Resistivity of High-T_c Superconductors in a Vortex-Liquid State

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The theory of pinning of a vortex liquid by weak disorder is developed. Two different vortex-liquid dissipative regimes are shown to exist: the flux flow above some crossover temperature T_k, where the vortex liquid is unpinned, and the thermally assisted flux flow below T_k. The activation barriers in the latter regime are those associated with the plastic motion of the vortices in the liquid.

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One of the most interesting features of the new high-T_c superconductors is the remarkable broadening of the resistive transition in the presence of a magnetic field.[1,2] The first detailed investigation of the resistive transition in the mixed state[2] revealed a current-independent and thermally activated resistance $\rho \sim \rho_0 \exp(-U_0/T)$ with U_0 ranging from 10^4 K at magnetic field $H \simeq 10$ T to 10^5 K ($H \simeq 0.1$ T) for Y-Ba-Cu-O. More recent transport measurements[3,4] showed a crucial change in the current-voltage (I-V) characteristics at some line $T_g(H)$ in the H-T plane: Above $T_g(H)$ a linear resistance was found which depends exponentially on temperature whereas below $T_g(H)$ the voltage V exhibited an extremely nonlinear current dependence: $V \propto \exp(-A/j^{\alpha})$. This behavior was attributed[3] to the transition from an unpinned viscous regime (vortex-liquid state) to the pinned regime (vortex-glass state) of motion of vortex lines. The line $T_g(H)$ coincides with the irreversibility line as measured from ac susceptibility.[4]

One can distinguish two different "vortex-liquid" regimes.[1-5] In the high-temperature regime the resistance versus temperature curves show a gradual decrease of the resistance down to some temperature T_k [$\rho(T_k) \sim 0.2\rho_n$, where ρ_n is the resistance in the normal state]. Below T_k, the resistance drops exponentially: $\rho \propto \exp(-U_0/T)$, $U_0(T_k) \gg T_k$. Sometimes this crossover manifests itself as a "kink" or a "shoulder" in the resistance curves.[4,5]

The melting of the vortex-line lattice (VLL) in the absence of pinning and the formation of a vortex liquid was studied by different authors[6-8] using the Lindemann criterion. Because of the high critical temperature, a large Ginzburg-Landau parameter κ, and a large anisotropy, the melting was predicted to occur well below the mean-field $H_{c2}^0(T)$ line.

In the absence of pinning both the vortex lattice and the vortex liquid move under an applied current, leading to a linear flux-flow resistivity $\rho_{\text{flow}} \simeq \rho_n B/H_{c2}$.[9] Disorder produces barriers for the vortex motion and three different situations can be distinguished. (i) The energy barriers U_0 are lower than the temperature and can be omitted, $\rho \approx \rho_{\text{flow}}$. (ii) The barriers $U_0 \gg T$, but do not depend on current j. This corresponds to the thermally assisted flux-flow[10] (TAFF) regime: $\rho \propto \exp(-U_0/T)$.[11] (iii) The barriers $U(j)$ grow unlimitedly with decreasing current j and the linear resistivity drops crucially ($\rho_{\text{linear}} \to 0$). This state is referred to as the vortex-glass state.

In Ref. 12 it was supposed that the pinned vortex solid is a vortex glass. The 3D collective-creep theory,[13] describing the dynamics of the vortex glass formed by weak short-range disorder, predicts the activation barriers $U(j)$ for the VLL motion to grow as $U(j) \propto j^{-\alpha}$; this results in an I-V curve of the form $V \propto \exp(-A/j^{\alpha})$. The exponent α has been calculated for different regimes of collective creep.[13] At large enough magnetic field and high temperatures, $\alpha = \frac{7}{9}$ has been predicted, which is in reasonable agreement with experiment.[3]

In this paper we investigate the influence of quenched disorder on the properties of a vortex liquid. In order to simplify formulas we do not take into account the anisotropy parameter in the calculations, but include it in our final results. We consider short-range disorder with the spatial scale of the random potential less than the vortex core radius ξ and assume this disorder to be weak and not to affect the melting transition.

