Motion of a Bloch Domain Wall

Djemoui Bouzidi and Harry Suhl
Department of Physics, B-019, University of California, San Diego, La Jolla, California 92093
(Received 14 August 1990)

We study the velocity versus applied-field relation of a moving Bloch domain wall, using the "collective coordinate" method employed in the theory of soliton motion. For a sufficiently large field, the wall emits spin waves, thereby self-limiting its velocity. That velocity is equal to the common value of the phase and group velocities of a particular spin wave, and may be above or below the Walker limit V_W, depending on the intrinsic damping.

PACS numbers: 75.60.Ch, 03.50.Kk

A stationary ferromagnetic domain wall represents a delicate balance of (crystalline or shape) anisotropy torque and exchange torque. An applied magnetic field upsets this balance, forcing the wall to move. The theory of this motion has long been a subject of study. Over thirty years ago, Walker[1,2] wrote down an exact solution for the motion of the simplest planar domain wall, in a material with uniaxial anisotropy, taking into account a phenomenological damping rate. According to that solution, the velocity of the wall varies like the ratio of the applied field to the damping rate, up to a certain maximum field, the so-called Walker limit (WL). Beyond that limit the solution breaks down.

Of the many nonlinear aspects of this problem, here we concentrate on one: the effect of the interaction of spin waves with the forward motion on the velocity versus field relation. Experimentally, a variety of these velocity-field relations are observed.[3,4] In some, the curve continues to rise with field, but changes slope (or even undergoes a discontinuous jump) at a certain critical field, possibly the WL. In others it saturates at a certain critical velocity and rises no further. We appear to have found a plausible reason for the latter result.

The stationary wall will support spin-wave excitations.[5,6] First, consider only the dependence of these modes on the coordinate normal to the wall. There is one mode, of zero excitation energy, that is bound to the wall. All others have plane-wave character remote from the wall, but are distorted in the vicinity of the wall, which acts as a reflectionless taper. The excitation energy of these modes has a finite minimum at wave number $k=0$, and rises, initially quadratically, with k.

Suppose the wall moves with velocity v. An observer moving with the wall will note frequencies $\Omega_k \mp vk$ of spin waves propagating with wave numbers $\pm k$ with and against him, respectively. Here Ω_k is the frequency of the wave at $v=0$. Consider the upper case. If the equation $\Omega_k = vk$ has a solution, the apparent frequency goes to zero, and the wave goes unstable. The smallest k for which this can happen is such that $d\Omega_k/dk = v$, so that the group and phase velocities of the wave then equal the wall velocity. Because of the finite energy gap, this equation has a root only for a certain minimum value of v. At that point, energy can be transferred from the motion of the wall to the spin wave. A further increase in v would result in more energy transfer and hence a further increase in damping of the wall motion. Thus v sticks at this critical value, in spite of further increases in the magnetic field H. This state represents a new fixed point, which presumably remains stable until a further instability threshold is reached. In addition, there is also a time-independent shift in each spin wave proportional to v, amounting to a wall distortion.

Our analysis is based on the method of "collective coordinates."[7] Since the position of the wall center is arbitrary, there must be a zero-frequency mode, precluding a direct perturbation treatment of the coupling of the wall to its small excitations. For this reason, the two conjugate variables of the zero-frequency mode are replaced by the position X of the wall, and an effective "momentum" P. The wave functions of the remaining small excitations are then measured relative to X, and their effect is then more accessible to perturbation theory. The ultimate objective is to find an equation of motion for X.

With $\theta(x,t)$ and $\Phi(x,t)$ denoting the polar angles of the magnetization vector at time t and position x, the Landau-Lifshitz equations can be written in Hamiltonian form,

$$\dot{\Psi} = \frac{\gamma}{M}\frac{\delta E}{\delta \Pi}, \quad \dot{\Pi} = -\frac{\gamma}{M}\frac{\delta E}{\delta \Psi}, \tag{1}$$

where $\Psi = \cos\theta$, and the "momentum" Π is the polar angle ϕ.[8] The energy is

$$\mathcal{E} = \int \left\{ J\left[\left(\frac{\partial\theta}{\partial x}\right)^2 + \sin^2\theta\left(\frac{\partial\phi}{\partial x}\right)^2\right] - K_u\cos^2\theta + 2\pi M^2 \sin^2\theta\cos^2\phi - HM\cos\theta \right\} dx .$$

