A-B Interface of Superfluid ^{3}He: Effect of Phase Coherence

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Because of phase coherence across the *A-B* interface, the hypercooled phase transition is not an onset phenomenon in this system, but rather the limiting behavior of very large interface velocities. At lower velocities, the chemical potential is continuous across the interface and the growing *B* phase is colder than the receding *A* phase.

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The first-order phase transition between the *A* and *B* phases of liquid ^{3}He has, in recent years, been investigated both experimentally and theoretically.[1-3] Our focus in this Letter is on those features of the interface dynamics that result directly from the presence of superfluidity and thus far have escaped attention: A phase transition can always be viewed as a mass current crossing an interface; the persistency of mass currents is the defining property of superfluidity. These two facts naturally lead us to the expectation of new, especially dissipationless, ways for superfluids to accomplish phase transitions. They therefore cast doubt on the unrestricted validity of the frequently employed equation[4] $g=K\Delta\mu$ (where g is the mass current, and $\Delta\mu$ the difference in chemical potential, both across the interface; the Onsager coefficient K is frequently referred to as the growth coefficient). The important point here is that this equation implies[5] an entropy production, at the interface and of the rate g^2/K. If we consider a torus filled in equal parts with ^{3}He-*A* and -*B* that are in equilibrium with each other and separated by two interfaces, we must falsely conclude that a persistent current is not possible here, since any mass current g would lead to an entropy production with twice the above rate. Hence, despite its fairly general validity[6] at a normal-normal or normal-superfluid interface (NNI or NSI), the Onsager relation $g=K\Delta\mu$ obviously fails at a superfluid-superfluid interface (SSI): It does not contain enough structure to account for the interface dynamics of a SSI.

In addition, but not coincidentally, there is also the question about the onset of the hypercooled phase transition (HPT). This phase transition consists only of interface motion, without any concurrent heat current, either via a counterflow v_s-v_n or via diffusion. With the appropriate values of the specific heat C_v, the initial temperature T_i, the coexistence temperature T_{AB}, and the latent heat L, the usual onset condition[1] $C_v(T_i-T_{AB})/L=1$ yields an exceptionally small value of 0.5% undercooling, at which the *A-B* interface supposedly enters the mode of HPT. Here, our objection is that this is only a necessary but not a sufficient condition. In fact, we show that HPT is the limiting behavior at large interface velocity $\dot{u}\gg 10$ cm/s, and is only continuously approached with increasing supercooling: If T_i is 10% below T_{AB}, $\dot{u}=10$ cm/s and the correction term from heat current is half the size of the term describing HPT; at 20% supercooling, $\dot{u}=30$ cm/s and the correction term is around 4% of the HPT one.

Starting from conservation laws and the positivity of the entropy production at the interface, we shall arrive at general connecting conditions valid across any interface between two superfluids. Despite their simplicity, they lead to three qualitatively different stages of the superfluid interface dynamics: (i) At equilibrium, $T=T_{AB}$, mass currents cross the interface with no dissipation. (ii) At small supercooling values, up to around 2%, with the resulting interface velocity $\dot{u}$ much below the second-sound velocity c_2, the superfluid has ample time to equalize the discontinuity in the chemical potential, $\Delta\mu=0$. Correspondingly, the temperature T_B of the growing *B* phase is lower than T_A of the *A* phase; see Fig. 1. Decreasing the initial temperature T_i of the *A* phase accelerates $\dot{u}$, and the equalization of the chemical potential becomes increasingly incomplete. As a result, T_B is gradually overpassed by T_A. (iii) In the limit of strong supercooling, $\Delta\mu$ rushes past the helium so fast