The observed exponential drop in resistance with decreasing temperature indicates that the vortex liquid is pinned in the interval $T_g < T < T_k$. From the naive point of view, however, there should be no pinning at all in the vortex-liquid state. The interaction between the vortices and the random potential is much weaker than the intervortex interaction and, since the latter is relatively small in the liquid state, the random potential seems to be even less important. Therefore, the existence of pinning producing large barriers and an exponential drop of resistance seems to be quite surprising and incompatible with the concept of weak pinning.

On the other hand, the single vortex line was found to be in a disorder-dominated pinned phase at *any temperature*;[14] this means that one vortex is always in a "glassy state." In fact, if there exists a finite barrier U_0 for vortex motion, the random potential would become irrelevant at temperatures $T > U_0$. One can ask the opposite question: Why are the barriers for vortex-liquid motion independent of the applied current, while for the single vortices they grow unlimitedly?

To understand the nature of the vortex pinning in the vortex-liquid state let us focus first on the role of thermal fluctuations in the pinning of the vortex-solid state.[14] Above the depinning temperature $T_p \sim (\Phi_0^3 Bm/M)^{1/2}/(2\pi\kappa)^2$, the mean-squared value of the thermal displacement $u_{ph} = \langle u^2 \rangle_T^{1/2} \approx \xi (T/T_p)^{1/2}$ becomes larger than the core size ξ and the thermal motion of the vortex lines averages the vortex core pinning over the area u_{ph}^2. Then the characteristic averaged range of the random potential can be approximated by $r_f \approx (\xi^2 + u_{ph}^2)^{1/2}$ and the critical current j_c decreases rapidly with increasing temperature.[14,15] The procedure employed in Ref. 14 to find $j_c(T)$ was in fact the result of first averaging over thermal fluctuations and then over randomness. Such an approach can be used only if the characteristic time of the thermal phononlike fluctuations τ_{ph} is much less than the characteristic time of pinning τ_{pin}. τ_{ph} can be estimated as[14] $\tau_{ph} \sim \Gamma a^4 \lambda^2/\Phi_0^2$, where the friction coefficient $\Gamma \simeq BH_{c2}/\rho_n c^2$,[9] a is the VLL constant, and Φ_0 is the flux quantum. We show below that the characteristic time of pinning is $\tau_{pin} \simeq r_f/v_c$, where $v_c = j_c B/c\Gamma$. One can easily find that $\tau_{pin} \sim \tau_{ph}(T\lambda^2/\Phi_0^2 a)^{1/2} j_0 \xi / j_c a$, where $j_0 \sim \Phi_0/\lambda^2 \xi$ is the depairing current. Since we will consider temperatures $T > T_p$ and weak collective pinning [the latter means that $j_c \ll j_0(\xi/a)^2$], we obtain $\tau_{pin} \gg \tau_{ph}$ and the procedure of Ref. 14 is justified.

The point to be noted here is that the thermal fluctuations smoothen the vortex cores considerably, *but the VLL still preserves its periodicity* and the interaction of the periodic structure with disorder provides pinning at temperatures lower than the melting temperature T_g.

Now we turn to the vortex-liquid state. Recall that in a "conventional" liquid all the characteristic times are of the same order as τ_{ph}. Therefore, on averaging over thermal fluctuations during the time $\tau_{pin} \gg \tau_{ph}$, one obtains *completely smoothened homogeneous* vortex structure and the pinning is absent.

