Quantum Color Transparency

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We propose a quantum description of color transparency in terms of light-cone operator matrix elements. The fundamental quantity needed is the distribution amplitude $\phi_A(x,Q^2)$ for each hadron that participates in the elastic scattering in the nuclear target. Two large scales, Q^2 and a mass scale going like $A^{1/3}$ for $A \gg 1$, occur in a perturbatively consistent description. We find that the limit of large A and moderate Q^2 may shed light on controversies over the applicability of perturbative QCD to exclusive processes.

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Much of the intuition of the high-energy limit is semiclassical, involving probabilities for partons to interact. The formalism of perturbative QCD is quantum mechanical, of course, but the marvelous property of factorization allows sets of diagrams to be organized into initial- and final-state interactions. Then the parton model follows for inclusive reactions. Factorization applies in a less direct way to exclusive reactions, where scattering amplitudes rather than probabilities are central. Semiclassical aspects are more or less anticipated, as in the arguments for color transparency by Brodsky and Mueller.[1] In color transparency it is claimed that a hadron undergoing a large-momentum-transfer elastic collision should be geometrically small. Consequently, the probability to interact with nuclear matter is small, and attenuation should decrease as the momentum transfer increases. The first data[2] show evidence for transparency, although the $pp \to pp$ case turned out to be subtle to compare the nuclear to free-space scattering.[3] Various ways to implement the semiclassical ideas have been introduced in Ref. 4. However, a quantum-mechanical description has been lacking.

In this paper we outline a description of quantum color transparency, or QCT. We put physical thinking ahead of mathematical proof and present an educated guess of what the quantum description should be. New issues similar to the need to prove factorization come in. Assuming such proofs could be made, experiments that measure color transparency can be interpreted in terms of certain well-defined matrix elements and not just as nuclear physics phenomenology. A second result of this paper is that the hadronic states probed by exclusive data depend in an intricate way on two large scales, one increasing with the size of the nucleus and the other being the momentum transfer Q^2. The size of the participating hadrons is small for two independent reasons. The states measured in nuclear transparency experiments are not ordinary hadrons, but new objects which we call minihadrons. There is a nonperturbative aspect to the nature of a minihadron due to the soft interactions of stripping an ordinary hadron down to a miniature one. By isolating the nonperturbative part and looking at the general quantum description we have an indication of how data should be analyzed.

In free-space exclusive processes[5,6] the fundamental object is the light-cone distribution amplitude $\phi_0(x,Q^2)$ of quarks in a participating hadron. The definition is[6]

$$\phi_0(x,Q^2)=\int^Q d^2k_T\,\psi_0(x,\mathbf{k}_T^2)\,. \tag{1}$$

Here $\psi_0(x,\mathbf{k}_T^2)$ is the minimal Fock-space projection amplitude to find quarks with longitudinal momentum xP and transverse momentum $\mathbf{k}_T$. For simplicity of notation we assume one pair of quarks. The formalism for the three-quark, proton case is a straightforward modification.[6] The dependence on the large momentum scale Q^2 should be emphasized. $\phi_0(x,Q^2)$ satisfies renormalization-group equations and thus has calculable evolution at large Q^2, going roughly like $\ln^\gamma(Q^2/\Lambda_{\rm QCD}^2)$.

For QCT the fundamental object is the analog of $\phi_0(x,Q^2)$ for each active hadron that has a hard scattering inside the nuclear medium. For definiteness consider the pion knockout reaction $\pi A \to \pi'\pi'' A$, where the nucleus has nuclear number A. We are concerned with soft elastic interactions of the quarks in the pions. At the end of a chain of soft collisions there is a hard-scattering kernel $H(x_i,Q^2)$. This is shown in Fig. 1(a). The hard scattering itself is perturbative, independent of the nu-

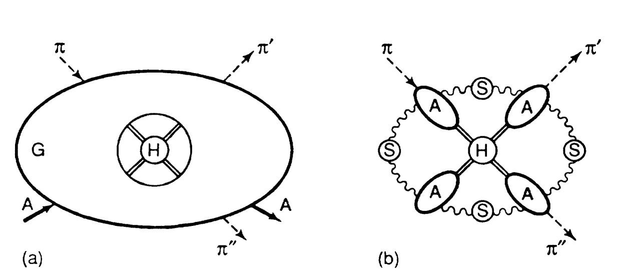

FIG. 1. (a) General decomposition of the scattering in nucleus A using a quark hard-scattering kernel $H(x,Q^2)$ and an effective multiquark wave function G. (b) Factorized form leading to Eq. (2).

clear number A, and independent of the location of the scattering inside the nucleus. In the sum over diagrams, $H(x_i,Q^2)$ will occur automatically at all allowed places in the nucleus.

