Aharonov-Bohm–Type Effect for Vortices in Josephson-Junction Arrays

B. J. van Wees
Department of Applied Physics, Delft University of Technology, 2600 GA Delft, The Netherlands
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The dynamics of a single vortex present in a ring-shaped (Corbino geometry) two-dimensional array of low-capacity Josephson junctions is studied. The vortex is treated as a macroscopic quantum particle, whose energy levels $E_n(Q_0)$ are periodic functions of the externally induced gauge charge Q_0 which is enclosed by the vortex, with a period $2e$. This Aharonov-Bohm–type effect may manifest itself as a persistent voltage $V_s = dE_0(Q_0)/dQ_0$ between interior and exterior contacts, or alternatively as Bloch oscillations. The relation with the Aharonov-Casher effect and the possibility for experimental observation are discussed.

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The Aharonov-Bohm (AB) effect[1] has important implications for the motion of an electron in an electromagnetic field. The energy levels $E_n(\Phi)$ of an electron in an isolated ring[2] are periodic functions of the magnetic gauge flux Φ which is enclosed by the ring, with a period given by the flux quantum h/e. Because the circulating current in the ground state is given by $I(\Phi) = dE_0(\Phi)/d\Phi$, this has the interesting consequence that a persistent current may flow in this system, which depends periodically on Φ.

In this paper I discuss the possibility for an AB-type effect for a macroscopic particle. The particle is a vortex present in a ring-shaped (Corbino geometry) two-dimensional array of low-capacity Josephson junctions. It is predicted that under certain conditions the energy levels $E_n(Q_0)$ of the vortex depend periodically on the induced gauge charge Q_0 which is enclosed by the vortex, with a period $2e$. This AB-type effect manifests itself as a persistent voltage $V_s = dE_0(Q_0)/dQ_0$, which may be detected between interior and exterior contacts of the array, or alternatively as Bloch oscillations with period $2e$, when the array is driven with an external current $I = dQ_0/dt$.

Aharonov and Casher[3] (AC) have introduced an AB-type effect for a neutral particle with a magnetic moment which moves around a line charge. In a recent paper[4] this AC effect was discussed for a fluxon inside a superconductor, which moves around a charge. Although there are clear similarities between this system and the system discussed in this Letter, there are also important differences. The AC effect arises from the electromagnetic interaction between a particle with a magnetic moment (e.g., a fluxon in a superconductor which carries a magnetic flux $h/2e$) and a charge. In contrast, the vortex in the system discussed in this Letter does not carry magnetic flux. Also the AC effect occurs in "real" space, whereas the AB-type effect described in this Letter occurs in an artificial two-dimensional space, formed by superconducting islands coupled by Josephson junctions and capacitors.

Figure 1(a) shows the layout of the system. The two-dimensional array consists of $N \times M$ superconducting islands, each coupled to its four neighbors by both Josephson tunnel junctions and capacitors C [see also Fig. 1(b)]. The array is enclosed by superconducting boundaries 1 and 2 on both outside and inside, which also serve as electrical contacts. A charge Q_0 can be induced on the interior contact 2 by means of a capacitor C_0,