This consideration does not hold for the *very viscous liquid*, where large "smoothening times" exist. If the characteristic smoothening time τ_{pl} is large ($\tau_{pl} \gg \tau_{pin}$), the thermal averaging during the pinning time τ_{pin} is not complete and the vortex configuration retains its inhomogeneous structure, which *is pinned effectively* by the random potential. Exponentially large smoothening times τ_{pl} in the vortex liquid can be provided by high energy barriers U_{pl} associated with thermally activated plastic motion of the vortex structure. In this case, $\tau_{pl} \sim \tau_{ph} \times \exp(U_{pl}/T)$. The characteristic plastic barriers have been estimated to be[15]

$$U_{pl} \sim \sqrt{m/M}\,\Phi_0^2 a/8\pi^2\lambda^2 \propto (T_c - T)/\sqrt{H}\,, \qquad (1)$$

where m and M are the masses in the a-b plane and along the c axis, respectively, and λ is the London penetration depth for $\mathbf{H} \| \mathbf{c}$. Large barriers in the vortex liquid can also arise due to entanglement of vortex lines.[8] The motion of the vortices with respect to each other in such an entangled liquid can be effected by means of cutting and reconnecting of the vortex lines. In the field $H \gg H_{c1}$ the reconnection barriers are also estimated to be of the same order as (1) but with an additional numerical factor. Note that the energy (1) is of the order of the energy of the vortex segment of length $\sim a$ and therefore this estimate can be applied to any vortex deformation with spatial scale $\sim a$.

The inhomogeneities in the vortex liquid are relevant as long as $\tau_{pin} < \tau_{pl}$. With increasing temperature the characteristic plastic barriers decrease, and a crossover from the pinned to the unpinned regime takes place at a temperature T_k, where $\tau_{pin} \approx \tau_{pl} \sim \tau_{ph}\exp(U_{pl}/T)$. This crossover can manifest itself as a "kink" or a "shoulder" in the resistive curve. Note that because of the weakness of the pinning, $\tau_{pin} \gg \tau_{ph}$ and, consequently, $U_{pl}(T_k) \gg T_k$, in agreement with experimental data.[2,4,5,16]

To find the characteristic time scales we explore the pinning of the vortex liquid more rigorously. To do this we use the dynamical approach developed first in Ref. 17 and modify it for the case of the vortex motion in a liquid state. We consider the motion of a vortex structure under the action of a constant Lorentz force $\mathbf{j} \times \mathbf{B}/c$ due to an applied current $j > j_c$ in the presence of a weak random potential

$$U_{pin} = \sum_i V(\mathbf{r}) p(\mathbf{r}_\perp - \tilde{\mathbf{r}}_{\perp i}(z,t))\,, \qquad (2)$$

which is treated as a small perturbation. Here $V(\mathbf{r})$ is the quenched short-range disorder potential, $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \gamma(2\pi)^3\delta(\mathbf{r}-\mathbf{r}')$, $\langle \cdots \rangle$ denotes the average over disorder, $p(\mathbf{r}_\perp)$ describes the interaction of the vortex core with disorder, $p(\mathbf{r}_\perp) \to 0$ at $\mathbf{r}_\perp > \xi$, we sum over all vortices, and the field lies along the z axis. The position $\tilde{\mathbf{r}}_i$ of the ith vortex with respect to the disorder potential can be written in the form $\tilde{\mathbf{r}}_i = \mathbf{r}_i + \mathbf{v}t + \mathbf{u}_{pin,i}$, where $\mathbf{r}_i$ is the undisturbed position of the ith vortex in the frame moving with constant velocity $\mathbf{v}$ and $\mathbf{u}_{pin,i}(t,z)$ is the small disturbance in the position of the ith vortex due to the random potential. The constant velocity is $\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v}$, where $\mathbf{v}_0 = \mathbf{j} \times \mathbf{B}/\Gamma c$ is the velocity of the unpinned liquid due to the Lorentz force and $\delta\mathbf{v}$ is the small deviation of the velocity caused by disorder. Pinning becomes relevant when $\delta v \sim v$ and the condition $\delta v(v_c) \sim v_c$ determines the critical current $j_c = v_c \Gamma c/B$. The correction $\delta\mathbf{v}$ can be found from the self-consistent equation

$$\Gamma \delta\mathbf{v} = \langle \mathbf{f}_{pin} \rangle \equiv \left\langle \sum_i V(\mathbf{r}) \nabla p(\mathbf{r}_\perp - \mathbf{r}_{\perp i} - \mathbf{v}t - \mathbf{u}_{pin}) \right\rangle . \qquad (3)$$