The reaction amplitude is thus the convolution of the hard scattering and a big wave function G to find a number of quarks in a small scattering region. The integrals for the amplitude M can be written as

$$M=\int\prod_i dl_i \int\prod_j dk_j\, G(l_i,k_j)H(k_j,Q^2)\,,$$

where k_i are the active parton momenta and l_i are loop momenta related to interactions of the participating quarks with spectators (not shown). But if the diagrams can be separated into soft and collinear initial- and final-state interactions by the momentum flow, then we should be able to cut apart Fig. 1(a) to get Fig. 1(b). In Fig. 1(b), the S blobs represent soft interactions which have strong cancellations because the hadrons are color singlets. The A blobs represent collinear jets along the distinct directions of the active high-energy hadrons.

If such a factorization exists, and ignoring the soft interactions for the moment, then the scattering simplifies to the form

$$\int\prod_j dk_j\, H(k_j,Q^2)\hat\psi_A^{(1)}(k_1)\psi_A^{(2)}(k_2)\psi_A(k')\psi_A(k'')\,. \quad (2)$$

This factorization represents the physical picture that the impulse approximation can be applied to the hard collision. This is not a general property of field theory but a special feature of coherence in the gauge theory.

Computing Eq. (2) requires the nuclear light-cone distribution for the hadron that has interacted with the nucleus, $\phi_A(x,Q^2)$:

$$\phi_A(x,Q^2)=\int^Q d^2k_T\,\psi_A(x,\mathbf{k}_T^2)\,. \quad (3)$$

We would like to relate ψ_0 to ψ_A. Consider the elastic scattering of a quark on the nucleus. First, take a quark beam with a wave function that is a plane wave in the $\hat{\mathbf{z}}$ direction, $\delta(1-x)\delta^2(\mathbf{k}_T)$. Let the scattering amplitude for quark energy s and momentum transfer t in target A be $F_A(s,t)$. If x comes in and $x',\mathbf{k}_T'$ goes out, then $-t=(x/x')\mathbf{k}_T'^2$.

For a wave packet given by amplitude $\psi_0(x,\mathbf{k}_T^2)$ coming in, by superposition we have a scattered wave

$$\psi_s(x',\mathbf{k}_T'^2)=\int dx\int d^2k_T\,F_A\left(s,\left[\frac{x}{x'}(k_T-k_T')\right]^2\right)\psi_0(x,\mathbf{k}_T^2)\,.$$

As usual, we assume that there are no severe small-x problems so that the x dependence is inessential. Moreover, since the x dependence of wave functions is not well known and model dependent, we suppress it, letting F_A be proportional to $\delta(x-x')$ for this exposition. Going to impact-parameter (b) space, with Fourier transforms indicated by tildes, we have a transmitted wave ψ_A which is the original wave minus the scattered wave,

$$\tilde\psi_A(x,b)=\tilde f_A(s,b)\tilde\psi_0(x,b)\,,$$

where $\tilde f_A=1-\tilde F_A$ will be denoted as the nuclear filtering amplitude.

