Isospin-Forbidden β-Delayed Proton Emission

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Shell-model calculations have been carried out for isospin-forbidden β-delayed proton emission for $1s$-$0d$-shell nuclei. The process in which a nucleus near the proton drip line β^+ decays by a superallowed transition to the isobaric analog state in the daughter nucleus is considered. The first complete set of calculations for the subsequent isospin-forbidden particle decay of the isobaric analog state is presented. The decay channels considered include the processes of single-proton, sequential two-proton, direct two-proton, and α decay. The calculations reproduce the known data and suggest further experiments.

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Nuclei near the proton drip line (i.e., the boundary beyond which nuclei are unbound to direct proton decay) exhibit exotic decay modes whose understanding is important for nuclear-structure physics.[1,2] They provide a unique test of the assumed wave functions for these nuclei far away from the valley of stability. One of the particularly interesting decay modes is superallowed β decay to the isobaric analog state (IAS) followed by isospin-forbidden multiparticle decay of the IAS. β-delayed two-proton emission from the IAS was first predicted[3] and then observed experimentally[4] about ten years ago. The relative importance of direct versus sequential decay was investigated in subsequent experiments.[5,6] With the advent of the new facilities such as those at GANIL,[7-9] it has become possible to extend these type of studies in light nuclei ($A<60$) out to the proton drip line where even more particle decay channels are open. I will discuss an example of the first complete theoretical calculation of the various decay modes.

The Q-value systematics are illustrated for a typical case of the ^{22}Al decay in Fig. 1. The Q-value systematics are such that the β^+ decay of the parent nucleus with atomic number A, charge Z, and isospin T can lead to highly excited states in the daughter nucleus $(A,Z-1)$ which are unbound to proton decay. The β^+ decay is dominated by Gamow-Teller transitions to states with $(A,Z-1,T-1)$ at low and medium excitation energy. Superallowed decay to the IAS with $(A,Z-1,T)$ is also possible. (The IAS is simply related to the parent state by the application of the isospin-lowering operator.) The IAS is usually relatively isolated in the β^+ decay compared to the weaker nearby and underlying $(A,Z-1,$ $T-1)$ states, and thus experimentally it is easy to isolate its decay properties.

The $(A,Z-1,T-1)$ states at low and medium excitation energy are then either bound with respect to single-proton emission (in which case one can observe γ transitions in the daughter nucleus) or unbound with respect to isospin-allowed single-proton emission (in which case one can observe the β-delayed protons). The IAS is unbound with respect to single-proton emission, but only to the $(A-1,Z-2,T-\frac{1}{2})$ states, so that the single-proton decay is isospin forbidden ($\Delta T=\frac{3}{2}$). Often the IAS is also unbound to isospin-forbidden multiparticle decay —direct two-proton (diproton) and α decay in the ^{22}Al example.

Lifetimes and β^+ branching ratios were obtained with the $1s$-$0d$-shell wave functions based on the USD interaction of Wildenthal.[10] Comparisons of individual theoretical and experimental Gamow-Teller (GT) matrix elements for all $1s$-$0d$-shell nuclei near the valley of stability have been made.[11,12] The experimental B(GT)

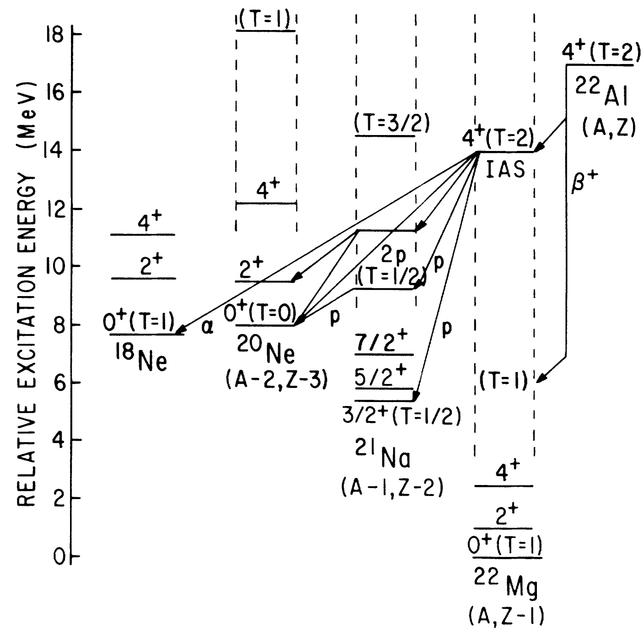

FIG. 1. Energy-level diagram for the β^+ decay of ^{22}Al. The dashed vertical lines indicate regions of high level density. Representative p, $2p$, and α decays of the 4^+, $T=2$ state in ^{22}Mg are shown. The energies of the lowest states with $T=T_{g.s.}+1$ are also indicated.

