Experimental Evidence of Chaotic Itinerancy and Spatiotemporal Chaos in Optics

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By increasing the Fresnel number F of a ring cavity with photorefractive gain, we show the transition between a low-F regime, where a few modes compete with a periodic alternation which becomes irregular with increasing F (chaotic itinerancy), and a high-F regime, where many modes oscillate simultaneously, giving rise to chaotic domains spatially correlated over a length much shorter than the wave-front size (spatiotemporal chaos).

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Since the proposal of Schawlow and Townes,[1] coherent optical oscillators have been considered as discrete physical systems where only one mode, or a few at most, can survive.[2] Even though photon statistics provides an accurate tool to test the noise dependence of the laser around the threshold of oscillation,[3] and even though a variety of instabilities and routes to optical chaos have been described,[4] so far optical devices have defeated the search for spatial dependence. On the other hand, hydrodynamic systems display a richness of behaviors as they are driven away from equilibrium. Beyond a spatially uniform dynamics, spatial structures are created, leading eventually to a situation of spatiotemporal chaos (STC) consisting of many uncorrelated chaotic domains.[5] The spatial coupling is provided by the gradient terms of the field equations ruling the instabilities. A spectral expansion reduces the dynamics to the interplay of a few modes only in the limit of small cells, that is, when the motion is strongly correlated across the cell size.

Being that the optical fields are ruled by a wave equation, it is surprising that up to now no clear evidence of spatial instabilities has been available in spite of the large amount of theoretical investigation. A wealth of experimental and theoretical information is available on (1+0)-dimensional systems,[6] as reported in Refs. 3 and 4; however, for the (1+1)- and (1+2)-dimensional cases, the experimental situation[7,8] is lagging with respect to the theory.[9,10] As for the (1+3)-dimensional case, no one has even theoretically considered the simultaneous presence of longitudinal and transverse modes.

In this Letter we report the first experimental evidence of (1+2)-dimensional physics in an extended optical medium. Precisely, we seed a ring cavity with a photorefractive gain medium pumped by an argon laser and study the temporal and spatial features of the generated field. By varying the size of the cavity pupil, we control the number of transverse modes which can oscillate. We report two different regimes, namely, one of a low-dimensional chaos, where a single mode at a time is oscillating, and a small number of modes alternate in a fashion which displays close similarities with the recently introduced concept of "chaotic itinerancy,"[11] and one of STC, where many modes oscillate simultaneously yielding a very small transverse correlation length and spectral fluctuations with Gaussian statistics.

The experimental setup, shown in Fig. 1(a), consists of a ring cavity with photorefractive gain.[12] The gain medium is a $5\times5\times10$ mm BSO (bismuth silicon oxide) crystal to which a dc electric field is applied. The crystal