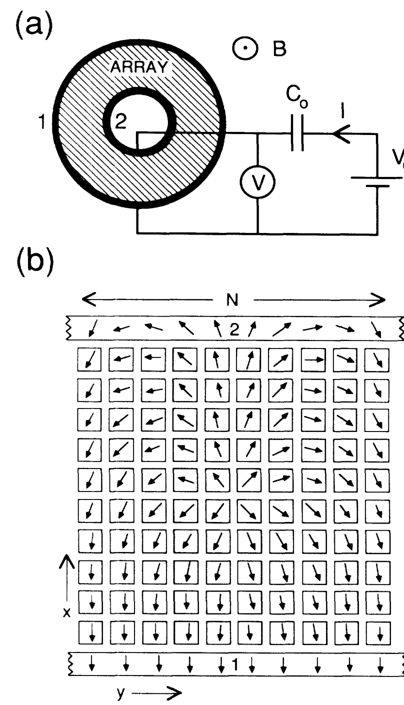

FIG. 1. (a) Schematic layout of the system. The two-dimensional array is enclosed by the superconducting boundaries 1 and 2. A gauge charge Q_0 can be induced on boundary 1 by means of a voltage source V_0 and capacitor C_0. The response of the system is measured by the voltage V. (b) Phase configuration of a vortex located in the center of the array. The Josephson junctions and capacitors which connect adjacent islands are not shown. The vector potential $\mathbf{A} = \mathbf{B} \times \mathbf{y}$. The array is periodic in the y direction, the last column of islands borders on the first column.

which is connected to a voltage source V_0. This technique was introduced to study the quantum dynamics of single junctions.[5] It was argued that when $C_0 \ll C$, the charge Q_0 can be treated as a classical variable, and acts as a gauge charge.

At zero temperature the dynamics of the array is determined by the ratio of the charging energy constant $E_c = e^2/2C$ and the Josephson coupling constant $E_J = \hbar I_c/2e$, with I_c the critical current of the junctions.[6] I am interested in the regime where E_c is sufficiently large, but does not yet dominate the dynamics of the array. I choose[7] $E_c = 0.5E_J$. The degrees of freedom of the array are the phases $\phi_{x,y}$ of each island, together with the phase ϕ of the interior contact (the phase of the exterior contact is set equal to zero). The dynamics of the array is determined by the condition of current conservation for each island. There are two types of solutions. The first category is formed by vortex-type solutions, for which the net Josephson current flowing to each island is zero[8] [a vortex is illustrated in Fig. 1(b)]. The second type of solutions can be illustrated by defining the phase differences

$$\Psi_{x,y} = 4\phi_{x,y} - \phi_{x+1,y} - \phi_{x-1,y} - \phi_{x,y+1} - \phi_{x,y-1} .$$

By linearizing the Josephson current-phase relation, the following differential equations are obtained:

$$d^2\Psi_{x,y}/dt^2 = -\omega_J^2 \Psi_{x,y} , \qquad (1)$$

with the Josephson frequency $\omega_J = (2eI_c/\hbar C)^{1/2}$. Equation (1) shows that the second type of solutions can be described by a set of harmonic oscillators.

In order to introduce a single vortex into the system, a perpendicular magnetic field is applied, which corresponds to a flux quantum $\Phi_0 = h/2e$ through the area of the array.[9] The system now minimizes its Josephson energy by introducing one vortex in the array [see Fig. 1(b)]. The vortex is confined in a potential well, which consists of a superposition of an approximately parabolic potential well[10] $E \approx 0.2E_J(x-x_0)^2$ (x_0 is the center of the array, and the distance between islands is put equal to unity), and a periodic potential with amplitude $0.07E_J$, which is associated with the crossing of the vortex from one cell to the next one.[11] Apart from its potential energy, the vortex can also have kinetic energy $E_{\rm kin} = \frac{1}{2} m_v v^2$, which is due to the charging energy associated with a moving vortex (the major contribution is from the junctions inside or near the vortex core). The mass of the vortex can be estimated[12] $m_v \approx 0.12h^2C/e^2$.

The voltage V across the array is approximately the sum of two parts: the voltage due to the dynamics of the array without a vortex plus the voltage due to the motion of the vortex alone. In the remainder of the paper I will ignore the first contribution[13] and focus on the motion of the vortex, which gives rise to the AB-type effect. I will now assume that this motion can be described in terms of the vortex coordinates (position and velocity of the vortex core) alone.[14] The oscillation frequency of the vortex in the potential well is then approximately given by[15] $\omega_V \approx (0.4E_J/m_v)^{1/2} \approx 0.2\omega_J$, and quantized energy levels $E_n = (n+\frac{1}{2})\hbar\omega_V$ are expected. Although the assumption of a one-dimensional system is not essential for the final result, I assume for the remainder of the paper that the motion of the vortex in the x direction is confined to its ground state.

