Nuclear-Structure Effects in Atomic Parity Nonconservation

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It has been suggested to measure ratios of atomic parity-nonconservation observables in strings of isotopes to cancel atomic-structure effects. Nuclear-structure effects nevertheless play a significant role in extracting weak-interaction parameters. Uncertainties in nuclear structure, especially in the neutron distribution, severely limit the precision of extracting the weak-interaction parameters. On the other hand, the sensitivity to the neutron distribution could provide a unique method for accurately determining rms radii of neutron distributions.

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Studies of parity nonconservation (PNC) in intermediate and heavy atomic systems have provided a test of the electroweak theory of Glashow, Weinberg, and Salam, known as the standard model. Interpretation of atomic experiments[1] requires high-precision calculations of atomic structure. In the most favorable case of interest, atomic cesium, calculations are now considered reliable to about 1%.[2] As experimental accuracy improves beyond this level, the following question emerges: Will it be possible to interpret measurements free of uncertainties in atomic structure, and thus provide crucial tests of the electroweak theory in the low-energy regime?

It has been suggested[3] to study parity-nonconservation effects in strings of isotopes of the same element. Techniques are becoming available to include moderately long-lived radioactive members. By taking ratios of the observables among various members, it might be expected that details of atomic structure should cancel, so that only dependence on N, Z, nuclear spin, and (of special interest) the Weinberg angle, or additional parameters of a more general electroweak theory, should survive.

We examine this in detail here. We find that purely atomic effects do indeed cancel, but that details of nuclear structure continue to play a significant role. In particular, variations in the (elusive) neutron, as well as proton, density distribution among the isotopes affect the results. Thus nuclear structure may become an interesting but limiting factor in the interpretation of PNC experiments of increasing accuracy.

In the standard model, the electron-nucleon interaction is mediated by both the photon and its partner, the intermediate boson Z^0. The latter does not conserve parity. The energies involved in atomic PNC experiments are usually only a fraction of an eV, while the mass of the Z^0 is ≈ 92 GeV, and so the parity-nonconserving interaction may be written as a contact potential. Treating the nucleon nonrelativistically we have

$$H_{\rm PNC}=\frac{G_F}{\sqrt{2}}\sum_{eB}\left[C_{1B}\int\psi_B^\dagger\psi_B\psi_e^\dagger\gamma^5\psi_e\,d^3r + C_{2B}\int\psi_B^\dagger\boldsymbol{\sigma}_B\psi_B\cdot\psi_e^\dagger\boldsymbol{\alpha}\psi_e\,d^3r\right], \quad (1)$$

where B stands for n (neutron) or p (proton) and

$$C_{1p}=\tfrac{1}{2}(1-4\sin^2\theta_W),\quad C_{2p}=\tfrac{1}{2}(1-4\sin^2\theta_W)g_A,$$
$$C_{1n}=-\tfrac{1}{2},\quad C_{2n}=-\tfrac{1}{2}(1-4\sin^2\theta_W)g_A.$$

The first term grows coherently with nucleon numbers N and Z. The second term, together with the anapole-moment[1] term (which also depends upon $\boldsymbol{\sigma}_B\cdot\boldsymbol{\alpha}$), amounts to at most a few percent of the first term in heavy atoms, and furthermore sums to zero when all hfs sublevels are combined,[1] since all directions of $\boldsymbol{\sigma}_B$ are then weighted equally. Thus in this paper we will consider the first term only. The effective interaction is

$$H_{\rm PNC,1}=\frac{G_F}{2\sqrt{2}}\int[-N\rho_n(\mathbf{r})+Z(1-4\sin^2\theta_W)\rho_p(\mathbf{r})]\times\psi_e^\dagger\gamma^5\psi_e\,d^3r, \quad (2)$$

where here the ρ_n and ρ_p are normalized to unity, and we ignore the short-range nucleon PNC form factors.