It is conventional[7] to write $\tilde f_A$ in terms of an eikonal function $\chi(b)$ as $\tilde f(s,b)=1-e^{i\chi(s,b)}$. Our analysis is general, and applies whatever the model for the scattering, but to focus the discussion we recall that in hadronic physics $\chi(s,b)$ typically goes like $\chi(s,b)\sim is^\alpha\exp(-b^2/2b_0^2)$. Expanding the eikonal to lowest order in χ reproduces the naive Born-level picture, which is instructive:

$$\lim_{|\chi|\ll 1} f_A(s,b)\cong s^\alpha e^{-b^2/2b_A^2}\,.$$

Here the exponential in b^2 represents the cutting off of large-impact-parameter quarks beyond a scale b_A^2. Evidently b_A^2 is of order $A^{-1/3}$ fm^2 for large A, since we could write

$$e^{-b^2/2b_A^2}\cong e^{-Z(A)/2\lambda},\quad \lambda\cong\frac{1}{n\sigma(b^2)},\quad \sigma(b^2)\cong b^2\,,$$

with n the average nuclear density seen in the problem and $Z(A)\cong A^{1/3}$, a distance scale in units of fermis.

The main effect of traveling through the nucleus should be filtering, i.e., the cutting off of quarks separated by a large impact parameter.[8] Let us examine the general effect of this on $\phi_A(x,Q^2)$. We now have

$$\phi_A(x,Q^2)=\int_0^Q d^2k_T\int d^2b_T\, e^{i\mathbf{b}_T\cdot\mathbf{k}_T} f_A(s,b^2)\psi_0(x,b)$$
$$=(2\pi)^2Q\int_0^\infty db\, J_1(Qb)f_A(s,b^2)\psi(x,b)\,, \quad (4)$$

upon doing the integrals to find a Bessel function $J_1(Qb)$. The content of this formula is in its Q dependence, which is entirely in the $QJ_1(Qb)$ combination. This just comes from Q in the upper k_T limits, i.e., factorization. The remaining content is in the A dependence, entirely from the filtering amplitude $\tilde f_A$. Even assuming the full $\tilde f_A$ is complicated and model dependent, we have the power-series expansion around $b=0$ of

$$\tilde f_A(s,b^2)=1-A^{1/3}nb^2\sigma'_{\rm eff}+\cdots\,,$$

where $b^2\sigma'_{\rm eff}$, by definition, is an effective cross section. We are assuming only that, at $b=0$, cancellation of the color dipole moment of the singlet minihadron occurs, so that $\tilde f_A(b=0)=1$. [Actually, we do not have to limit ourselves to a filtering amplitude that is analytic at $b=0$. There is a negligible perturbative component to the scattering that should go like $\alpha_s(1/b^2)$.]

The only physical input so far has been to propose factorization and incorporate filtering in the nucleus. Nevertheless, transparency is so general that this is all we need to assume. The point is that even though $\tilde{f}_A$ has a filtering effect with a large cross section, say, $\sigma'_{\rm eff} \sim 1$ fm^2, at large enough Q the integral in (4) is *independent of A;* in fact, it gives the free-space value

$$\lim_{Q\to\infty} \phi_A(x,Q^2) = \phi_0(x,Q^2)\,.$$

This comes about because the integral as $Q\to\infty$ over the Bessel function $J_1(Qb)$ only requires the value of $\tilde{\psi}_A(x,b)$ as $b\to 0$.

To show this quantitatively we develop an asymptotic series in large Q^2 from (4). It is not safe to expand $\tilde{\psi}_A(x,b^2)$ around $b=0$ (a possible method) because this region is renormalization-group determined to diverge like powers of $\ln(b^2\Lambda^2_{\rm QCD})$. We use a Mellin transform which is systematic. Let

$$\phi_{A,N}(x) = \int_0^\infty \frac{dQ}{Q} Q^{-N} \phi_A(x,Q) = (2\pi)^2 2^{-N} \frac{\Gamma(1-N/2)}{\Gamma(1+N/2)} \tilde{\psi}_{A,N}(x)\,, \qquad (5)$$

using (4). Here $\tilde{\psi}_{A,N}$ is the impact-parameter moment

$$\tilde{\psi}_{A,N} = \int_0^\infty \frac{db}{b} b^N \tilde{\psi}_A(x,b)\,.$$

Note the b moments are defined with the negative $-N$ of the Q moments; this is because powers of b will turn out to map into powers of $1/Q$ (modulo logarithms).