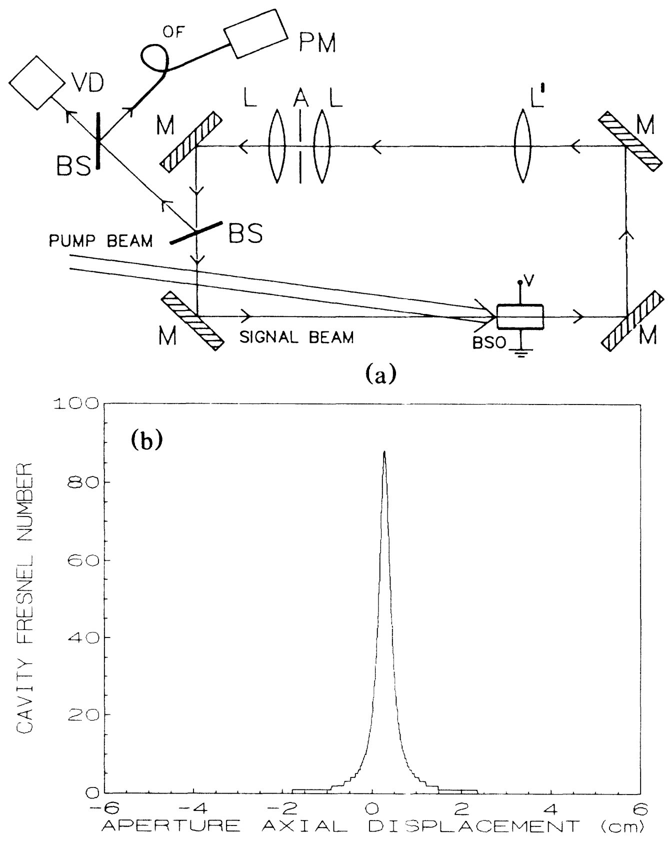

FIG. 1. (a) Experimental setup. Video camera VD records the wave-front pattern; photomultiplier PM measures the time evolution at a point selected by fiber OF. (b) Effect of the variable pupil on the cavity Fresnel number F. The horizontal axis reports the displacement of aperture A away from the focus of lens L. The slight asymmetry is due to the presence of lens L'.

is pumped by a cw argon laser with an intensity around 1 mW/cm^2. The basic cavity configuration consists of four high-reflectivity dielectric mirrors and a lens L' of 500-mm focal length, which enhances the cavity mode stability, providing a near-confocal configuration.

The Fresnel number of the cavity is controlled by a variable aperture. A pinhole of 300 μm diameter is inserted in the optical path between two confocal lenses L of short focal length (50 mm). Small displacements of the pinhole along the optical axis yield a continuous change of the aperture to spot size ratio, and consequently inhibit a different number of transverse modes. The effective Fresnel number F is the ratio of the area of the diffracting aperture that limits the system (pupil) to the size of the fundamental-Gaussian-mode spot, evaluated in the plane where the aperture is placed. F can be varied in the range from 0 to approximately 100 [Fig. 1(b)]. This corresponds roughly to the variation of the number of transverse modes that can oscillate. The mechanical and thermal stabilities are ensured on time intervals longer than those of the measurements (half an hour).

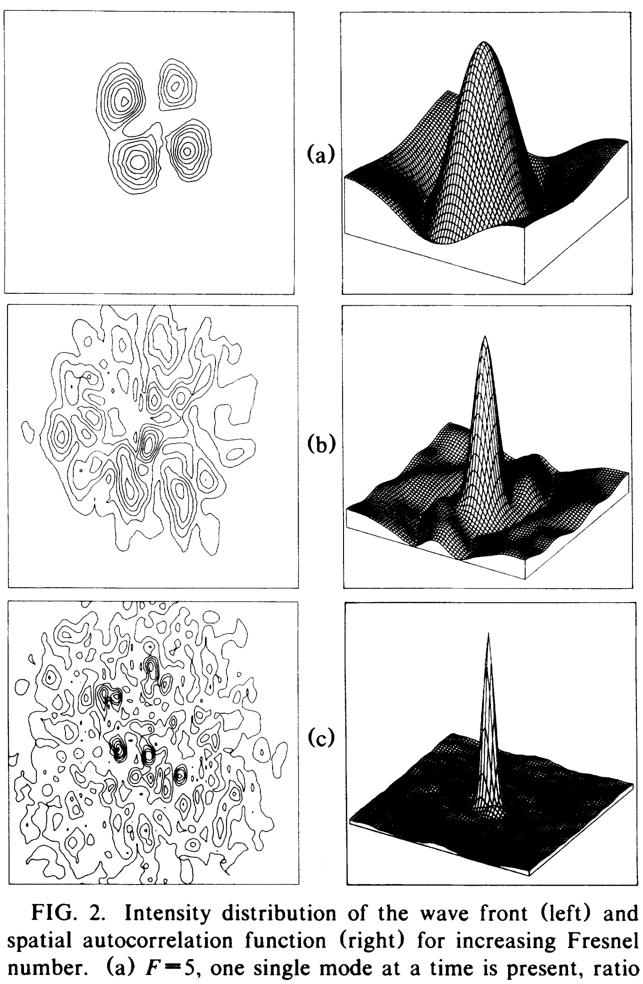

FIG. 2. Intensity distribution of the wave front (left) and spatial autocorrelation function (right) for increasing Fresnel number. (a) $F=5$, one single mode at a time is present, ratio between coherence length ξ and frame size D is $\xi/D \simeq 1$; (b) $F=20$, $\xi/D \simeq 0.25$; (c) $F=70$, $\xi/D \simeq 0.1$.