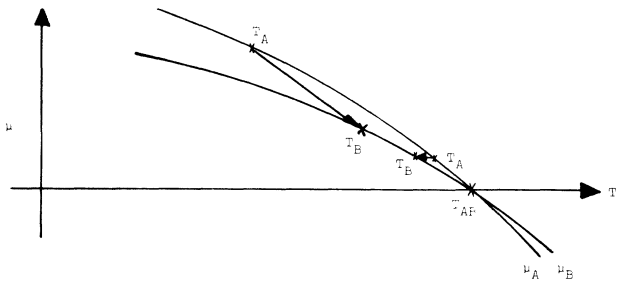

FIG. 1. The two curves depict the chemical potential of the *A* and *B* phases, respectively, as functions of the temperature. Below T_{AB}, $\mu_A>\mu_B$ and the *B* phase is stable. At weak undercooling values, the phase transition takes place at constant chemical potential; therefore $T_B<T_A$. If strongly undercooled, the transition takes place at (approximately) constant entropy, i.e., at two points of the curves with the same derivative $-\partial\mu/\partial T=\sigma$. As a result, $T_B>T_A$.

that it hardly has time to accelerate the superfluid velocity (or rather the countercurrent, since the fluid can be considered incompressible on the time scale of phase transitions). Only now does the transition occur without heat transfer and is what we have abbreviated as HPT. At this stage, the SSI dynamics does not differ from that of an interface involving at least one normal system (NNI or NSI). Neglecting the interface entropy production, and since no heat transfer takes place during HPT, the entropy density of the two phases must be equal, $\sigma_B(T_f)=\sigma_A(T_i)$, T_f being the final temperature. Expanding this equation around $T_f=T_{AB}$ (where considerable counterflow completely invalidates the equation), one can obtain the onset condition, mentioned and criticized above. Instead, one should rather view this equation as an operating condition in the temperature range where it is valid, yielding T_f for given T_i, with $T_i<T_f$; see Fig. 1. Its more complete form, including the interface dissipation, is one of two equations governing the HPT. Both arise from our general connecting conditions in the limit of high interface velocities.

This sketch of physics shall now be put into quantitative terms. Without a magnetic field, there are five general connecting conditions, formulated across the interface. Three conservation laws: for mass, momentum, and energy; one is the equation of motion for the phase difference across the interface, a Josephson equation; and the last is an Onsager relation, the appropriately generalized version of the usual $g=K\Delta\mu$. The conservation laws for mass and momentum determine the two amplitudes of first sound, leaving the interface in both directions. On the time scale relevant to phase transitions, we can take the first-sound pulses to have reflected back and forth and settled down to the equilibrium pressure. If, in addition, we neglect the difference between the A and B phases in the equilibrium densities, $\Delta\rho/\rho\approx 10^{-8}$, no (laboratory-frame) mass flow is being generated by the phase transition. In other words, $g=-\rho\dot{u}$ in the rest frame of the interface, and no more information can be obtained from these two conservation laws. More relevant to our problem is the conservation law for energy. It is obtained by equating the energy currents[7] of the two superfluids,

$$-\langle T\rangle\Delta f=\langle f\rangle\Delta T+g\Delta\mu+\Delta(v_n\Pi)\,, \tag{1}$$

where f is the entropy current, $\Pi=v_ng+v_sj_s$ is the nonlinear part of the momentum current, and $j_s=\rho_s(v_s-v_n)$. All quantities are given in the rest frame of the interface; all vectors are perpendicular to the interface and pointing into the A phase at the right if positive. The symbols $\langle\,\rangle$ and Δ denote average and difference across the interface, respectively. For instance, $\langle T\rangle=\frac{1}{2}\times(T_A+T_B)$ and $\Delta T=T_B-T_A$. Dissipative terms and broken symmetries, in spin and orbital spaces,[8] are neglected here and below. We shall comment upon them at the end of the paper.