Inverting ϕ_N, complex N-plane singularities at $N=0$ are leading twist; those at $N=-2$ give $1/Q^2$ higher twist, and so on. The wave function $\tilde{\psi}_A(b)$ determines the $N=0$ singularity:

$$\tilde{\psi}_{A,N}(x) = \int_0^\infty \frac{db}{b} b^N \tilde{\psi}_0(b)[1 - b^2 A^{1/3} n\sigma'_{\rm eff} + \cdots]\,.$$

Note that the $N\to 0$ behavior is fixed by the *free-space* wave function $\tilde{\psi}_0(b)$. Here the effect of interaction with the nucleus is a factor "1." The $A^{1/3}\sigma'_{\rm eff}$ dependence affects the $N\to -2$ behavior, and is thus suppressed by $1/Q^2$. This is transparency. It is remarkably general in the asymptotic limit $Q^2\to\infty$.

The expansion in powers of $A^{1/3}$ and $1/Q^2$ is tricky. It can be clarified by the physical picture that as $Q^2\to\infty$, with fixed A, the active minihadrons get smaller and smaller and attenuation is turned off. That has been shown above. However, for fixed large Q^2, one could always take $A\to\infty$, a huge nucleus, and there should be filtering and attenuation for large enough A. So the large-A and large-Q^2 limits cannot be uniform. Both are good perturbative limits, nonetheless.

The large-A limit, like large Q^2, has the effect of cutting off large impact parameters. We note that it has been controversial whether or not such "soft physics" regions contribute a significant amount to the data. It is a general feature of QCD that the scale at which the perturbative treatment begins to work has to be determined experimentally. Isgur and Llewellyn Smith,[9] for example, have argued vehemently that soft physics dominates even the highest-energy data available. For this question we return to the soft-exchange blobs we postponed discussing above. In general, infrared effects will be regulated by the effective transverse size of the hadrons. The effective infrared cutoff scales like the *larger* of the incoming hadron intrinsic k_T, the scale Q^2, and the scale from the nuclear filtering amplitude $A^{1/3}\sigma'_{\rm eff}$. For large enough A the nuclear filtering wins. Thus, even if Q^2 is not large enough to guarantee a perturbative treatment, moderate Q^2 and large nuclear number A should provide the infrared protection to justify the perturbative picture. Evidence for this has already been found in the filtering away of large-impact-parameter-independent scatterings, as argued intuitively in Ref. 3. The consequences of eliminating soft physics at large A are quite significant.

There is a last loophole to be closed. Consider the hadron that resides inside the nucleus before being struck. For very large Q^2 it will be small and there is no problem. For moderate Q^2 and very large A, however, this hadron's wave function is not filtered by the $A^{1/3}$ attenuation and, in fact, should be independent of A. Thus the argument seems to break down. However, in QCD the struck hadron is protected by the smallness of the other participating hadrons. It must be small to interact with them. The struck-hadron distribution amplitude $\hat{\phi}_A(x,Q^2)$ remains rather different from the other amplitudes in encoding different initial-state interactions. All of the effects of interactions with the nucleus are contained in $\hat{\phi}_A$ and ϕ_A because of factorization. Fermi motion of the nucleons, for example, is automatically included in the complete ϕ_A.

An example illustrates several points. In our formula (4) we insert a typical soft wave function

$$\tilde{\psi}_0(b) = e^{-mb}\,.$$

For the nuclear effects we want a filtering amplitude that is 1 at $b=0$ and has powers of $b^2A^{1/3}\sigma'_{\rm eff}$. The integrals are doable for

$$\tilde{f}_A(b) = \exp[-(b^2A^{1/3}\sigma'_{\rm eff})^{1/2}]\,.$$

Now, letting $\tilde{\psi}_A(b) = \tilde{\psi}_1(b)\tilde{f}_A(b)$, we evaluate (4) to obtain

$$\phi_A(Q^2) = \phi_0(Q^2)\left[\left(1 - \frac{m + (A^{1/3}\sigma'_{\rm eff})^{1/2}}{[m + (A^{1/3}\sigma'_{\rm eff})^{1/2}]^2 + Q^2}\right) \Big/ \left(1 - \frac{m}{(m^2+Q^2)^{1/2}}\right)\right]. \qquad (6)$$

Here we see the following explicitly: as $Q^2 \gg A^{1/3}\sigma'_{\rm eff}$, $\phi_A(Q^2)\to\phi_0(Q^2)$. This is general. As Q^2 is fixed, but $A^{1/3}\sigma'_{\rm eff} \gg Q^2$, the very large nucleus limit, there is nevertheless attenuation.