values are systematically reduced by a factor of 0.60 relative to theory.[12] This "quenching" arises from the effects of higher-order configuration mixing and Δ-particle nucleon-hole admixtures.[13] All subsequent comparisons will be based on theoretical B(GT) values which incorporate this quenching factor. The β^--decay properties of the neutron-rich $1s$-$0d$-shell nuclei are in good agreement with theory[14] except for a small region of nuclei in the "island of inversion" near ^{32}Mg, where the $1p$-$0f$-shell intruder states dominate.[15] The proton-rich mirrors of the nuclei in the island of inversion lie outside the proton drip line. The β^+-decay properties of the proton-rich $1s$-$0d$-shell nuclei with $|T_z| \leq \frac{3}{2}$ are also in good overall agreement with theory.[12,16] In Table I, I give the calculated half-lives for all stable proton-rich $1s$-$0d$-shell nuclei with $|T_z| \geq 2$ together with the β-decay properties to the IAS. The agreement with experiment[7,8,17–22] is excellent, although the experimental errors in the lifetimes are often too large for a quantitative comparison. (These results are based on complete β^+-decay spectra.[23] The largest dimensions involved are those for the decay of ^{27}S which has about 200 final states below the IAS and about 15 000 total final states.)

In this paper I concentrate on the β-delayed multiparticle decay of ^{22}Al and make some comments on the general features of the process. Results for ^{22}Si and ^{31}Ar will be reported together with new experimental data.[24] Related calculations have been previously reported for isospin-allowed multiparticle decay[25] and isospin-forbidden single-proton decay.[7] Experimentally, β-delayed single-proton decay of the IAS in ^{22}Mg to the ground and first excited states of ^{21}Na was first observed.[22] Subsequent experiments detected the delayed two-proton emission and indicated that this was mainly sequential rather than direct.[4–6] Direct emission would result in the observation of correlated protons with equal energies coming out back to back in the two-proton rest frame. The existing data were consistent with protons which were uncorrelated. No α-particle branch was detected. A J^π of 4^+ is assumed for the ^{22}Al ground state on the basis of its assignment in the mirror nucleus ^{22}F.[12,26] However, the 3^+ state in the mirror nucleus lies at only 72 keV in excitation,[26] and it is possible, but unlikely, that a Thomas-Erhman shift could cause the 3^+ and 4^+ states to cross, resulting in a J^π of 3^+ for the ^{22}Al ground state.

Since the particle decays of the IAS are all isospin forbidden, the calculations were based on isospin-mixed $1s$-$0d$-shell wave functions obtained by adding the isospin-nonconserving (INC) interaction of Ormand and Brown[27] onto Wildenthal's isospin-conserving interaction.[10] The INC interaction consists of a Coulomb interaction between protons, plus charge-dependent and charge-asymmetric nuclear interactions with strengths adjusted to fit energy shifts of isobaric analog states.[27] The strengths obtained are close to those expected from the nucleon-nucleon scattering data.[27] The average features of isospin-forbidden proton and neutron decays of $T=\frac{3}{2}$ states in the $1s$-$0d$ shell are well reproduced by these INC wave functions.[28] The isospin-allowed matrix elements are changed very little with the addition of the INC interaction. For example, the superallowed ^{22}Al 4^+ to IAS transition is reduced by only 0.8% when the INC interaction is added. There is little experimental data on the absolute β-decay branching to the IAS. Data for decay of ^{24}Al^m suggest a (30 ± 8)% reduction in the Fermi matrix element[12,16] (assuming that the calculated Gamow-Teller contribution is correct), only about 5% of which can be explained by isospin mixing with a nearby $T=0$ (antianalog) state.[29] The data for

TABLE I. Calculated and measured β-decay properties of all stable proton-rich $1s$-$0d$-shell nuclei with $|T_z| \geq 2$.