The next connecting condition is the Josephson equation, $\Delta\dot{\varphi}+\Delta(\mu+v_nv_s)=0$, taken in the interface frame and obtained by subtracting across the interface. Since the A and B phases are strongly coupled in the sense that the stiffness constant ρ_s of the interface region is hardly softer than in the bulk, the gradients of the phase should also be comparable. Now, the thickness d of the interface is of the same order as the correlation length; hence both $\Delta\varphi\approx(\mathbf{d}\cdot\nabla)\varphi$ and $\Delta\dot{\varphi}$ are vanishingly small quantities. Therefore, we can take the second connecting condition as

$$\Delta(\mu+v_nv_s)=0\,. \tag{2}$$

Equations (1) and (2) are the same connecting conditions that one employs to describe second-sound shock waves.[7] [However, note the notational difference: The chemical potential in Eq. (2) is the one in the interface frame. In Ref. 7, μ denotes the chemical potential in the frame of vanishing v_s.] For the interface dynamics one needs an additional connecting condition, a circumstance caused by the following fact: In the former case, the discontinuity in the temperature ΔT is an input, and one calculates the shock front velocity and the countercurrent as its functions. Two connecting conditions are therefore sufficient. In the interface dynamics, the discontinuity in the temperature is also a variable to be determined, and all three variables are functions of the initial temperature of the supercooled liquid. The third connecting condition is an Onsager relation. The surface entropy production $R_s/\langle T\rangle$ is the change in the entropy current at the interface, or Δf. Obviously, R_s is given by the three terms on the right-hand side of Eq. (1). We may parametrize this positive-definite quantity as

$$R_s=g^2/K=\langle f\rangle^2/\kappa\,, \tag{3}$$

with K denoting the growth coefficient and κ the Kapitza conductance. Since, for the weak and strong limits of undercooling, we shall find $R_s=\langle f\rangle\Delta T$ and $R_s=-gT\Delta\sigma$, respectively, this parametrization yields $\langle f\rangle=\kappa\Delta T$ and $g=-TK\Delta\sigma$, representing two limits of an Onsager relation.[9]

We first examine the weakly supercooled case, in which the resulting interface velocity $\dot{u}$ and the counterflow velocity $w\equiv-j_s/\rho$ are much smaller than the second-sound velocity c_2. In this case, we can linearize Eqs. (1)–(3) (with respect to ΔT, $\Delta\mu$, and all the velocities) to yield

$$\Delta f=0,\quad \Delta\mu=0,\quad \langle f\rangle=\kappa\Delta T\,. \tag{4}$$

In equilibrium, at T_{AB}, these equations are trivially satisfied for $f_A=f_B=0$, since $\Delta T=\Delta\mu=0$. And there is no constraint on the magnitude of the supercurrent crossing the interface (as long as v_s does not exceed its critical value). Next, we consider the initial configuration of constant temperature, $T_A=T_B=T_i$, and an initial discontinuity in the chemical potential $\Delta\mu_i$

$\approx(\sigma_B-\sigma_A)(T_{AB}-T_i)$. Equations (4) represent a set of three linear equations for the variables $\dot{u}$, δT_A, and δT_B. The latter two are the respective amplitudes of the second-sound step functions being sent out by the moving interface in both directions. The amplitudes are such that $\Delta\mu=0$ is achieved at the price of a finite ΔT. In the wake of the two second-sound step functions, the interface moves with the velocity

$$g=-\rho\dot{u}=(\kappa/\langle\sigma\rangle^2)\Delta\mu_i\,, \tag{5}$$

while the second-sound pulses have the amplitudes

$$\delta T_{A,B}=\frac{1}{2}\frac{\Delta\mu_i}{\langle\sigma\rangle}\left(\frac{\kappa}{\langle c_2\rangle}\left\langle\frac{\partial T}{\partial\sigma}\right\rangle\frac{\Delta\sigma}{\langle\sigma\rangle\rho}\mp 1\right). \tag{6}$$

Terms of higher order in $\Delta\sigma/\langle\sigma\rangle$, $\Delta c_2/\langle c_2\rangle$, etc., have been neglected. Despite the similarity in appearance between Eq. (5) and the formerly employed $g=K\Delta\mu$, their physical content is rather different. Especially striking is the lack of a discontinuous $\Delta\mu$ at the interface.

