Orientation Fluctuations and the Angular Distribution of the Giant-Dipole-Resonance γ Rays in Hot Rotating Nuclei

Y. Alhassid and B. Bush

Sloane Laboratory, Center for Theoretical Physics, Yale University, New Haven, Connecticut 06511 and A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06511

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A recent macroscopic approach to the giant dipole resonances in hot rotating nuclei is extended to include the angular distributions of the γ rays emitted in the resonance decay. It provides a uniform description of thermal fluctuations in all quadrupole shape degrees of freedom within the framework of the Landau theory. In particular, the inclusion of fluctuations in the nuclear orientation with respect to the rotation axis is crucial in reproducing the observed attenuation of the angular anisotropy. The theory is applied to recent precision measurements in ^{90}Zr and ^{92}Mo and is the first to reproduce well both the observed giant-dipole-resonance cross sections and the angular anisotropies.

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The main experimental probe in the study of the shapes of hot rotating nuclei is the giant dipole resonance (GDR) built on nuclear excited states.[1,2] Most of the data available in the past were restricted to the GDR absorption cross section. However, recent measurements have also determined the γ-ray angular distributions.[2-5] These provide additional information on the role played by the deformation in heated rotating nuclei. A deformed nucleus is expected to have an anisotropic γ-ray angular distribution, and the magnitude of the anisotropy should increase with deformation.

Theoretically, we have introduced[6] the Landau theory of shape transitions as a mean-field theory in terms of which one can study the universal features of the equilibrium shape evolution versus temperature and spin. The importance of shape fluctuations around the equilibrium configuration in the finite nuclear system was recognized by several authors.[7-9] Their existence makes the relationship between the equilibrium shape and the data more complex and an accurate theory is required for a successful interpretation of experiments. We have developed a fluctuation theory within the framework of the Landau theory in which all five quadrupole shape degrees of freedom $\alpha_{2\mu}$ are treated uniformly.[10,11] This means that fluctuations in the nuclear orientation relative to the rotation axis are included in addition to fluctuations in the intrinsic shape. The former were neglected by other authors.[8,9]

With all parameters fixed by the zero-temperature nuclear properties, our fluctuation theory reproduces very well existing experimental GDR absorption-cross-section measurements in hot nuclei.[10,11] These cross sections are found to be dominated by the intrinsic-shape fluctuations. Although the inclusion of the orientation degrees of freedom was important in determining the metric for the fluctuations and its dependence on the intrinsic deformation, the orientation fluctuations themselves, if taken into account, were found to have negligible effects on the cross section in the rare-earth nuclei.[11]

In this Letter we show, however, that the orientation fluctuations play a very important role in the determination of the GDR γ-ray angular anisotropies. Furthermore, our unified fluctuation theory is shown to reproduce well both the GDR cross sections and the angular anisotropies of recent precision measurements in ^{90}Zr and ^{92}Mo nuclei[12] at finite temperature and spin where their equilibrium shape is expected to be a noncollective oblate. To the best of our knowledge it is the first time that both the cross section and the angular distribution of the GDR γ rays are well reproduced by a theory.

We start with the expression for the differential cross section for a nucleus of energy E and spin J to emit an electric dipole γ ray of energy ϵ and direction Θ (with respect to an axis to be specified later):

$$\frac{d\Gamma_{\rm em}}{d\epsilon\, d\Omega}=\frac{1}{2\pi\hbar}\left(\frac{\epsilon}{\hbar c}\right)^3\frac{1}{\rho(E,J)}\sum_{\substack{i,f\\ \mu,M_i}}|\langle fJ_fM_f|D_\mu|iJM_i\rangle|^2F_\mu(\Theta)\delta(E-E_i)\delta(E'-E_f)\,, \tag{1}$$

where $E'=E-\epsilon$. The sum in (1) represents an average over all initial states at energy E and spin J and a sum over all final states at energy E'. $\rho(E,J)$ is the initial density of states, D_μ is the dipole operator, and the angular functions $F_\mu(\Theta)$ are given by $F_0=2(1-P_2)/3$ and $F_{\pm1}=2(1+P_2/2)/3$, with $P_2(\cos\Theta)$ the Legendre polynomial of second degree.