The random displacement $\mathbf{u}_{pin}(r,t)$ is related to the pinning force $\mathbf{f}_{\rm pin}$ by

$$\mathbf{u}_{\rm pin}(\mathbf{r},t)=\int d\mathbf{r}_1\,dt_1 G(\mathbf{r},\mathbf{r}_1;t,t_1)\mathbf{f}_{\rm pin}(\mathbf{r}_1,t_1)\,, \qquad (4)$$

where G is the response function of the vortex liquid. Since the pinning potential is weak, $\mathbf{u}_{\rm pin}$ varies slowly from vortex to vortex and the subscript i is omitted.

Substituting (4) into (3), taking the Fourier transform, and performing averaging we obtain

$$\frac{\delta v}{v}=\frac{\gamma}{\Gamma a^2}\int\frac{d^2k\,dt}{(2\pi)^2}k_{\parallel}k^2|p(k,t)|^2 \times S(k,t)G(0,t)\frac{\sin(k_{\parallel}vt)}{v}\,, \qquad (5)$$

where $k_{\parallel}$ is the $\mathbf{k}$-vector component along the $\mathbf{v}_0$ direction, $G(0,t)\equiv G(\mathbf{r},\mathbf{r},t)$, and the structure factor $S(k,t)$ is given by

$$S(k,t)=\frac{1}{L}\int dz\frac{1}{N}\sum_{ij}\exp\{i\mathbf{k}\cdot[\mathbf{r}_i(z,0)-\mathbf{r}_j(z,t)]\}\,,$$

where N is the total number of vortices.

At low temperatures one has a well-defined vortex lattice and the structure factor is simply the sum of δ functions reduced by the Debye-Waller factor $\exp(-k^2u_{\rm ph}^2/2)$. Then the general result (5) reproduces the results of Refs. 14 and 17. Note that in a vortex solid the structure factor $S(k,t)$ is finite and ceases to depend on time for $t\gg\tau_{\rm ph}$. The response function decays like $G(0,t)\propto t^{-3/2}$. Then the divergence in the integral at large t is cut off by the $\sin(k_{\parallel}vt)$ factor at $t\approx 1/kv\approx u_{\rm ph}/v$ [because of the presence of the Debye-Waller factor $\exp(-k^2u_{\rm ph}^2/2)$ the main contribution comes from $k_{\parallel}\sim u_{\rm ph}^{-1}$]. The ratio $\delta v/v$ grows as $v^{-1/2}$ for $v\to 0$; therefore at any temperature the condition $\delta v/v\simeq 1$ can be satisfied and the critical current does exist. The cutoff time characterizing pinning is $\tau_{\rm pin}=u_{\rm ph}/v_c$. The existence of the critical current implies that disorder, however weak, is relevant at any temperature and the vortex solid is in the vortex-glass state. [We would like to stress once again that if the pinning barriers $U(j)$ for vortex motion remain limited for any current $U(j)<U_0$, then at temperatures $T>U_0$ pinning becomes irrelevant.]

In the liquid state $S(k,t)\approx S(k)$ for small times, where $ku(t)\ll 1$. Here $u^2(t)\equiv\langle[\mathbf{r}(0)-\mathbf{r}(t)]^2\rangle$ and the static structure factor $S(k)\propto k^{\delta}$, $\delta\geq 1$, for small k.[18] For large times where $ku(t)\gg 1$, $S(k,t)$ drops rapidly to zero. The cutoff time in the integral (5) follows then from the condition $ku(t)\sim 1$.

