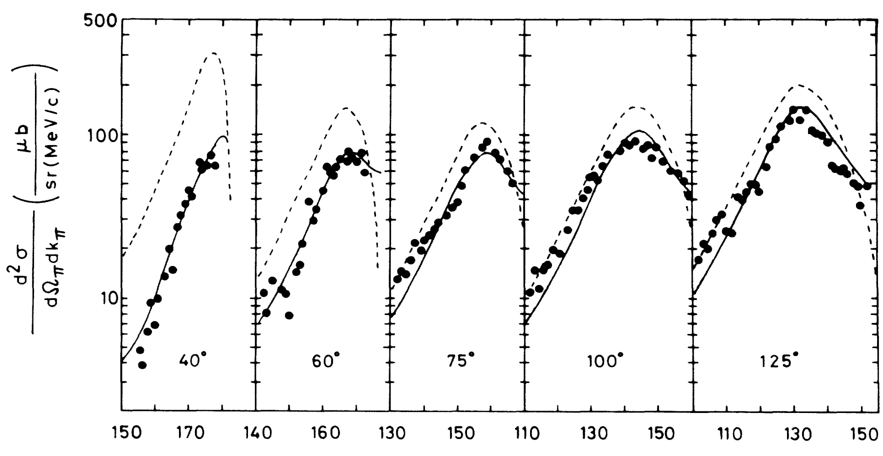

FIG. 3. Double-differential cross section of the reaction $\pi^+d \rightarrow \pi^+X$ at 96.5 MeV for five different pion angles. The labeling of the curves and the data are the same as in Fig. 2.

(dashed lines) with the double-differential cross sections of Khandaker *et al.*[2,3] As expected from the previous discussion, the multiple-scattering effects are largest as one approaches the forward direction and they are clearly needed in order to bring the theory in agreement with the data at the quasielastic peak.

To conclude, we have found that the standard prescription to relate the breakup cross sections to the free πN cross sections is not valid in the forward direction. In the case of the charge-exchange reaction one must at least take into account the Pauli principle for the two final neutrons and in the case of the normal breakup reaction the effect of orthogonality between the initial and final neutron-proton wave functions.

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