Equation (1) can be expressed as[11]

$$\hbar\frac{d\Gamma_{\rm em}}{d\epsilon\, d\Omega}=\frac{1}{4\pi}\left(\frac{\epsilon}{\pi\hbar c}\right)^2\sigma(\epsilon)[1+a_2(\epsilon)P_2(\Theta)]\,, \tag{2}$$

where

$$\sigma(\epsilon)=\frac{2\pi\epsilon}{3\hbar^2 c}\int_{-\infty}^{\infty}dt\, e^{i\epsilon t/\hbar}\sum_{\mu}\langle D_\mu^\dagger(t)D_\mu(0)\rangle \qquad (3)$$

is the GDR absorption cross section and

$$a_2(\epsilon)=\frac{1}{2}-\frac{3}{2}\frac{\int_{-\infty}^{\infty}dt\, e^{i\epsilon t/\hbar}\langle D_0^\dagger(t)D_0(0)\rangle}{\int_{-\infty}^{\infty}dt\, e^{i\epsilon t/\hbar}\sum_{\mu}\langle D_\mu^\dagger(t)D_\mu(0)\rangle} \qquad (4)$$

is the angular-anisotropy parameter. In Eqs. (3) and (4), $D_\mu(t)$ is the dipole operator in the Heisenberg representation and the average is over the microcanonical ensemble at a given energy E and spin J.

As usual, we replace the microcanonical average by a canonical one with the corresponding temperature T and angular velocity $\boldsymbol{\omega}$. In doing so we have chosen a preferred direction $\boldsymbol{\omega}$, so that the γ-ray angular distributions are measured with respect to the rotation axis (i.e., approximately with respect to the spin direction). Experimentally a_2 is usually measured with respect to the beam direction which is perpendicular to the spin direction. We then have to multiply (4) by $-\frac{1}{2}$.

The laboratory dipole equilibrium correlation functions $\langle D_\mu^\dagger(t)D_\mu(0)\rangle$ in Eqs. (3) and (4) are calculated as in Ref. 11: For fixed quadrupole deformation $\alpha_{2\mu}$ ($\mu=-2,-1,\ldots,2$), we assume harmonic oscillations of a dipole which is rotating with angular velocity $\boldsymbol{\omega}$. The vibrational frequencies E_j^0 (at $\omega=0$) are as usual assumed to be inversely proportional to the corresponding semiaxis length. In terms of the normal-mode variables of the rotating oscillator, the correlation tensor is assumed to be diagonal and its corresponding Fourier transform in (3) and (4) gives Lorentzian functions with centroid $E_j(\boldsymbol{\omega})$ and width Γ_j. The width satisfies the power law $\Gamma_j=\Gamma_0(E_j/E_0)^\delta$, where Γ_0 and E_0 are the width and energy of the resonance built on a spherical nucleus. The resonance parameters E_0, Γ_0, and δ are assumed to be temperature independent and they are determined from the known ground-state GDR experimental cross sections. For δ we take[11] $\delta=1.6$. Transforming the correlation functions to the laboratory frame we find the contribution to σ and a_2 from a given $\alpha_{2\mu}$. Instead of $\alpha_{2\mu}$ we can use the Hill-Wheeler intrinsic-shape parameters β,γ and the Euler angles $\Omega=(\psi,\theta,\phi)$. The latter characterize the orientation of the principal frame with respect to the laboratory frame (in which $\boldsymbol{\omega}$ is parallel to z).

To account for thermal fluctuations we then average [both the numerator and the denominator in (4)] over all possible $\alpha_{2\mu}$, using the unitary invariant metric[10,11]

$$D[\alpha_{2\mu}]=\prod_{\mu}d\alpha_{2\mu}=\beta^4|\sin 3\gamma|\,d\beta\, d\gamma\, d\Omega\,, \qquad (5)$$

and the Boltzmann probability distribution

$$P[\alpha_{2\mu}]\propto\exp[-F(T,\omega;\alpha_{2\mu})/T]\,. \qquad (6)$$

In (6), $F(T,\omega;\alpha_{2\mu})$ is the free energy as a function of the deformation $\alpha_{2\mu}$ at temperature T and angular velocity ω. From the Landau expansion of the free energy we have,[6,11] to second order in ω,

$$F=F(T,\omega=0;\beta,\gamma)-\tfrac{1}{2}(I_{x'x'}\sin^2\theta\cos^2\phi+I_{y'y'}\sin^2\theta\sin^2\phi+I_{z'z'}\cos^2\theta)\omega^2\,, \qquad (7)$$

where $I_{x'x'}(T;\beta,\gamma)$, $I_{y'y'}(T;\beta,\gamma)$, and $I_{z'z'}(T;\beta,\gamma)$ are the nuclear moments of inertia along the principal axes x', y', and z'.