Here J is the exchange constant times a lattice spacing, K_u is the anisotropy energy per unit volume. The third term is the demagnetizing energy, M is the saturation value of magnetization, and γ is the gyromagnetic ratio. To allow for intrinsic losses, Eqs. (1) are supplemented by $-\alpha\,\partial E/\partial\phi$ and $-\alpha\,\partial E/\partial\theta$, respectively, where α is a damping rate. However, and because of prior practice,[9] we shall equivalently supplement Eqs. (1) by Gilbert damping, $\alpha(1-\Psi^2)\dot{\Pi}$ and $[\alpha/(1-\Psi^2)]\dot{\Psi}$, respectively. For $\alpha=0$, these equations of motion are also the Euler-Lagrange equations minimizing the action $A=\int\int dt\,dx \times(\Pi\dot{\Psi}-E)$. The shape of the stationary Bloch domain wall centered on X is $\Psi=\Psi_0(x-X)=-\tanh[(x-X)/\Delta]$, $\Pi=\Pi_0=\text{const}=\pm\frac{1}{2}\pi$. Here Δ is the width $\sqrt{J/K_u}$ of the wall. Consider first the stationary wall. The expressions for the spin waves are usually stated in terms of the small deviations of the Cartesian magnetization components from their unperturbed values.[5,6] Here we need to restate them in terms of deviations from the unperturbed Ψ_0 and Π_0. The result is

$$\delta\Psi=\sum_k a_k\psi_k(x),\quad \delta\Pi=\sum_k b_k\pi_k(x)\,, \tag{2}$$

where the a_k and b_k are the amplitudes, and where ψ_k,π_k are given in Eqs. (5). For the wall in motion, we write

$$\Psi=\Psi_0+\delta\Psi,\quad \Pi=\Pi_0+p+\delta\Pi\,, \tag{3}$$

which are evaluated at $x-X$ and substituted into the action integral. Then X and its quasiconjugate (Π_0+p), along with the spin-wave amplitudes, are allowed to vary with time. We will use reduced units defined by

$$[x]=\sqrt{J/K_u},\quad [t]=1/\gamma M\,. \tag{4}$$

For the applied dc magnetic field we write $h=HM/K_u$.

In the expansions of $\delta\Psi$ and $\delta\Pi$ the spacial dependence is given by

$$\psi_k(x)=W_k(x)/(\omega_k\cosh x)\,,$$
$$\pi_k(x)=W_k(x)(\cosh x)/\omega_k\,, \tag{5}$$

where $W_k(x)$ are the same functions as used by Winter[5] and Thiele:[6]

$$W_k(x)=e^{ikx}(-ik+\tanh x)\,. \tag{6}$$

The zero-frequency eigenmode $1/\cosh x$ is excluded. The rest of the modes have the form given by (5) where the appearance of "cosh$x$" is due to the change from rectangular to polar coordinates; the factor $\omega_k=k^2+1$ is due to the normalization of the $W_k(x)$.[10] The frequencies for these modes are $\Omega_k=[\omega_k(\omega_k+1/Q)]^{1/2}$, where $Q=K_u/2\pi M^2$ is the well known quality factor. The action integral is then seen to be

$$A=\int dt\Bigg[2\dot{X}(\Pi_0+p)+\sum_k\frac{\dot{a}_k b_{-k}}{\omega_k}-\sum_k\frac{\omega_k+1/Q}{\omega_k}b_k b_{-k}-\sum_k a_k a_{-k}$$
$$-\frac{2p^2}{Q}-\dot{X}\sum_{kk'}\frac{a_k b_{k'}}{\omega_k\omega_{k'}}J_{k'k}+\frac{2p^2}{Q}\sum_k\frac{b_k b_{-k}}{\omega_k}-\frac{p^2}{Q}\sum_{kk'}\frac{a_k a_{k'}}{\omega_k\omega_{k'}}G_{k'k}$$
$$+\frac{p^2}{2Q}\sum_k\frac{a_k}{\cosh\frac{1}{2}\pi k}+\frac{4p}{Q}\sum_{kk'}\frac{a_k b_{k'}}{\omega_k\omega_{k'}}i_{k'k}+h\int dx\,\Psi_0(x-X(t))\Bigg]\,. \tag{7}$$