Equations (4)–(6) are valid up to about 2% undercooling. At lower initial temperatures, nonlinear terms such as $\dot{u}^2$, $\dot{u}w$, or $\dot{u}\Delta\mu$ become successively more important, leading to the alteration described in the introduction. In the limit of a strongly undercooled A phase, the interface velocity $\dot{u}$ is by far the fastest, while $w=w_B$ is small in comparison. (Just as in a shock wave, $w_A\equiv 0$.) Neglecting therefore terms of second and third order in w, we obtain from Eqs. (1)–(3)

$$\begin{aligned}&-\langle T\rangle(\Delta\sigma-\sigma_B w/\dot{u})=(\langle\sigma\rangle-\tfrac{1}{2}\sigma_B w/\dot{u})\Delta T+\Delta\mu^0\,,\\ &\Delta\mu^0+\dot{u}w(\rho_n/\rho_s)=0\,,\\ &\dot{u}=(K\langle T\rangle/\rho)(\Delta\sigma-\sigma_B w/\dot{u})\,,\end{aligned} \tag{7}$$

where μ^0 denotes the chemical potential of the helium in which $v_s=v_n=0$. It is therefore only a function of the temperature, $\Delta\mu^0=\mu_B^0(T_f)-\mu_A^0(T_i)$.

To explore the physical content of Eqs. (7), we observe that the Josephson equation lets $w\to 0$, if $\dot{u}\to\infty$ for a given $\Delta\mu^0$. So $w/\dot{u}\ll\Delta\sigma/\sigma_B$ can always be achieved by an initial temperature that is low enough. Then we can solve these equations iteratively, neglecting w in the first and third of Eqs. (7) as the zeroth step,

$$-\langle T\rangle\Delta\sigma=\langle\sigma\rangle\Delta T+\Delta\mu^0,\quad \rho\dot{u}=K\langle T\rangle\Delta\sigma\,. \tag{8}$$

The first equation comes from energy conservation and yields T_f as a function of T_i, and the second equation is the Onsager relation and yields $\dot{u}$, proportional to $\Delta\sigma=\sigma_B(T_f)-\sigma_A(T_i)$. One can now use T_f and $\dot{u}$ to calculate the first-order quantity w. But it is Eqs. (8) that merit attention, because they describe HPT, the phase transition with no heat transfer. Studying the motion of a NNI while assuming the absence of heat transfer, one would arrive at exactly the same equations. The range of validity, however, is quite different for these two systems. For a SSI, one can neglect the terms containing w only when $\Delta\sigma/\langle\sigma\rangle\gg w/\dot{u}$; compare Eqs. (8) to (7). Extrapolating from T_{AB}, we estimate $\Delta\sigma/\langle\sigma\rangle\approx 2w/\dot{u}$ for $T/T_{AB}=0.9$ and $\Delta\sigma/\langle\sigma\rangle\approx 25w/\dot{u}$ for $T/T_{AB}=0.8$. For a NNI, on the other hand, w is identically zero, while the diffusive heat current is too slow to ever become important. Therefore, the HPT is an onset phenomenon for a NNI. It occurs when it is possible, and the onset condition can be obtained from the first of Eqs. (8) (the approximate version of which was discussed in the introduction).

Expanding Eqs. (8) around an initial temperature T_i that is low enough for a SSI, in terms of $\Delta T=T_f-T_i$, we arrive at

$$\begin{aligned}&\Delta T_1=-(\Delta\sigma+\Delta\tilde{\mu}^0/\langle T\rangle)\partial T_B/\partial\sigma\,,\\ &\Delta T_2=-\tfrac{1}{2}[\langle T\rangle^{-1}+(\partial^2\sigma_B/\partial T^2)\partial T_B/\partial\sigma]\Delta T_1^2\,,\\ &\rho\dot{u}/K=-\Delta\tilde{\mu}^0(1+\Delta T/2\langle T\rangle)-\tfrac{1}{2}\Delta T^2\partial\sigma_B/\partial T\,,\end{aligned} \tag{9}$$