Parent nucleus	$\lvert T_z \rvert$	J^π	$T_{1/2}$ (msec) Theory[a]	Expt.	E_x of IAS (MeV) Theory	Expt.	Branching ratio to IAS (%)[b]	IAS decay modes	Expt. Ref.
^{20}Mg	2	0^+	102	95^{+80}_{-50}	6.46	6.57	3.5	p	20
^{22}Al	2	4^+	85	70^{+50}_{-35}	13.97	14.04	4.0	$p,2p,\alpha$	22
^{24}Si	2	0^+	147	103 ± 42	5.87	5.91	11.1	p	18
^{26}P	2	3^+	38	20^{+35}_{-15}	13.37	13.08	2.8	$p,2p,\alpha$	17
^{28}S	2	0^+	129	125 ± 10	5.96	5.95 ± 0.02	14.5	p	7
^{32}Ar	2	0^+	87	98 ± 2	5.00	5.04	21.0	p	19
^{36}Ca	2	0^+	104	≈ 100	4.24	4.26	40.7	p	20
^{23}Si	$\frac{5}{2}$	$\frac{5}{2}^+$	38		11.59		4.9	$p,2p,3p,\alpha$	
^{27}S	$\frac{5}{2}$	$\frac{5}{2}^+$	9		12.62		3.5	$p,2p,3p,\alpha$	
^{31}Ar	$\frac{5}{2}$	$\frac{5}{2}^+$	12	15 ± 3	12.06		4.1	$p,2p,3p,\alpha$	8
^{35}Ca	$\frac{5}{2}$	$\frac{1}{2}^+$	23	50 ± 30	8.82	9.05	12.1	$p,2p,3p$	21
^{22}Si	3	0^+	28		8.95		3.8	$p,2p,3p$	

[a]There is an uncertainty of about 10% due to the uncertainty in the β-decay Q value.
[b]These are the theoretical values. All experimental values quoted in the literature are inferred from the E_x(IAS) and $T_{1/2}$ or have a very large error.

the decay of ^{33}Ar (Ref. 30) suggest a $(10\pm10)\%$ reduction in the Fermi matrix element. More precise experiments on these and other nuclei are needed.

The spectroscopic factors θ^2 for the proton decay of the IAS in ^{22}Mg are given in Table II. The partial widths were then estimated in the standard way[31] from the expression $\Gamma=2\theta_l^2\gamma^2 P_l(Q_p)$, where Q_p is the proton-decay Q value. γ^2 is the Wigner single-particle width given by $\gamma^2=3\hbar^2c^2/2\mu R_0^2$, where μ is the reduced mass. The penetrabilities were calculated from Coulomb wave functions obtained using the method of Steed as described by Barnett.[32] The partial widths and branching ratios obtained with $R_0=3.5$ fm are given in Table II. The total calculated width is 1.0 keV. The branching ratios are insensitive to reasonable variations in R_0, but that the total width increases to 1.3 keV when $R_0=4.0$ fm is used.

The calculated branching ratios are in good agreement with experiment. All levels in ^{21}Na below 2.4 MeV are bound to further proton decay. The weak single-proton branches to the ground and first excited states correspond to those observed.[22] It appears that the strong single-proton decay to the 1.77-Mev $\frac{7}{2}^+$ state may not have been observed in the experiment of Ref. 22 because of the background contamination. All levels in ^{21}Na between 2.4 and 4.1 MeV should decay by further proton emission to the ground state of ^{20}Ne. The only important level in this group is the 2.78-MeV $\frac{9}{2}^+$ state which would give rise to a second proton with an energy which is too low to be observed in Ref. 5. The levels between 4.1 and 6.3 MeV in ^{21}Na can decay either to the 0^+ ground state or to the 2^+ first excited state of ^{20}Ne. The strongest level in this group is the 6.19-MeV $\frac{9}{2}^+$ state.

TABLE II. Calculated spectroscopic factors θ^2, partial widths Γ, and branching ratios for the INC proton decay of the 4^+, $T=2$ IAS state in ^{22}Mg to states in ^{21}Na.

	E_x	$10^4\theta^2$		Γ (keV)		Branching
J^π	(MeV)	$l=0$	$l=2$	$l=0$	$l=2$	ratio (%)
$\frac{3}{2}^+$	0.000		0.13		0.055	5.3
$\frac{5}{2}^+$	0.239		0.18		0.071	6.9
$\frac{7}{2}^+$	1.773	0.04	1.71	0.041	0.371	40.0
$\frac{9}{2}^+$	2.779	0.09	0.56	0.073	0.071	14.0
$\frac{13}{2}^+$	2.780		0.03		0.004	0.4
$\frac{5}{2}^+$	3.664		0.24		0.017	1.6
$\frac{11}{2}^+$	4.449		1.22		0.041	4.0
$\frac{5}{2}^+$	4.502		1.34		0.043	4.1
$\frac{3}{2}^+$	4.753		0.49		0.012	1.1
$\frac{7}{2}^+$	5.341	0.00	0.11	0.000	0.001	0.1
$\frac{3}{2}^+$	5.700		0.07		0.000	0.0
$\frac{13}{2}^+$	5.810		0.78		0.004	0.4
$\frac{9}{2}^+$	6.066	0.10	0.07	0.008	0.000	0.8
$\frac{9}{2}^+$	6.191	2.29	5.69	0.157	0.012	16.5
$\frac{7}{2}^+$	6.299	0.40	11.90	0.023	0.020	4.1
	>6.3					0.6