In one revolution of the vortex around the array, the phase difference ϕ between the array boundaries increases with 2π. From this, one obtains a relation between the vortex velocity v in the y direction and the rate of change of ϕ: $d\phi/dt = 2\pi v/L$, with L the circumference of the array.[16] The power supplied by the external current source is given by $P = IV = Fv$, with F the force on the vortex, and V the voltage across the array. With the Josephson relation $V = (\hbar/2e)d\phi/dt$, one obtains the known result for the force on the vortex $\mathbf{F} = \Phi_0 \mathbf{j} \times \mathbf{n}$, with $j = I/L$ the sheet current density, and $\mathbf{n}$ the unit vector normal to the array.

The ring geometry implies that the vortex coordinates y and $y+L$ are identical. This means that the force on the vortex cannot be written as the gradient of a scalar potential $\mathbf{F} = -\nabla\Phi(\mathbf{r})$, since the line integral along the circumference of the array $\oint \nabla\Phi(\mathbf{r}) = 0$. The situation is similar to that of an electron on a ring, driven by a time-dependent gauge flux.[2] The force on the electron is given by Faraday's law: $\mathbf{F} = -e\mathbf{E} = -e\,d\mathbf{A}/dt$, with $\mathbf{A}$ the magnetic vector potential. To describe the force on the vortex I therefore introduce a charge vector potential $\mathbf{A}_Q$, and write the force on the vortex as $\mathbf{F} = \Phi_0 \mathbf{j} \times \mathbf{n} = \Phi_0\, d\mathbf{A}_Q/dt$. It should be noted here that there is a relation between this charge vector potential and the so-called modular electric field which was employed to demonstrate the nonlocality of the Aharonov-Casher effect in Ref. 4. The charge vector potential is related to the enclosed charge Q_0 by the line integral along the circumference of the array:

$$\oint \mathbf{A}_Q \cdot dl = Q_0 . \qquad (2)$$

The vortex Hamiltonian is now given by

$$H_v = \frac{(\mathbf{p} + \Phi_0 \mathbf{A}_Q)^2}{2m_v} + E_p(x,y) , \qquad (3)$$

with $\mathbf{p}$ ($=p_x,p_y$) the canonical momentum of the vortex. The potential energy $E_p(x,y)$ describes the spatial dependence of the Josephson energy of the vortex. It contains the parabolic confinement potential and the periodic potential. It may also contain a random contribution due to disorder in E_J for the individual junctions, which cannot be avoided in fabricated systems.

I now briefly discuss the physical meaning of the charge vector potential. The energy levels of a single junction are periodic functions of the induced charge Q_0, with a period $2e$. Similarly it can be shown[17] that the energy levels of an array of coupled Josephson junctions are also periodic when the gauge charge on a particular island is increased with $2e$. It follows from (2) and (3) that a vortex represents a special case, since its energy levels are periodic functions of the sum of the gauge charges induced on the islands which are enclosed by the vortex, irrespective of the distribution of these charges over the enclosed islands. This illustrates the topological nature of the effect. (Note that in the particular case discussed in this Letter, the charge Q_0 is induced exclusively on the center island.)

The spectrum of (3) consists of discrete energy levels $E_n(Q_0)$, which depend periodically on Q_0 with period $2e$. The presence of the $E_p(x,y)$ term gives rise to the formation of gaps at $Q_0=+e$, 0, and $-e$. The first two levels are schematically given in Fig. 2(a). When the disorder is not too strong, and the gaps are small, the energy of the vortex in the ground state is approximately given by the free-particle dispersion relation:

$$E_0(Q_0)=\Phi_0^2 Q_0^2/2m_v L^2 . \quad (4)$$

From this the maximum persistent voltage can be estimated: $V_s=dE_0(Q_0=e)/dQ_0 \approx 2e/CL^2$. A typical experimental value[18] is $C \approx 10^{-15}$ F, which gives $V_s \approx 3$ μV. In order to observe the vortex band structure, kT should be reduced below the energy-level spacing between the ground state and first excited level, which is about $8E_c/L^2$. This requires that the temperature is below 80 mK.