To compare our theory with experiment we have applied it to three cases for which precision measurements were recently taken:[12] ^{90}Zr at $T=1.6$ MeV, $J=9\hbar$; ^{90}Zr at $T=1.7$ MeV, $J=22\hbar$; and ^{92}Mo at $T=2$ MeV, $J=33\hbar$. The free-energy surfaces were constructed[13] from a cranked Nilsson-Strutinsky Hamiltonian. The cranking calculations were done only for $\boldsymbol{\omega}$ parallel to a principal axis from which the moments of inertia were determined as a function of β and γ. The free energy for a general orientation Ω is then determined by the expression (7). We have calculated the phase diagrams of these nuclei and found that at the equilibrium configuration $\omega=0.47$, 1.03, and 1.36 MeV, respectively, in each of the above three cases. The equilibrium shape is a noncollective oblate[13] ($\gamma=-180°$, $\theta=0$) whose deformation increases with spin; $\beta=0.02$, 0.08, and 0.16, respectively. When the metric (5) is included with the distribution (6), the resulting most probable shapes are triaxial ($\gamma\approx-150°$) and their deformation is significantly larger; $\beta=0.24$, 0.31, and 0.46, for the above three cases. From the ground-state GDR data[14,15] in ^{90}Zr and ^{92}Mo we have determined $E_0=16.82$ MeV and $\Gamma_0=5$ MeV. Both σ and a_2 were then calculated from (3) and (4) where fluctuations in the intrinsic deformation (β,γ) were included as well as those in the orientation (θ,ϕ). The results are shown by the solid curves in Fig. 1. They agree very well with the experiment as is demonstrated for the $J=22\hbar$ case where the data[12] are shown by the error bars. In particular, the theory reproduces accurately the observed broadening of σ at finite temperature. In judging the quality of the agreement between the calculated and experimental a_2, the region $E_\gamma\lesssim 11$ MeV should be disregarded since there the γ rays from daughter nuclei contaminate the signal.[12] The latter have lower spin and energy and tend to drive the observed a_2 to zero. Also at the high-energy side we are at the tail of the resonance and the error bars are large. Thus the range to consider for the a_2 fit is $11\lesssim E_\gamma\lesssim 20$ MeV. The experimental data for the other two cases also agree well with our predictions.[12]

The dashed lines in Fig. 1 are the results of similar calculations but with no orientation fluctuations. The

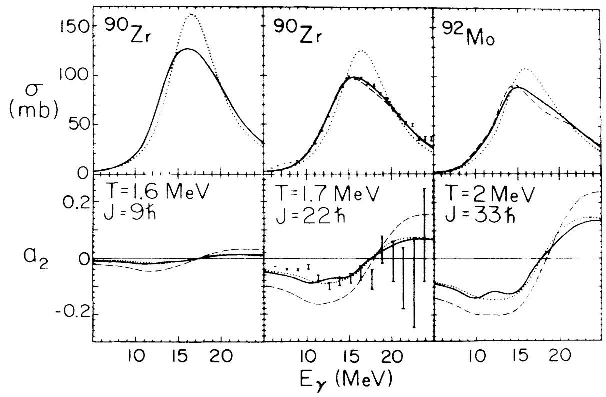

FIG. 1. The GDR cross sections σ (top row) and angular anisotropies a_2 (bottom row) vs γ-ray energy E_γ for ^{90}Zr and ^{92}Mo at finite temperature and spin. The solid lines show the results of the calculations based on Eqs. (3) and (4) where fluctuations in both the intrinsic shape and the nuclear orientation were included according to the metric (5). They agree very well with the experiment as is demonstrated for the $J=22\hbar$ case, where the experimental data (Ref. 12) points are shown by error bars. The agreement with the other two cases is of similar quality. The dashed lines show the results of similar calculations but with no orientation fluctuations. Notice the attenuation in a_2 when orientation fluctuations are included. Finally, the dotted lines show the results when the metric (8) is used (Ref. 8) instead of (5), and with no orientation fluctuations.

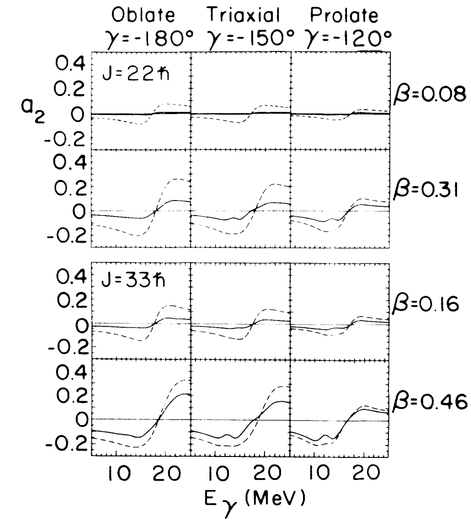

FIG. 2. Attenuation of a_2 due to orientation fluctuations. Top: The a_2 averaged over orientation at fixed deformation (β,γ) (solid lines) compared with a_2 at the equilibrium orientation $\theta=0$ (dashed lines) for ^{90}Zr at $T=1.8$ MeV and $J=22\hbar$. Shown are two values of β, the equilibrium value ($\beta=0.08$) and the most probable value ($\beta=0.31$), and three values of γ. Notice that the attenuation is stronger for oblate shapes (than for prolate) and at smaller deformations. Bottom: As above but for ^{92}Mo at $T=2$ MeV and $J=33\hbar$ where the equilibrium value of β is $\beta=0.16$ and the most probable value is $\beta=0.46$. Notice that at the higher spin the attenuation is weaker and the a_2 before orientation averaging is larger (for either the equilibrium or the most probable deformation).

