Energy Exchanges between a Rotating Retardation Plate and a Laser Beam

A frequency shift of a laser beam due to its interaction with a rotating quarter-wave plate has been reported by Simon, Kimble, and Sudarshan.[1] This shift was then claimed to be new and interpreted as a dynamical manifestation of Berry's phase shift. We wish to point out the fact that such frequency shifts are well known and can be interpreted as energy exchanges. Indeed, since Beth,[2] it has been shown that a circularly polarized light exerts a torque on a retardation plate, as predicted by Poynting and Kastler.[3] Consequently, when such a plate rotates, it exchanges work with the light, as more easily shown in microwave experiments.[4] In the experiment of Ref. 1, the light passes twice through a rotating quarter-wave plate, that is equivalent to a single half-wave plate. Consequently, we send a linearly polarized beam from a monomode 3.39-μm laser onto a rotating half-wave plate (see Fig. 1). The Jones vector of the light emerging from this plate is

$$\mathbf{E}=\begin{bmatrix}\cos(2\Omega t)\\ \sin(2\Omega t)\end{bmatrix} =\frac{1}{2}\left\{\exp(2i\Omega t)\begin{bmatrix}1\\ -i\end{bmatrix}+\exp(-2i\Omega t)\begin{bmatrix}1\\ i\end{bmatrix}\right\}. \quad (1)$$

E is the superposition of two oppositely frequency-shifted circular polarizations, equivalent to a rotating linear polarization. The intensity detected through a polarizer in experiment 1 (see Fig. 1) exhibits then a modulation at angular frequency 4Ω [see Figs. 2(a) and 2(b)]. These results are the same as those of Ref. 1 though our frequency shifts are twice as large because we kept both circular components. One must notice that such frequency shifts have already been observed in microwave experiments[4] and many years ago in optics[5] and agree with a classical Jones matrix calculation.

However, a deeper interpretation of this phenomenon can be given thanks to energy conservation. The incident beam contains $N\sigma^+$ photons with angular momentum $\hbar$ and $N\sigma^-$ photons with angular momentum $-\hbar$. When the half-wave plate is turned with the torque of the wave, the wave produces work and loses a part of its energy,

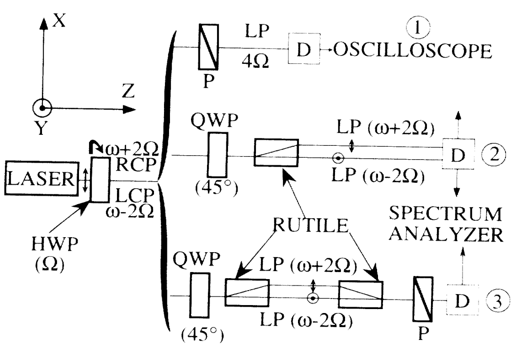

FIG. 1. Experimental setups.

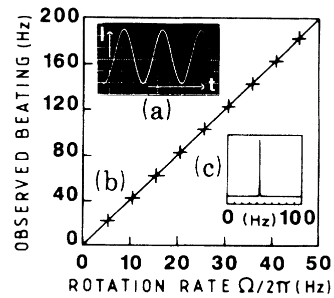

FIG. 2. (a) Typical observed modulation at angular frequency 4Ω. (b) Frequency shift vs mechanical rotation rate Ω. (c) Typical signal obtained from a spectrum analyzer in the case of the optical beating experiment 3.

and vice versa. The calculation of the energy exchanges during this process leads to

$$\tfrac{1}{2}J\Omega^2+N\hbar\omega+N\hbar\omega=\tfrac{1}{2}J\Omega^2+N\hbar(\omega-2\Omega)+N\hbar(\omega+2\Omega), \quad (2)$$

where J is the moment of inertia of the plate, showing that the half-wave plate transfers some energy from one beam to the other. This energy exchange is still more strikingly proved by experiments 2 and 3. In experiment 2, a quarter-wave plate changes the two frequency-shifted circular waves into two orthogonal linear waves that are spatially separated by a rutile birefringent crystal. Both ordinary and extraordinary beams separately exhibit *no more modulation.* However, their frequencies are shifted, as shown by their mixing with a second crystal that exhibits an optical beating at angular frequency 4Ω [see experiment 3 in Figs. 1 and 2(c)].

This series of simple experiments has shown that the unavoidable frequency shift of a circularly polarized beam incident on a rotating birefringent plate is a simple consequence of the energy-conservation law and is associated with an angular momentum exchange. Moreover, this consequence of the energy-conservation law throws light on many papers dealing with the manifestation of Berry's phase shift in optics.[6]

F. Bretenaker and A. Le Floch
Université de Rennes I
35042 Rennes CEDEX, France

Received 9 April 1990
PACS numbers: 03.65.Bz, 42.10.Jd, 42.10.Nh

[1]R. Simon, H. J. Kimble, and E. C. G. Sudarshan, Phys. Rev. Lett. **61**, 19 (1988).
[2]R. A. Beth, Phys. Rev. **50**, 115 (1936).
[3]J. H. Poynting, Proc. Roy. Soc. London A **82**, 560 (1909); A. Kastler, Soc. Sci. Phys. Nat. Bordeaux **1932**, 55.
[4]P. J. Allen, Am. J. Phys. **34**, 1185 (1966).
[5]G. E. Somargren, J. Opt. Soc. Am. **65**, 960 (1975); B. A. Garetz and S. Arnold, Opt. Commun. **31**, 1 (1979).
[6]R. Bhandari and J. Samuel, Phys. Rev. Lett. **60**, 1211 (1988); R. Y. Chiao *et al.*, Phys. Rev. Lett. **60**, 1214 (1988); T. H. Chyba *et al.*, Opt. Lett. **13**, 562 (1988); R. Bhandari, Phys. Lett. A **133**, 1 (1988).

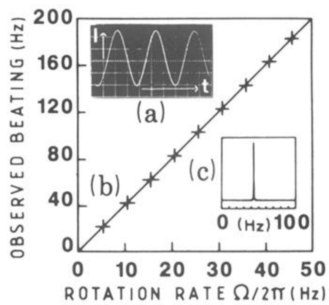

FIG. 2. (a) Typical observed modulation at angular frequency 4Ω. (b) Frequency shift vs mechanical rotation rate Ω. (c) Typical signal obtained from a spectrum analyzer in the case of the optical beating experiment 3.