We need the spatial variation of $\psi_e^\dagger\gamma^5\psi_e$ over the nucleus, its normalization, and its dependence on nuclear structure. PNC effects are dominated by s electrons ($\kappa=-1$) coupled to p electrons ($\kappa=+1$). Because the electric potential is very strong near the nucleus, we can safely neglect atomic binding energies. We define

$$\rho_5(r)\equiv\psi_p^\dagger(\mathbf{r})\gamma^5\psi_s(\mathbf{r}), \quad (3)$$

which turns out to depend only on the magnitude of $\mathbf{r}$. $\rho_5(r)$ can be factored conveniently as follows:

$$\rho_5(r)=C(Z)\mathcal{N}(Z,R)f(r), \quad (4)$$

where $C(Z)$ contains all atomic-structure effects for a *point* nucleus including many-body correlations; $\mathcal{N}\equiv\psi_p^\dagger(0)\gamma^5\psi_s(0)$ is the normalization factor for a single electron; $f(r)$ contains the spatial variation and is normalized to $f(0)=1$. To a very good approximation (see Fig. 1)

$$\mathcal{N}=R^{-\gamma}, \quad (5)$$

where $\gamma=2\{1-[1-(Z\alpha)^2]^{1/2}\}$ and R, often called the *equivalent charge radius,* is given by

$$R=[\tfrac{5}{3}\langle r^2\rangle_{\rm charge}]^{1/2}. \quad (6)$$

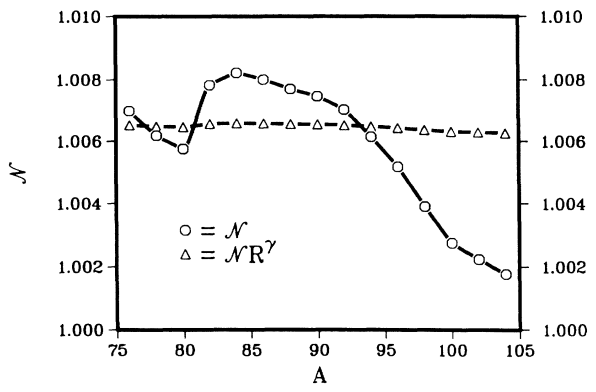

FIG. 1. The normalization factor, defined by Eq. (4), in arbitrary units, for a single electron in the Coulomb field of $_{40}$Zr nucleus. To a good approximation $\mathcal{N}=R_p^{-\gamma}$.

Observable PNC effects are proportional to the matrix element between two atomic states i and j,

$$\langle i|H_{\mathrm{PNC},1}|j\rangle = \frac{G_F}{2\sqrt{2}} C_{ij}(Z)\mathcal{N}[-Nq_n + Z(1-4\sin^2\theta_W)q_p]\,. \quad (7)$$

All effects of nuclear structure on PNC are contained in $\mathcal{N}$ and the two quantities

$$q_n = \int \rho_n(r) f(r) d^3r\,, \quad (8a)$$

$$q_p = \int \rho_p(r) f(r) d^3r\,. \quad (8b)$$

Let us abbreviate $1-4\sin^2\theta_W$ by x. This is a small number; from high-energy experiments, $x=0.08\pm0.02$. The value of x can also be deduced from atomic experiments with an accuracy that will be limited in part by nuclear-structure effects, as we now discuss.

The proton (charge) form factors needed for q_p and $\mathcal{N}$ are generally well known from measurements of the charge distribution of nuclei close to the stable valley, but are not so well known for unstable nuclides. Neutron form factors are needed for q_n, and are not well determined experimentally, and statements about them are quite model dependent, involving both specific-shell and systematic effects. To first approximation, neutron and proton form factors are often taken to be proportional to each other, scaled by N and Z. However, neutron-rich nuclei have larger neutron distributions than the protons and the reverse is true for proton-rich nuclei. In an isotopic sequence, the $A^{1/3}$ law is not followed for either the charge or the neutron distribution separately.

To estimate the importance of such uncertainties in nuclear structure, we consider a simple model based on a uniform nuclear charge distribution of radius R. (The actual calculations use a more realistic charge distribution.) This charge produces an electric potential

$$V_c(r) = Ze^2 \times \begin{cases} (-3+r^2/R^2)2R, & r<R, \\ -1/r, & r>R\,. \end{cases} \quad (9)$$

The essential results of our calculations can be seen from this simple model. A power series for the Dirac wave function inside the nucleus (to second order in $Z\alpha$) yields

$$f(r) = 1 - \tfrac{1}{2}(Z\alpha)^2[(r/R)^2 - \tfrac{1}{5}(r/R)^4] + \cdots\,. \quad (10)$$

Although there is no necessity in doing so we make the simplifying approximation here and below that, as in the uniform-density approximation, for either n or p, $\langle r^4\rangle = \frac{3}{7}R^4$, where $R^2 \equiv \frac{5}{3}\langle r^2\rangle$. From (8a) and (8b) we find

$$q_p = 1 - \tfrac{9}{35}(Z\alpha)^2 + \cdots\,, \quad (11a)$$

which is insensitive to nuclear structure *to this order*, and

$$q_n = 1 - \tfrac{3}{70}(Z\alpha)^2[1 + 5R_n^2/R_p^2] + \cdots\,, \quad (11b)$$

which does depend on the neutron form factor. Here we have introduced equivalent neutron and proton radii of the form (6).