metric (5) is used. Comparing with the solid lines, we see that the effect of the orientation fluctuations on the cross section is small. Only in the high-spin case ($J=33\hbar$) is this effect measurable, and providing a correction at the high-energy side of the resonance. However, the effect of the orientation fluctuations on a_2 is large; it causes a considerable attenuation in a_2 in agreement with the experiments.

The dotted lines in Fig. 1 show the results of the calculations where the fluctuations are evaluated with a metric used by other authors[8,9]

$$D[\alpha]=\beta d\beta d\gamma\,, \tag{8}$$

which includes no orientation fluctuations. We see that these results are in strong disagreement with the measured GDR cross sections. The GDR widths are considerably underestimated by the calculations and their shape is different from the experimental ones. The a_2's, on the other hand, are close to the ones calculated with the metric (5) and with orientation fluctuations. If a temperature-dependent width[16] is added to fit the experimental width of the GDR cross sections, then a disagreement with a_2 will result. Also, if orientation fluctuations are included with the metric (8), then a_2 will be strongly attenuated relative to the experiment. Thus only a theory with intrinsic-shape and shape-orientation fluctuations according to the unitary metric (5) can reproduce both the observed cross sections and the a_2's. The good agreement between experiment and theory is also an indirect confirmation of the existence at finite temperatures of noncollective oblate shapes whose deformation increases with spin.

The effects of fluctuations on a_2 can be understood by calculating $a_2(\epsilon;\beta,\gamma)$ at a fixed intrinsic deformation β,γ through an average over all orientations $\Omega=(\theta,\phi)$. In Fig. 2 (top panel) we compare the averaged $a_2(\epsilon;\beta,\gamma)$ (solid lines) with a_2 of the equilibrium orientation $\theta=0$ (dashed lines) for two values of β, the equilibrium and the most probable ones, and three values of γ, oblate, triaxial, and prolate, all at $J=22\hbar$. The net effect of the orientation fluctuations is to reduce the a_2 anisotropy. This attenuation is stronger in the oblate case than in the prolate case. For the equilibrium deformation $\beta\approx0.08$ the attenuated a_2 is almost zero. However, the most probable β is 0.31, for which the resulting anisotropy is significant. It is seen that the enhancement in a_2 due to intrinsic-shape fluctuations is counteracted by its suppression due to orientation fluctuations, so that the observed a_2 is an indirect indication of the equilibrium shape.

To understand the spin dependence of the a_2 attenuation due to orientation fluctuations, we show in Fig. 2 (bottom panel) similar calculations but for $J=33\hbar$.

Now the equilibrium deformation ($\beta=0.16$) and the most probable deformation ($\beta=0.46$) are larger. We see from Fig. 2 that for higher spin, a_2 is less attenuated. The main reason is that the distribution (6) becomes more peaked around the most probable orientation $\theta=0$ (due to the ω^2 factor in the exponent) so that the fluctuations are reduced.

The anisotropy parameter a_2 is more sensitive to the spin (than it is to temperature) and its magnitude increases with spin (see Fig. 1). This is mostly due to the increase of the equilibrium and most probable deformations with spin and partly due to the weaker attenuation at higher spins as explained above.

To conclude, we have demonstrated that a macroscopic fluctuation theory that treats uniformly all five quadrupole degrees of freedom in the framework of the Landau theory can simultaneously provide a good description of the GDR γ-ray cross section and their angular anisotropy. This unified description of fluctuations leads to the unitary metric $\beta^4|\sin 3\gamma| d\beta\, d\gamma\, d\Omega$. In particular, we obtain good agreement with recent precision experiments[12] in excited ^{90}Zr and ^{92}Mo nuclei. The success of our theory implies that the quadrupole degrees of freedom $\alpha_{2\mu}$ are the correct order parameters for the hot rotating nucleus. In this paper we have assumed that these degrees of freedom are adiabatic. To study nonadiabatic effects[13] on a_2 we can still use Eq. (4) but now the correlation functions should be calculated by the methods of Ref. 17.

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