The factor $T(Q,A)$ multiplying $\phi_0(Q^2)$ in (6) can serve as a crude representation of the effects of QCT. Roughly speaking we have one factor of $T(Q,A)$ for each hadron that must cross the nucleus in the scattering. The real situation is more complicated, involving integrations over the x dependence of the hard scattering and the wave functions. In Fig. 2 we plot $T(Q,A)$ vs Q^2 and A. The plots are not to be compared directly with the data, but to show that everything is in accord with the physical arguments. The plots show the same features as the semiclassical model of Ref. 4 in which the idea of transparency has been imposed by hand.

We now turn to the problem of analyzing the data. For definiteness suppose a given electromagnetic form factor of the proton inside a nuclear target $F_A(Q^2)$ has been measured in $ee'p$ reactions. A natural measure of transparency is the ratio $F_A(Q^2)/F_0(Q^2)$, where $F_0(Q^2)$ is the free-space case. This ratio can be studied from a lower limit $Q_0 \sim 1$ GeV to the upper values measured. A fit by the ratio in powers of $\ln(Q^2/\Lambda_{QCD}^2)$ and powers of Q_0^2/Q^2 is good for relating data to the minihadron nuclear distribution amplitudes. That is,

$$\frac{F_A(Q^2)}{F_0(Q^2)} = \sum_{M,J} \frac{\ln^J(Q^2/\Lambda^2)}{(Q^2/Q_0^2)^M} C_{M,J}$$

has the form that is good to compare to theory. The form is good because it arises naturally from Q moments. Depending on models of the x dependence, theory can relate the small-impact-parameter behavior of $\tilde{\psi}_A(b)$ to the first few coefficients $C_{M,J}$. Conversely, experiments measuring $C_{M,J}$ can give information on the filtered wave functions and tell us the filtering cross section. The process of relating $C_{M,J}$ to an expansion of $\tilde{\psi}_A(b)$ around $b=0$ is somewhat involved and need not be presented here. Conceptually, we separate the model-dependent details of predicting $\tilde{f}_A(s,b)$ and $\phi_A(x,Q^2)$ from their use in the perturbative formalism of QCT.

In conclusion, we have shown how color transparency can be obtained with very general quantum-mechanical assumptions. The fundamental object is $\phi_A(x,Q^2)$, the nuclear light-cone distribution amplitude. An explicit formula relates $\phi_A(x,Q^2)$ to the free-space $\phi_0(x,Q^2)$. We anticipate that, with models of the x dependence, the Q^2 dependence of $\phi_A(x,Q^2)$ can be related to the data. Thus this fundamental object will be measured by upcoming experiments.

We have emphasized the interplay of large nuclear number A and large Q^2. By studying exclusive processes at large A, long-standing controversies over the applicability of perturbative QCD may be approached from a new point of view.

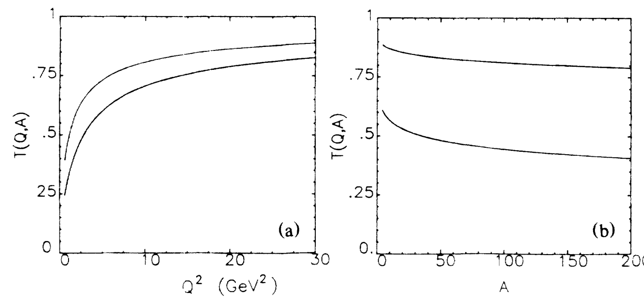

FIG. 2. Model QCT transparency factor $T(Q,A)$ relating the nuclear light-cone distribution to the free-space one [Eq. (6)] with parameter $m = 0.3$ GeV. (a) At fixed A, transparency increases with increasing Q^2; $A=12$ (200) for the upper (lower) curve. (b) At fixed Q^2, transparency decreases with increasing A; $Q^2 = 2$ (20) GeV^2 for the lower (upper) curve.

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