From (7) we can derive the following equations for p, X, a_k, and b_k:

$$\dot{p}=h+\frac{1}{2}\sum_{kk'}\frac{J_{kk'}}{\omega_k\omega_{k'}}\frac{d}{dt}(a_k b_{k'})\,, \tag{8}$$

$$\dot{X}=\frac{2p}{Q}\left[1-\frac{1}{4}\sum_k\frac{a_k}{\cosh\frac{1}{2}\pi k}\right]-\frac{p}{Q}\left[2\sum_k\frac{b_k b_{-k}}{\omega_k}-\sum_{kk'}\frac{a_k a_{k'}}{\omega_k\omega_{k'}}G_{k'k}\right]-\frac{2}{Q}\sum_{kk'}\frac{a_k b_{k'}}{\omega_k\omega_{k'}}I_{k'k}\,, \tag{9}$$

$$\dot{a}_k=\left[\omega_k+\frac{1}{Q}\right]b_k-\frac{2p^2}{Q}b_k+\dot{X}\sum_{k'}\frac{a_{k'}}{\omega_{k'}}J_{-k,k'}-\frac{4p}{Q}\sum_{k'}\frac{a_{k'}}{\omega_{k'}}I_{-k,k'}\,, \tag{10}$$

$$\dot{b}_k=-\omega_k a_k+\frac{p^2\omega_k}{2Q\cosh\frac{1}{2}\pi k}-\dot{X}\sum_{k'}\frac{b_{k'}}{\omega_{k'}}J_{k',-k}+\frac{4p}{Q}\sum_{k'}\frac{b_{k'}}{\omega_{k'}}I_{k',-k}-\frac{p^2}{Q}\sum_{k'}\frac{a_{k'}}{\omega_{k'}}G_{k',-k}\,, \tag{11}$$

with overlap integrals (see, for example, Ref. 11)

$$J_{k',k}=2\pi ik\omega_k\delta(k'+k)+\frac{\pi i(k^2-k'^2)}{2\sinh\frac{1}{2}\pi(k'+k)}-I_{k',k}\,, \tag{12}$$

$$I_{k',k}=\frac{\pi i(k+k')^2}{2\sinh\frac{1}{2}\pi(k'+k)}+i(1-kk')\left[2P\left(\frac{1}{k'+k}\right)-\frac{2}{k'+k}+\frac{\pi}{\sinh\frac{1}{2}\pi(k'+k)}\right], \tag{13}$$

$$G_{k,k'}=2\pi\omega_k\delta(k'+k)-2\int dx\frac{W_k(x)W_{k'}(x)}{\cosh^2 x}\,. \tag{14}$$

All the terms on the right-hand sides of (10) and (11) can be treated as perturbations except the two that are synchronous at frequency Ω_k (at constant $\dot{X}$). Because of their secular behavior, they must be treated nonperturbatively, and seriously change the natural frequency. The new frequencies are given by

$$\Omega^2=\left[[\Omega_k^2-\tfrac{1}{4}v^2(Q\omega_k-1)]^{1/2}\pm vk\left(1+\frac{1}{\pi\omega_k}\right)\right]^2-v^2\left(\frac{q(k)}{\omega_k}\right)^2, \tag{15}$$

representing the four roots of the corresponding determinant, where

$$q(k)=\frac{k^2}{2\sinh\pi k}+\frac{1-k^2}{2\pi}\left[P\left(\frac{1}{k}\right)-\frac{1}{k}+\frac{\pi}{\sinh\pi k}\right],$$

with $P(1/k)$ the principal value function of $1/k$. The kv term on the right-hand side of (15) is simply the "moving-frame" effect discussed at the beginning. The terms $vq(k)/\omega_k$ and $vk/\pi\omega_k$ are a modification of the moving-frame effect due to the distortion of the modes relative to simple plane waves. The term involving v^2 under the radical will be explained in a future publication.

Equations (8)-(11) show that we have to take into account the intrinsic damping α_0 to avoid any unstable solutions. Gilbert damping can then be added by rewriting Eqs. (1) as

$$\dot\Psi+\alpha_0(1-\Psi^2)\dot\Pi=\frac{\gamma}{M}\frac{\delta E}{\delta\Pi},$$
$$\dot\Pi-\frac{\alpha_0}{1-\Psi^2}\dot\Psi=-\frac{\gamma}{M}\frac{\delta E}{\delta\Psi}. \tag{16}$$

For example, Eq. (8) becomes, without the spin-wave term, $\alpha_0 v=h$, which, when combined with (9), gives $2\alpha_0 p=-Qh$. This expression represents the *linearized* version of the Walker condition represented by Eq. (5a) of Ref. 2.