An alternative way to observe the AB effect for vortices is given in Fig. 2(b). This experiment is related to the interference between two fluxon beams, which is discussed in Ref. 4. The phase difference between the vortex paths A and B is equal to $2\pi Q_0/2e$, with Q_0 the gauge charge induced on the center superconducting island. The resulting interference between these paths may lead to a modulation of the voltage V measured between the top and bottom superconducting boundaries of the array.

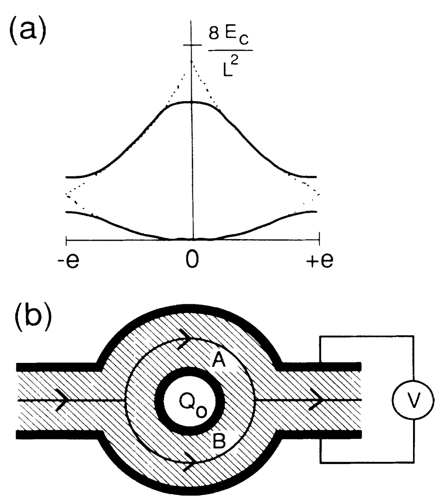

FIG. 2. (a) Energy levels of the vortex (for $N=M=10$) as a function of the gauge charge Q_0 (solid lines). Only the two lowest bands are shown. The dashed lines indicate the free-vortex band structure. (b) Alternative geometry for the observation of the AB effect for vortices (see text).

In the presence of (weak) quasiparticle tunneling, the effective gauge charge Q_0^* can differ from the externally induced Q_0 due to the tunneling of single electrons to the interior island. When Q_0 is changed adiabatically and $|Q_0|$ exceeds $\frac{1}{2}e$, the quasiparticle tunneling will reduce Q_0^* to the interval $-\frac{1}{2}e<Q_0^*<\frac{1}{2}e$, because in this way the ground-state energy is reduced [see Fig. 2(a)]. This means that the vortex energy levels will become periodic in Q_0 with period e. However, for $|Q_0| \ll \frac{1}{2}e$, the ground-state energy and the associated persistent current will not be affected by the quasiparticle tunneling. When Q_0 is changed fast enough,[17,19] so that the probability of a quasiparticle tunneling event is negligible in the time interval in which Q_0 increases with $2e$, it will still be possible to observe the vortex energy-level structure as oscillations of the voltage V (Bloch oscillations) with period $2e$.

It is clear from the above analysis that the vortex band structure is very brittle. Too large disorder in $E_p(x,y)$ will localize the vortex wave function, and the energy levels will become insensitive to Q_0. A coarse estimate can be made to discuss the effect of dissipation. Classically, the motion of the vortex is damped with a time constant $\tau=RC$, with R the subgap resistance of the junctions. This will induce a broadening of the energy levels by an amount $\Delta E \approx \hbar/\tau$. The broadening should be less than the level spacing, which gives the criterion $R>L^2\hbar/e^2$. Also the effect of the discrete lattice should be taken into account. The effect of the discrete lattice on the Josephson energy has already been included in the vortex Hamiltonian [Eq. (3)] as a periodic potential[20] with amplitude $0.07E_J$. The effect of the discrete lattice on the vortex mass is more difficult to understand. Eckern and Schmid[12] have nevertheless obtained the result that a vortex can move through the array as an almost free particle. However, in their model they did not treat the vortex core exactly.

An important question is how the vortex motion is affected by the interaction with the harmonic oscillators. To investigate this interaction, a computer calculation of the classical vortex motion was performed. It was found that a vortex, when released near a superconducting boundary, did not perform a harmonic oscillation. Also it was found that when an external current was applied, the supplied energy was divided equally among Josephson and charging energy, which shows that energy was transferred to the oscillators. However, these results, which indicate strong interaction, do not necessarily mean that the AB-type effect is destroyed. Recently, Geigenmüller and Schön[21] calculated the energy levels of a ring-shaped array, similar to that described in this Letter. Although the calculations were limited so far to systems with a small number (6 or 15) of junctions, the results confirm that the energy levels of the array are dominated by the presence of the vortex in the system, and also that the levels are periodic functions of the gauge charge on the interior island. Calculations on larger systems are in progress.