Figure 2 shows the transverse (x,y) intensity pattern recorded by the video camera (left) and its spatial autocorrelation function (right). The latter is measured by averaging over 200 statistically independent frames. For low F ($F \simeq 5$) one single mode at a time oscillates and the wave front is wholly correlated; indeed the correlation length ξ is of the same order as the cross size D of the beam [Fig. 2(a)]. For high F ($F \simeq 70$) many modes oscillate simultaneously, yielding a specklelike pattern [Fig. 2(c)] whose correlation length is very small ($\xi/D \simeq 0.1$). The correlation test is crucial; otherwise one might suppose that the intensity pattern at the left refers to a pure mode with a large mode number. Between these two asymptotic limits, we have a smooth variation of the ratio ξ/D, with intermediate situations as shown in Fig. 2(b).

The low-F limit corresponds to a periodic alternation of a few pure modes TEM_{0q} of the diffraction-limited propagation followed by a dark period. The radial quantum number is always 0 and the azimuthal quantum number changes from $q=0$ to q around 10. An example is shown in Fig. 3 (from now on, we identify the modes by their azimuthal quantum number).

To study the time behavior, the input of an optical fiber picks up the intensity at a generic point on the wave front (the signal level is a suitable code of each mode). The time plot shows fine details on a time scale of seconds, corresponding to the dielectric relaxation time of BSO. This time scale is typical of the fluctuations in a pure mode and of the intermode switches. Each mode persists for a time of the order of a few minutes. The mode pattern [e.g., 7,6,5,4,3,2 in Fig. 4(b)] repeats almost periodically. To improve the selectivity we change from the pinhole (low-pass filter) to the pinhole plus an axial stop (band-pass filter). For the same aperture size, introduction of the axial stop cuts off the lowest modes (1 and 0) and produces the regularization shown in go-

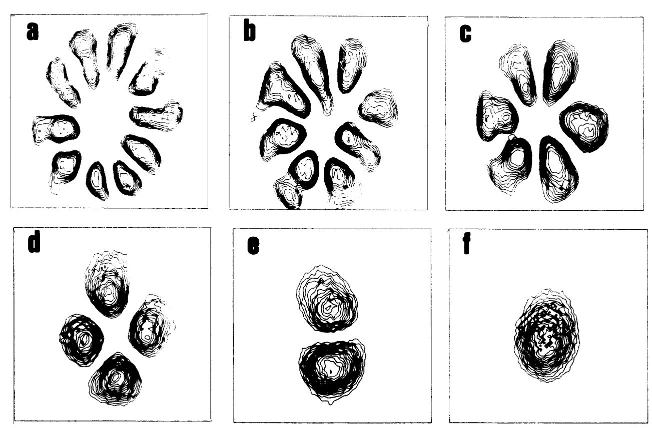

FIG. 3. Intensity patterns of the pure modes in their order of consecutive appearance in a cycle of periodic alternation at $F=5$.