Note the appearance of a dc term quadratic in p, namely, $p^2\omega_k/2Q\cosh\frac{1}{2}\pi k$, in the equations of motion for the spin waves. When the part of the action arising from the demagnetization is expanded, a term linear in the spin-wave amplitudes arises. This causes a displacement, $a_k^0=p^2/2Q\cosh\frac{1}{2}\pi k$, of the spin waves, found by setting $\dot a_k=\dot b_k=0$ and keeping only terms quadratic in p. This results [according to Eqs. (2) and (3)] in a modification of the wall shape, which can be reexpressed as the usual wall contradiction[9] (expanded to order p^2).

The effect of the critical spin waves will now be described by explicitly evaluating their thermal excitation. Alternatively, the same result follows from a fixed-point analysis, without the benefit of thermal agitation, as will become clear later. However, the use of a thermal field helps in visualizing the manner in which fixed points are attained.

Keeping just the secular terms in the equations for the spin-wave amplitudes, we use the substitutions $a_k=A_ke^{i\Omega t}+A^*_{-k}e^{-i\Omega t}$ and $b_k=B_ke^{i\Omega t}+B^*_{-k}e^{-i\Omega t}$, and allow these waves to be driven by the thermal fields whose Fourier components in k-Ω space are $h_k^{\rm th}(\Omega)$. At $\Omega=0$, the amplitudes become

$$A_k=f(k)h_k^{\rm th}/\Delta_k,\quad B_k=-g(k)h_k^{\rm th}/\Delta_k, \tag{17}$$

with

$$\Delta_k=\left[\Omega_k^2-v^2\left(k+\frac{k}{\pi\omega_k}-\frac{q(k)}{\omega_k}\right)^2\right]\times\left[\Omega_k^2-v^2\left(k+\frac{k}{\pi\omega_k}+\frac{q(k)}{\omega_k}\right)^2\right].$$

The two functions $f(k)$ and $g(k)$ are irrelevant for the discussions of the consequences of Eqs. (17) on the wall velocity.

From Eqs. (8), (9), and (14) we can write an equation for $\ddot X$ as follows:

$$\ddot X+(2/Q)(\alpha_0+\lambda)\dot X=(2/Q)h, \tag{18}$$

with λ equal to the time average of

$$\frac{1}{2}Q\sum_k\frac{1}{\omega_k}\frac{d}{dt}(b_kb_{-k}-\tfrac{1}{2}a_{-k}a_k),$$

which comes from the quadratic contribution of the spin waves to the dipolar demagnetizing energy; thus the terminal velocity is $\dot X=h/(\alpha_0+\lambda)$, or

$$v=h\left(\alpha_0+\sum_k\frac{3Q\omega_k+1}{\omega_k}\frac{\mathrm{Re}[f(k)g^*(k)]}{\Delta_k^2}h_k^{\rm th2}\right)^{-1}. \tag{19}$$

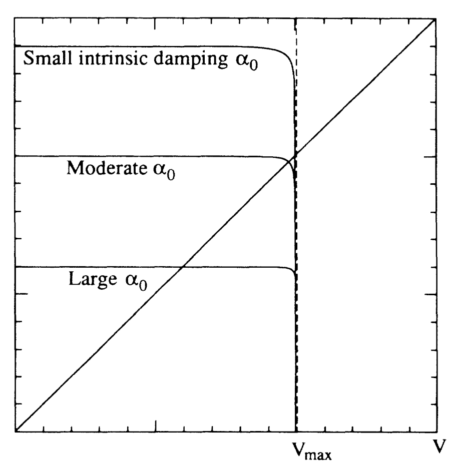

FIG. 1. Plot of the left-hand side of Eq. (19) (represented by the diagonal line), and of the right-hand side for different values of α_0. The solution to Eq. (19) is given by the intersection of those curves with the diagonal line. The abscissa of the dashed vertical line is $V_{\rm max}=\Omega_k/[k(1+1/\pi\omega_k)\pm q(k)/\omega_k]|_{\rm min}$. In the approximation $V_{\rm max}=\min(\Omega_k/k)$, we have $V_{\rm max}=(V_{\rm w}/2\pi)(1+\sqrt{1+1/Q})$, in excellent agreement with the case of $Gd_{0.3}Y_{2.7}F_{3.97}Ga_{1.03}O_{12}$ reported in Ref. 4, Fig. 1 and Table I.