Let us first estimate the uncertainty in the value of x, which one extracts from Eq. (7), if we have an uncertainty in R_n. Assume we know R_p and atomic structures with sufficient accuracy. From Eq. (7) we find

$$\delta x = \frac{N}{Z}\frac{\delta q_n}{q_p} \approx -\frac{3}{7}\frac{(Z\alpha)^2}{q_p}\frac{N}{Z}\frac{R_n}{R_p}\frac{\delta R_n}{R_p}\,. \quad (12)$$

For Pb, $Z=82$, this reduces to

$$\delta x \sim 0.3\,\delta R_n/R_p\,,$$

so an uncertainty of 10% in R_n/R_p (which is a typical uncertainty from pion scattering) results in $\delta x \sim 0.03$. At the present time, the uncertainties in the atomic structure are generally greater, and so the atomic structure will dominate the uncertainties in extracting the value of x, with the possible exception of Cs.[2]

Now if we take the *ratio* of PNC effects between two isotopes, we see that the purely atomic-structure effects, which are contained in the factor $C_{ij}(Z)$ in Eq. (7), indeed cancel in the ratio

$$\mathcal{R} = \frac{\mathcal{N}[Nq_n - Zq_p x]}{\mathcal{N}'[N'q_n' - Zq_p' x]}\,, \quad (13)$$

where the unprimed and primed quantities refer to the two different isotopes. Let us now determine the sensitivity of x to a variation (uncertainty) in the ratio of R_n/R_p between isotopic pairs. We find

$$\delta x \approx q_n' \delta q_n \frac{NN'/Z}{N'q_n'q_p - Nq_nq_p'} \approx -\frac{3}{7}(Z\alpha)^2\frac{NN'}{Z}\delta\left[\frac{\Delta(R_n/R_p)}{\Delta A}\right]. \quad (14)$$

Comparing with Eq. (12), we can see that the uncertainty in δx is now enhanced by a factor of $N/\Delta A$, unless $\Delta(R_n/R_p)$ between isotopes is less than the uncertainty

in R_n/R_p for individual isotopes. For two isotopes separated by $\Delta A=10$, this is about a factor-of-10 enhancement. Thus the uncertainty in the neutron distribution can affect the interpretation of the PNC experiments at a significant level when taking ratios between isotopes.

We can also estimate the effect of a deviation in R_p from an assumed value. To the order of the approximations we make here, this quantity enters only in $\mathcal{N}$ and q_n. We obtain

$$\delta x \approx \frac{N}{Z}\frac{1}{q_p}\left[-\gamma+\frac{18}{35}(Z\alpha)^2\frac{R_n^2}{R_p^2}\right]\frac{\delta R_p}{R_p}. \tag{15}$$

As remarked already, the charge distribution can be checked in many cases by measured isotope shifts.

We turn now to a discussion of how well the knowledge of the proton (charge) distribution can be used to estimate the neutron distribution. The difference between neutron and proton distributions is a very interesting one. A detailed description of each distribution involves both shell and systematic effects. The shell effects appear to be given quite well, for the protons at least, by deformed Hartree-Fock calculations. Systematic (A-dependence) effects are less precisely tested because of the limited number of isotopes available along the stable valley. A number of calculations appear to be fairly successful in reproducing isotope effects involving charge distributions. We restate in another way what was pointed out above: For constant Z, the neutron size increases more rapidly than the proton size, and for constant N, the charge size increases more rapidly than the neutron size.

If $\rho_n(r)$ were to equal $\rho_p(r)$, then to order

$$\langle r^2\rangle_A \equiv \langle r^2\rangle = [Z\langle r^2\rangle_p + N\langle r^2\rangle_n]/A \tag{16}$$

we would have $q_n=q_p=1-\frac{9}{35}(Z\alpha)^2$, independent of nuclear structure, and the only effect of nuclear structure would be in $\mathcal{N}=R^{-\gamma}$. We know from atomic isotope shifts that changes in the equivalent charge radii deviate considerably from that expected by the $A^{1/3}$ law. There are several nuclear effects at play here.