Since the decay of this state to the ^{20}Ne ground state is l forbidden in the $1s$-$0d$ shell, it must decay to the 2^+ excited state. There is also a nearby $\frac{9}{2}^+$ state at 6.07 MeV which is weakly populated in the calculation. When the spacing between levels is this close, one usually cannot rely on the calculation to give the correct mixing between them. Thus, the most one can confidently predict is that there are two close-lying $\frac{9}{2}^+$ states which together will take a total of 16.5% of the ^{22}Mg 4^+ proton decay. These states will subsequently decay to the 2^+ state of ^{20}Ne. There are many states with large θ^2 above 6.3 MeV (not listed in Table II) but because of the small (or negative) Q_p value the branching to these states is less than 1%. The above scenario for the strong ^{22}Mg(IAS)$\to$ ^{21}Na branches, along with the weaker branches given in Table II, accounts for the sequential two-proton decay seen in Ref. 5.

The general features of the INC proton decay of the IAS are relatively simple. The $l=2$ ($0d_{5/2}$ and $0d_{3/2}$) contributions to the width are small relative to the $l=0$ ($1s_{1/2}$) contributions because of the smaller penetration factors associated with the $l=2$ centrifugal barrier. The spectroscopic factors to the low-lying states in ^{21}Na are relatively small compared to those at higher excitation, but the branches to the low-lying states are significant due to the large Q_p value.

I have not investigated the specific origins of the state mixing which give rise to the INC proton decay. But, as in the case of the previously studied $T=\frac{3}{2}$ states,[28] it is probably dominated by the mixing of the IAS with nearby states with $T_<=T_{\rm IAS}-1$ in ^{22}Mg. Also, as in the case of the $T=\frac{3}{2}$ decay calculations, there is at least an uncertainty of several hundred keV in the location of these $T_<$ states. (The IAS is the 26th 4^+ state in the calculated spectrum, and the average spacing between 4^+, $T=1$ states is about 200 keV.) Therefore, one should interpret the calculated results qualitatively and not quantitatively.

The diproton decay and α decay of the ^{22}Mg 4^+ state was calculated by taking the overlap of the relevant initial and final nuclear states with two- and four-particle cluster wave functions ψ_c generated with an SU(3)-conserving interaction.[33-35] The cluster spectroscopic factor takes the form

$$\theta_c^2=G^2[A/(A-k)]^\lambda|\langle\psi_f|\psi_c|\psi_i\rangle|^2,$$

where $k=2$, $\lambda=4$, and $G^2=\frac{3}{8}$ (Ref. 36) for the diproton, and $k=4$, $\lambda=8$, and $G^2=0.038$ (Ref. 37) for the α. The overlap factors as well as most other calculations presented were carried out using the shell-model code OXBASH.[38]

The results are summarized in Table III. The widths for multiparticle decay turn out to be smaller by about 2 orders of magnitude than those for single-proton decay, again in agreement with the experimental limit for diproton decay.[6] The spectroscopic factors θ^2 for mul-

TABLE III. Calculated INC multiparticle decay properties of the 4^+, $T=2$ IAS state in ^{22}Mg.

Final nucleus	J^π	E_x (MeV)	ΔL	$10^4\theta^2$	Γ (keV)
^{20}Ne	0^+	0.000	4	0.032	6.2×10^{-5}
^{20}Ne	2^+	1.634	2	0.71	1.2×10^{-2}
^{20}Ne	4^+	4.247	0	0.092	3.3×10^{-5}
^{18}Ne	0^+	0.000	4	0.080	3.8×10^{-4}
^{18}Ne	2^+	1.887	2	0.77	7.1×10^{-3}
^{18}Ne	4^+	3.376	0	0.015	3.0×10^{-5}

tiparticle and single-proton decay are comparable in size. The main reason for the small widths for multiparticle emission compared to single-proton emission is due to the reduced penetration factors. They are reduced because ΔZ is twice as large. In addition, the decays with the largest Q values also have large ΔL values and hence relatively small penetrabilities. The $\Delta L=0$ decays to the excited 4^+ states have a small Q value.

In summary, I have calculated widths for the β-delayed particle decay of the isobaric analog state in ^{22}Mg. The calculations agree with experiment and suggest at least one strong decay channel which has not yet been experimentally identified. More accurate and more complete data on the half-life and absolute particle-decay branching ratios of all $1s$-$0d$-shell nuclei are needed.

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