The fluctuation dissipation theorem enables us to relate $u(t)$ to the response function $G(0,t)$:

$$u^2(t)=T\int\frac{d\omega}{\pi}\frac{\mathrm{Im}G(\omega)}{\omega}(1-\cos\omega t) =\tfrac{1}{2}T\int_{-t}^{t}d\tau\,G(\tau)\,.$$

We take the response function in the form $G(\tau)=G_0\times\theta(\tau)(\tau/\tau_{\rm ph})^{1-\beta}$,[18] where G_0 is some constant. If $\beta<0$, short times $t\approx\tau_{\rm ph}$ are relevant, corresponding to the case of a vortex solid: $u^2\to u_{\rm ph}^2$. For the vortex liquid $\beta>0$ and $u(t)$ diverges at large t: $u^2(t)\approx u_{\rm ph}^2\times(t/\tau_{\rm pl})^{\beta}$ at $t>\tau_{\rm pl}$. This result incidentally defines $\tau_{\rm pl}$: $u(\tau_{\rm pl})\approx u_{\rm ph}$.

Consider the behavior of the ratio $\delta v/v$ as a function of v. At large v, $\delta v/v$ is small. For $v>u_{\rm ph}/\tau_{\rm pl}$ the main contribution in (5) comes from $t\sim u_{\rm ph}/v<\tau_{\rm pl}$. In this case the divergence in $u(t)$ is irrelevant and the ratio $\delta v/v$ grows with decreasing v similarly to the case of a vortex solid. For $v<u_{\rm ph}/\tau_{\rm pl}$ the divergence of $u(t)$ should be taken into account. Performing the integration over k in (5) with a cutoff at $k_{\max}\simeq u^{-1}(t)$, using $S(k)\propto k^{\delta}$ and the above expressions for $G(t)$ and $u(t)$, one finds that for $\beta>1/(2+\delta/2)$ the integral over t converges and the main contribution comes from $t\approx\tau_{\rm pl}$. In this case the ratio $\delta v/v$ saturates with decreasing v and for pinning weak enough $\lim_{v\to 0}\delta v/v\ll 1$ (we specify the exact criterion below). The critical current does not exist and the vortex liquid is unpinned. If the disorder is not so weak and $\tau_{\rm pl}$ is large, the ratio $\delta v/v$ becomes as large as unity in the region $v>u_{\rm ph}/\tau_{\rm pl}$. In this case the vortex liquid is pinned and j_c is determined from $\delta v(v_c)=v_c$, analogously to the case of the vortex solid. The crossover between the pinned and unpinned regimes takes place when $\tau_{\rm pl}\simeq\tau_{\rm pin}=u_{\rm ph}/v_c$. In a pinned state $\tau_{\rm pin}\ll\tau_{\rm pl}$. $\tau_{\rm pin}$ grows with temperature, whereas $\tau_{\rm pl}$ drops exponentially and thus the increase of temperature gives rise to a crossover from a pinned to an unpinned vortex liquid.

If $\beta\leq 1/(2+\delta/2)$ then the integral over t diverges, the ratio $\delta v/v$ grows unlimitedly with decreasing v, and the system is in a pinned state. For a single vortex $S(k)=1$ ($\delta=0$) and $\beta=\frac{1}{2}$. This is the marginal case and $\delta v/v$ grows logarithmically. As a result, the critical current drops exponentially with increasing temperature.[14] It can be shown that for the vortex liquid with zero shear modulus but nonzero constant tilt modulus, $\beta=\frac{1}{2}$ as in the case of a single vortex, but $\delta=1$.[18] The condition $\beta>1/(2+\delta/2)$ is satisfied and at large enough temperatures the vortex liquid becomes unpinned. The different behavior of a single vortex and of a vortex liquid is due to the different structure factors.

The difference between the behavior of the vortex lattice and single vortex, on the one hand, and of the vortex liquid, on the other hand, can be understood in terms of the symmetry properties. In the cases of a single vortex and a vortex lattice the continuous two-dimensional translational symmetry breaks down, giving rise to pinning and a glass-state formation when turning on the random potential.

Since the vortex liquid becomes unpinned at large enough temperatures, one concludes that the pinning barriers remain limited at all currents. This implies that the vortex motion in the liquid state is thermally assisted flux flow.[10] The characteristic time which controls the motion in the liquid state is $\tau_{pl} \simeq \tau_{ph} \exp(U_{pl}/T)$; therefore the activation barriers can be identified as U_{pl}.