where $\Delta T=\Delta T_1+\Delta T_2$. Deviating from our usual notation, all expressions here are to be taken at T_i, e.g., $\Delta\mu^0=\mu_B^0(T_i)-\mu_A^0(T_i)$. The A-B interface dynamics should be well described by Eqs. (9) for $T_i<0.8T_{AB}$. The heating ΔT of the B phase is small, yet becomes more pronounced at low temperatures. We estimate $\Delta T/T_i=0.03$ (0.06) at $T_i=0.8T_{AB}$ ($0.7T_{AB}$). Neglecting ΔT, Eqs. (9) reduce to the equation of motion employed by Yip and Leggett.[2]

Now we list our sins of omission. We are aware of three serious ones, the neglect of which has considerably simplified our task. We are working hard to understand them and pledge amends in future publications. Our omissions are the following: (1) the dynamics of the interface at intermediate values of undercooling, $\dot{u}\approx c_2$; (2) the inclusion of dissipation such as from lateral walls or the damping of second sound; (3) the consideration of the interface-induced spin dynamics. (In contrast, orbital dynamics is probably too slow to be interesting.) Because of dissipation, second sound decays exponentially. This limits the range of heat transport, making it relevant again to consider the entropy balance $\Delta\sigma=0$ outside this range. Calculating the magnetization's response to the moving interface, $m=m(\dot{u})$, would enable us to make direct contact with experiments, especially the bizarre and most intriguing magnetic signals of the more recent paper of Ref. 1.

Our theory provides a phenomenological framework for the A-B interface dynamics. Aside from the growth coefficient K and the Kapitza conductance κ, the parameters in our formulas are thermodynamic in origin, some of them known, others obtainable. Therefore, our predictions, such as the temperature of the growing phase T_B or T_f, the interface velocity $\dot{u}$, the countercurrent w, or the amplitudes of the two second-sound pulses should be readily accessible to experimental probes. The existing experimental data for $\dot{u}(T)$, while certainly consistent with our theory, hardly present a convincing verification, since K (or κ) is itself a function of the tem-

perature. We can of course determine $K(T)$ from the experimental $\dot{u}(T)$, and compare the values with previous theoretical results.[2] Since K was calculated while taking $v_n = v_s = 0$, the comparison should probably exclude experimental data above $0.8T_{AB}$, with unfortunately only two remaining data points. A rough estimate, neglecting the heating of the B phase, shows that the warmer point agrees to within an accuracy of 15%, and the colder one to about 40%.

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[4]The explicit form employed in the works of Ref. 2 is $\dot{u} = (-K/\rho)\Delta\mu$. This can be derived from $g = -\rho\dot{u}$, where $\dot{u}$ is the interface velocity in the frame of vanishing mass current, usually also the laboratory frame.

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[6]Its validity is general, if one can neglect the effect of temperature; cf. M. Grabinski and M. Liu, J. Low Temp. Phys. **75**, 271 (1989).

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[9]The restriction to a "strong-coupling interface" is not obligatory and one can, more generally, derive an entropy production $R_s = f\Delta T + (g - J_s)\Delta\mu$ in the linear regime. $J_s \sim \Delta\varphi$ is the supercurrent crossing the interface. So $f, g - J_s \sim \Delta T, \Delta\mu$ are the two boundary conditions for a SSI, replacing $f, g \sim \Delta T, \Delta\mu$ for a NNI or NSI; cf. Ref. 5. Only in the first case do we have the possibility of a dissipationless phase transition with $f = 0$ and $g = J_s \neq 0$. The A-B interface consists of two strongly coupled superfluids; hence we can deduce $\Delta\mu = 0$ and $J_s = 0$ from $\Delta\varphi = 0$, as argued in the text, and are left with a single Onsager relation $f \sim \Delta T$.