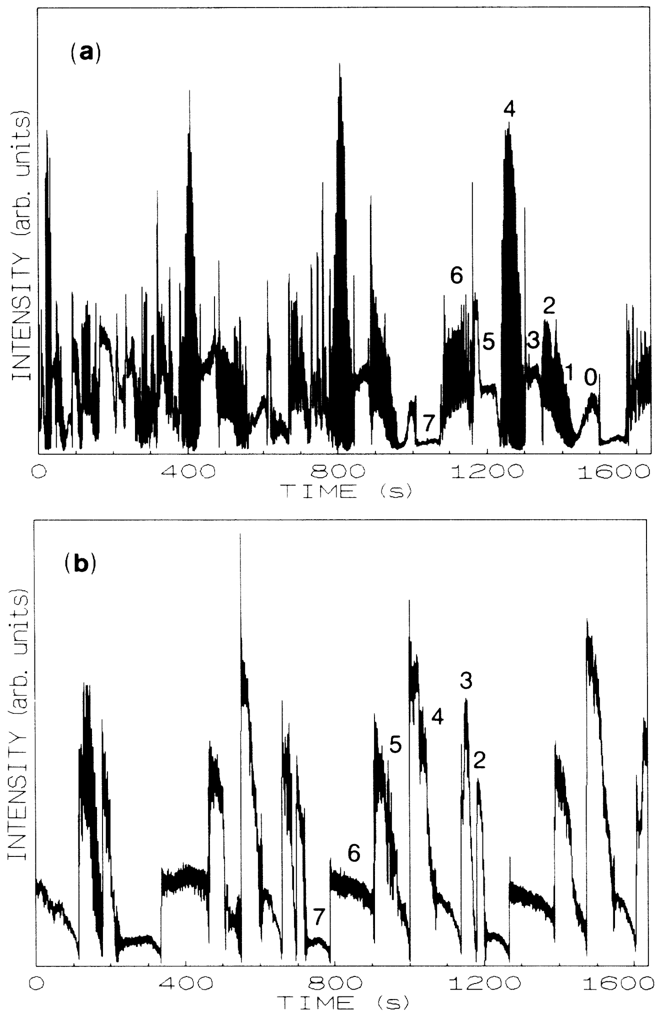

FIG. 4. Time records of the local intensity (samples collected at 10-Hz rate) at $F=8$: (a) with the low-pass filter; (b) with the band-pass filter.

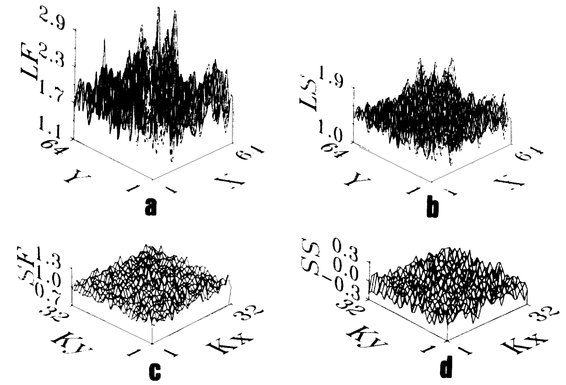

FIG. 5. (a),(b) Normalized local flatness (LF) and skewness (LS) for intensity fluctuations in (x,y) space. (c),(d) Normalized spectral flatness (SF) and skewness (SS) for intensity fluctuations in (k_x,k_y) space.

ing from Fig. 4(a) to 4(b).

At the minimum Fresnel number for which some signal is observed (F around 2) four different modes still oscillate one at a time, followed by a dark interval, in a very regular periodic sequence. We call such a behavior "periodic alternation." Increasing the pump intensity, the frequency of the alternation increases but it remains regular. For a slight increase of F above 5 ($F \simeq 8$) the regularity is lost; that is, the duration of each mode is no longer repeatable. This is an experimental evidence of the phenomenon called "chaotic itinerancy."[11] The heuristic explanation[11] is that the combination of a space gradient and nonlinearity is equivalent in Fourier representation to an interplay between dispersion and mode-mode coupling. As a consequence, the limited amount of energy provided by the pump is used successively by different modes. In Ref. 11 such a conjecture is confirmed by numerical experiments. Here, replacing the axial gradient with the transverse gradient due to diffraction, we have a similar phenomenology. To our knowledge, this is the first experimental evidence of chaotic itinerancy. Previous reports[12] on photorefractive oscillators do not contain information on the temporal sequences.

In the high-F limit, when $\xi/D \ll 1$, we expect spatiotemporal chaos. Considering the local intensity fluctuations at each point (x,y) of the wave front, a distinctive feature of STC is that, even though a collection of local fluctuation samples has non-Gaussian statistics, its Fourier transform in (k_x,k_y) has a Gaussian distribution.[5] STC can therefore be seen as a coarse-grained set of uncorrelated "pixels," each one of size ξ. To prove such a conjecture, we have measured (Fig. 5) the normalized skewness $M_3/M_2^{3/2}$ and flatness $M_4/3M_2^2$ for local and spectral intensity fluctuations (M_i being the ith-order moment of the statistical distributions). It is evident that local flatness (LF) and skewness (LS) have deviations from the Gaussian values (1 and 0, respectively) which are much larger than the residual fluctuations. On the contrary, in k space, both spectral flatness (SF) and skewness (SS) are centered at 1 and 0, respectively, showing evidence of STC.

In conclusion, we have reported experimental evidence of periodic alternation and STC as two asymptotic limits for very small and large Fresnel numbers in a (1+2)-dimensional optical system. At the lower edge of the intermediate region we have observed chaotic itinerancy. For still larger F we should expect transition phenomena which are not simply a mathematical bifurcation as in the case of the usual laser threshold,[3] but which display the scaling properties of phase transitions in extended media. We plan to explore this issue in a forthcoming work.

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