The resistivity in the TAFF regime has the form $\rho = \rho_0 \exp(-U_{pl}/T)$. One can estimate the preexponential factor by making use of the fact that the crossover to flux-flow resistivity $\rho_{flow} \simeq \rho_n H/H_{c2}$ takes place when

$$\exp(U_{pl}/T) \simeq \tau_{pin}/\tau_{ph} \simeq u_{ph}/v_c \tau_{ph}$$

and thus

$$\rho_0 \simeq \rho_{flow} \frac{j_0}{j_c(T)} \frac{\xi}{a} \left(\frac{T}{U_{pl}} \right)^{1/2} . \qquad (6)$$

As we have already pointed out, the weakness of the pinning gives rise to the condition $\tau_{pin} \gg \tau_{ph}$ and consequently $\rho_0 \gg \rho_{flow}$, which is consistent with the experimental findings.[2,16]

In conclusion, we have developed the theory of pinning of a vortex liquid by weak disorder. We have established that there exist two different regimes of dissipative motion of the very viscous liquid. Above the crossover temperature T_k the vortex liquid is unpinned leading to flux flow. Below T_k the vortex liquid is in the pinned state and its motion is governed by the TAFF mechanism for $T_g < T < T_k$. The activation barriers U_{pl} for this TAFF can be associated with the plastic motion of the vortices. These barriers do not depend on the pinning potential and are given by formula (1). At the crossover temperature T_k, $U_{pl}(T_k) \gg T_k$. The most probable origin of the large viscosity of the vortex liquid is the entanglement of the vortices.

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[1]Y. Iye, T. Tamegai, H. Takeya, and H. Takei, Jpn. J. Appl. Phys. **26**, 1057 (1987).

[2]T. T. M. Palstra, B. Batlogg, R. B. van Dover, L. F. Schneemeyer, and J. V. Waszczak, Appl. Phys. Lett. **54**, 763 (1989).

[3]R. H. Koch, V. Foglietti, W. J. Gallagher, G. Koren, A. Gupta, and M. P. A. Fisher, Phys. Rev. Lett. **63**, 1511 (1989).

[4]T. K. Worthington, F. H. Holtzberg, and C. A. Feild (to be published).

[5]W. K. Kwok, U. Welp, G. W. Crabtree, K. G. Vandervoort, R. Hulscher, and J. Z. Liu, Phys. Rev. Lett. **64**, 966 (1990).

[6]A. Houghton, R. A. Pelcovits, and A. Sudbo, Phys. Rev. B **40**, 6763 (1989).

[7]E. H. Brandt, Phys. Rev. Lett. **63**, 1106 (1989).

[8]D. R. Nelson and S. Seung, Phys. Rev. B **39**, 9153 (1989).

[9]J. Bardeen and M. H. Stephen, Phys. Rev. **140**, A1197 (1965).

[10]P. H. Kes, J. Aarts, J. van den Berg, C. J. van der Beek, and J. A. Mydosh, Supercond. Sci. Technol. **1**, 242 (1989).

[11]M. Tinkham, Phys. Rev. Lett. **61**, 1658 (1988).

[12]M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).

[13]M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).

[14]M. V. Feigel'man and V. M. Vinokur, Phys. Rev. B **41**, 8986 (1990).

[15]V. B. Geshkenbein, M. V. Feigel'man, A. I. Larkin, and V. M. Vinokur, Physica **162-164C**, 239 (1989).

[16]J. N. Liu, K. Kadowaki, M. J. V. Menken, A. A. Menovsky, and J. J. M. Franse, Physica **161C**, 313 (1989).

[17]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **65**, 704 (1973) [Sov. Phys. JETP **38**, 854 (1974)].

[18]V. M. Vinokur, V. B. Geshkenbein, A. I. Larkin, and M. V. Feigel'man (to be published).