Note that $h_k^{\rm th}$ satisfies $(h_k^{\rm th}/\alpha_0)^2 \sim k_B T/\hbar\,\Omega_k^{\rm lab}$, with $\Omega_k^{\rm lab} = \gamma M \Omega_k$.

Equation (19) for v can be solved graphically (see Fig. 1). For small fields, the expression for the velocity is still given by a Walker-type relation $v = h/\alpha_0$, until it reaches values within $h_k^{\rm th2}/k\,\Omega_k$ of the smaller of

$$\Omega_k \Bigg/ \left[\left(k + \frac{k}{\pi\omega_k} \pm \frac{q(k)}{\omega_k} \right)^2 + \frac{Q\omega_k - 1}{4} \right]^{1/2} \Bigg|_{\rm min},$$

at which point the excitation becomes large and temperature independent. The velocity "sticks" close to the value at which the right-hand side of (19) goes to zero.

That saturation velocity can also be obtained from a fixed-point analysis by equating all time derivatives and thermal fields to zero.

Thus we conclude that the velocity of a moving Bloch domain wall should reach a limiting value, corresponding to a new "fixed point" of the system. Physically this is due to power being diverted from the forward motion by amplification of thermal spin waves to large nonthermal values (and by spontaneous emission of these waves, but normally T is not low enough for this quantum effect to be comparable). It is possible that upon further increases in dc driving field, the new fixed point goes unstable, and yet another state is reached, perhaps corresponding to a limit cycle. These questions are currently being examined.

These results are not substantially changed when propagation perpendicular to the x direction is included. The synchronicity condition $\Omega_k = vk$ is then replaced by a vanishing of the energy gap between the spin-wave spectrum and the band of modes clinging to the wall, i.e., localized in x.

The authors gratefully acknowledge numerous discussions with Professor W. Baltensperger's group at Eidgenössische Technische Hochschule (ETH), Zurich, particularly with H. B. Braun and J. Broz. One of the authors (H.S.) further acknowledges the hospitality of ETH during the summer of 1989. Finally, we thank Professor A. Manohar for helpful discussions on soliton motion. This research was supported by the Office of Naval Research Contract No. N00014-89-J-1202 and by the Center for Magnetic Recording Research, University of California, San Diego.

[1] L. R. Walker (unpublished); see J. F. Dillon, in *A Treatise on Magnetism,* edited by G. T. Rado and H. Suhl (Academic, New York, 1963), Vol. III.

[2] N. L. Schryer and L. R. Walker, J. Appl. Phys. **45**, 5406 (1974).

[3] R. W. DeBlois, J. Appl. Phys. **29**, 459 (1958).

[4] B. E. Argyle, J. C. Slonczewski, and A. F. Mayadas, in *Magnetism and Magnetic Materials—1971,* edited by D. C. Graham and J. Rhyne, AIP Conference Proceedings No. 5 (American Institute of Physics, New York, 1971), p. 175.

[5] J. M. Winter, Phys. Rev. **124**, 452 (1961).

[6] A. A. Thiele, Phys. Rev. B **7**, 391 (1973).

[7] R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982), p. 237.

[8] Π is not a true momentum in that its quantum version is not a generator of infinitesimal translations. See F. D. M. Haldane, Phys. Rev. Lett. **57**, 1488 (1986).

[9] T. H. O'Dell, *Ferromagnetodynamics* (MacMillan, London, 1981), p. 40.

[10] That normalization proceeds as follows: $(W_k, W_{k'}) = \int_{-\infty}^{\infty} dx\, e^{i(k+k')x}[1 - kk' - 1/\cosh^2 x - i(k+k')\tanh x]$. The last two terms cancel out, and the remaining term gives $2\pi\omega_k \delta(k+k')$.

[11] Y. Wada and J. R. Schrieffer, Phys. Rev. B **18**, 3897 (1978).