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[1]Y. Aharonov and D. Bohm, Phys. Rev. B **3**, 485 (1959).

[2]M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983); R. Landauer and M. Büttiker, Phys. Rev. Lett. **54**, 2049 (1985).

[3]Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).

[4]B. Reznik and Y. Aharonov, Phys. Rev. D **40**, 4178 (1989).

[5]F. Guinea and G. Schön, Europhys. Lett. **1**, 585 (1986); M. Büttiker, Phys. Rev. B **36**, 3548 (1987); T. D. Clark, T. P. Spiller, D. A. Poulton, R. J. Prance, and H. Prance, J. Low Temp. Phys. **78**, 315 (1990).

[6]It is assumed that the junction normal-state resistance $R_n \gg h/4e^2$, so that the effect of dissipation on the junction dynamics can be ignored.

[7]In a two-dimensional array the (zero-temperature) phase transition between superconducting and nonsuperconducting phases takes place at $E_c \approx 2E_J$ [S. Chakravarty, S. Kivelson, G. T. Zimanyi, and B. I. Halperin, Phys. Rev. B **35**, 7526 (1987)], which indicates that the Josephson coupling dominates at $E_c = 0.5E_J$.

[8]Strictly speaking the Josephson current does not have to be conserved in or near the core of the vortex. The absence of current conservation, and the consecutive charging of the capacitors near the vortex core constitutes the microscopic mechanism for the acceleration of the vortex when an external current is applied.

[9]Because of the small critical currents I_c, the magnetic field penetrates the array uniformly, and there is no flux quantum attached to the vortex.

[10]The energy of the vortex was calculated (for $M = N = 10$) as a function of its position by fixing the phases of two adjacent islands, and adjusting all other phases to minimize the energy.

[11]C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1983).

[12]U. Eckern and A. Schmid, Phys. Rev. B **39**, 6441 (1989); A. I. Larkin, Yu. N. Ovchinnikov, and A. Schmid, Physica (Amsterdam) **152B**, 266 (1988); T. P. Orlando, J. E. Mooij, and H. S. J. van der Zant (to be published).

[13]It is very difficult to calculate the exact energy levels of the array, and their dependence on the gauge charge Q_0. A possible argument may be that the maximum persistent voltage V_s is small because the array has many junctions in series and in parallel, and possibly behaves like a large Josephson junction with large effective E_J^* and small effective E_c^*, and an exponentially small [K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985)], $V_s \approx \exp(-8E_J^*/E_c^*)^{1/2}$.

[14]This will be correct when there is no interaction between the vortex and the harmonic oscillators, which is the case away from the vortex core where the current-phase relation can be linearized. However, in or near the vortex core the phase differences can be large, and the current-phase relation can be highly nonlinear.

[15]Because of the presence of a periodic potential the actual energy levels will be slightly modified, compared to those calculated for a strictly parabolic potential.

[16]Because of the periodic boundary conditions, the circumference of the array is constant, and is equal to N.

[17]For a review, see G. Schön and A. D. Zaikin (unpublished).

[18]L. J. Geerligs, M. Peters, L. E. M. de Groot, A. Verbruggen, and J. E. Mooij, Phys. Rev. Lett. **63**, 326 (1989).

[19]The rate (at $T=0$) for quasiparticle tunneling is given by $r = V/eR$, with R the subgap resistance. The voltage across the array is of the order of $V = 2e/CL^2$, which implies that the current $I = dQ_0/dt$ should be larger than e/RCL^2 to observe the Bloch oscillations.

[20]The periodic potential will lead to the formation of gaps in the vortex band structure. This only occurs at energies much higher than those considered in this paper.

[21]U. Geigenmüller and G. Schön (to be published).