First, nuclear shape deformations increase the nuclear mean-square radius according to[4]

$$\langle r^2\rangle_\beta = \langle r^2\rangle_0[1+(5/4\pi)\langle\beta^2\rangle], \tag{17}$$

where β is the nuclear shape parameter, proportional to the quadrupole moment. β can attain values of the order of $\frac{1}{3}$ and changes in β among isotopes can produce deviations in isotope shifts by an order of magnitude from the $A^{1/3}$ law. One might expect the neutron and proton distributions to be proportional to each other and thus not affect the values of q_n and q_p, but this requires further consideration.

Second, isotope shifts are systematically smaller than predicted by the $A^{1/3}$ law by about 50%. Let us define the *systematic* reduction α,

$$\Delta R_p/R = \alpha\Delta A/3A. \tag{18}$$

Although the charge distribution increases more slowly than the $A^{1/3}$ law for fixed Z, we know that the law is quite good over the entire range of nuclides along the stable valley. To obtain a handle on changes in neutron densities, we might assume that total nuclear density is reasonably constant in an isotopic sequence and that $\langle r^2\rangle^{1/2}$ as given by (16) obeys the $A^{1/3}$ law,

$$\Delta R/R = \Delta A/3A. \tag{19}$$

This implies that the *systematic* neutron radius increases more rapidly, namely,

$$\frac{\Delta R_n}{R}=\frac{\Delta A}{3A}\left[1+\frac{Z}{N}(1-\alpha)\right], \tag{20}$$

and that

$$\frac{\Delta(R_n/R_p)}{\Delta A}=\frac{1}{3A}\left[1+\frac{Z}{N}\right](1-\alpha). \tag{21}$$

In the case of Pb, for example, the variation in R_n as given by (21) yields about 0.001, so that the effect on x as given by (14) is 0.04. This is to be compared with the experimental uncertainty of 0.02. If we could rely on theory to give the variation of R_n, this would not be serious. However, we cannot say how reliable nuclear Hartree-Fock calculations of these quantities are at present.

At this point we do not have clean experimental information on neutron distributions, in contrast to the nuclear charge distributions which are based on weakly interacting electromagnetic probes (electron and muon scattering, electronic and muonic atoms). Hartree-Fock calculations of nuclear structure provide a useful *guide* to neutron and proton distributions. As one example, we refer to deformed Hartree-Fock calculations on isotopes of $_{40}$Zr by Bonche *et al.*,[5] exhibited in Fig. 2. We note that R_p deviates widely from the $A^{1/3}$ law, and is not even monotonic. This can be attributed to variation in nuclear deformation. Nevertheless, the ratio R_n/R_p varies smoothly through the range shown: The neutrons tend to *track* the protons. R_n increases more rapidly than the $A^{1/3}$ law, which on this curve would correspond to $R_n/R_p=1$. From formula (21), this corresponds to $\alpha\sim 0.65$. These results are model dependent, and are presented for illustrative purposes only. Furthermore, we are particularly interested in heavier atoms, cesium ($Z=55$) on up.

We conclude the following.

(1) Before one can achieve significant improvement in the PNC parameters (i.e., Weinberg angle) utilizing ratios of precision measurement of the isotopes, it is necessary to reduce uncertainties in neutron distributions beyond what is currently available.

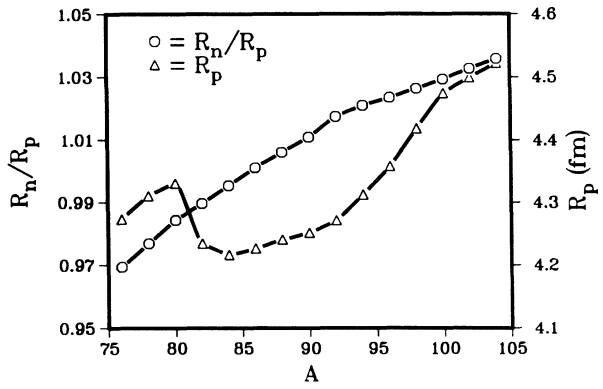

FIG. 2. Results from a deformed Hartree-Fock calculation (Ref. 5) for $_{40}$Zr. The charge radius R_p deviates from the $A^{1/3}$ law, and is not even monotonic. Nevertheless, the ratio R_n/R_p varies smoothly over the range.

(2) Atomic PNC experiments provide a model-independent tool for studying variations in neutron distributions. It is the only tool available at present. It thus provides a critical test of nuclear models. We urge further tests of nuclear modeling for charge distributions over chains of isotopes, particularly those which are likely to be used in PNC experiments.

(3) If (2) can be used to validate nuclear models for reproducing neutron distributions, or more specifically R_n/R_p, then the proposed isotope PNC experiments hold promise of providing more precise measurements of the PNC parameters.

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