A Solution of the Solar-Neutrino Problem

John N. Bahcall
Institute for Advanced Study, Princeton, New Jersey 08540

H. A. Bethe
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

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Comparison of the results from the Kamiokande neutrino-electron scattering experiment with those from the chlorine experiment and with solar models shows that the explanation of the solar-neutrino problem probably requires physics beyond the standard electroweak model with zero neutrino masses. The experimental results, including the shape of the electron-recoil energy spectrum measured by Kamiokande, are in excellent agreement with a nonadiabatic solution of the Mikheyev-Smirnov-Wolfenstein effect, yielding a neutrino mass difference of $\Delta m^2 = 1\times 10^{-8} \sin^{-2}\Theta_V$ eV^2.

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Recently,[1] the Kamiokande II Collaboration has reported on 1040 days of observations of solar neutrinos via neutrino-electron scattering. These results are of fundamental significance since the angular dependence of the scattered electrons shows that the detected neutrinos originate in the Sun and since the observation provides a specific measurement of the scattering rate of the highest-energy ^{8}B solar neutrinos. Moreover, the Kamiokande II results show that the ^{8}B neutrino flux is independent of time, despite strong hints from the chlorine observations of a time dependence.[2]

In this paper we assume the correctness of the following experimental results, a chlorine detection rate[3] of

$$\langle \phi\sigma \rangle_{\mathrm{Cl,expt}} = 2.1 \pm 0.3\ \mathrm{SNU}\ \ (1\sigma\ \mathrm{error}) \tag{1}$$

(where SNU denotes solar-neutrino units) for neutrinos above the 0.81-MeV threshold energy and a neutrino-electron scattering rate of[1]

$$\langle \phi\sigma \rangle_{e\text{-}\nu} = [0.46 \pm 0.05(\mathrm{stat}) \pm 0.06(\mathrm{syst})] \langle \phi\sigma \rangle_{\mathrm{stnd}} \tag{2}$$

for recoil electrons with energies greater than 7.5 MeV. Here stnd refers to the rate calculated[4] assuming the correctness of the standard solar model and the standard electroweak theory with zero neutrino masses. The chlorine detector is sensitive to lower-energy neutrinos ($E < 2$ MeV) from the *pep* reaction, from ^{7}Be electron capture, and from the decay of ^{13}N, ^{15}O, and ^{17}F, as well as the higher-energy ^{8}B neutrinos. The theoretical expectation for the event rate in the chlorine detector is[4]

$$\langle \phi\sigma \rangle_{\mathrm{Cl,theory}} = 7.9 \pm 2.6\ \mathrm{SNU}\,, \tag{3}$$

where the indicated error refers to the total theoretical uncertainty. The difference between the values given in Eqs. (1) and (3) has constituted for two decades the "solar-neutrino problem." The measurement cited in Eq. (2) points the way to a solution of this long-standing puzzle.

We do not know of any modifications of the astrophysical calculations of the state of the solar interior that could lead to the reconciliation of Eqs. (1)-(3) without requiring new physics for the neutrino. To see the reasons for this situation, suppose that electroweak theory with zero neutrino masses is exactly correct. Then the shape of the neutrino energy spectrum produced in the Sun can be calculated accurately from laboratory data. Using the known relative efficiencies of the two detectors,[4] Eq. (2) implies an event rate of 2.8 ± 0.3 SNU in the chlorine detector from ^{8}B neutrinos alone. In addition, the *pep* neutrinos, whose flux can be calculated with an accuracy of 5%, contribute another 0.2 SNU. The ^{7}Be neutrinos contribute 1.1 SNU in the standard solar model and are much less sensitive to changes in the model than are the ^{8}B neutrinos (total uncertainty 15% compared to 37% for the ^{8}B neutrinos). Crudely speaking, the ^{7}Be neutrinos depend upon T_c^8 whereas the ^{8}B neutrinos depend upon T_c^{18}, where T_c is the central temperature of the solar model. We conclude that solar models, including models with weakly interacting massive particles, that are consistent with Eq. (2) will predict a chlorine rate near 4 SNU (without including the effects of CNO neutrinos). This rate is outside the range observed in the chlorine experiment. Our conclusion that the solar-neutrino problem requires new physics does not depend upon knowledge of the results from gallium experiments or from Mikheyev-Smirnov-Wolfenstein (MSW) calculations.

The MSW solution[5] with a relatively large mass difference can be excluded because Kamiokande II observed a significant neutrino flux above 7.5 MeV. The large-mass version of the MSW effect predicts[6] that these higher-energy ^{8}B neutrinos are almost entirely missing.

The situation is different for the MSW effect with a small squared mass difference Δm^2. As is well known, in this case the conversion of electron neutrinos ν_e to a second flavor neutrino that we shall call ν_x is nonadiabatic.[7] The observed flux of electron neutrinos can then be written[8]

$$\phi\nu_e(E) = \phi\nu_e(E)\exp(-C_{\text{jump}}/E)\,, \tag{4}$$

and the observed flux of other neutrinos satisfies

$$\phi\nu_x(E) = \phi\nu_e(E)[1-\exp(-C_{\text{jump}}/E)]\,. \tag{5}$$

The constant C_{jump} is given by

$$C_{\text{jump}} = \pi\beta^{-1}\Delta m^2 \sin^2\Theta_V\,, \tag{6}$$

where Θ_V is the neutrino mixing angle in vacuum and β is the absolute value of the logarithmic derivative of the electron density with radius, i.e., $\beta = -n_e^{-1}\,dn_e/dr$. For the inner region of the Sun, $\beta = 10.54$ per solar radius.[4]

The value of C_{jump} can be fixed by requiring that the calculated rate for the chlorine experiment be equal to the observed rate, Eq. (1). For the range of ^{8}B neutrino fluxes allowed by the standard solar model,[4]

$$C_{\text{jump}} = 10.5 \pm 5.5 \text{ MeV}\,. \tag{7}$$

The best-estimate value of C_{jump} can be used to calculate the rate for the Kamiokande neutrino-electron scattering experiment. We find

$$\langle\phi\sigma\rangle_{\text{MSW}} = 0.46\langle\phi\sigma\rangle_{\text{stnd}}, \quad E_e > 7.5 \text{ MeV} \tag{8}$$

for electrons with recoil energies above 7.5 MeV and

$$\langle\phi\sigma\rangle_{\text{MSW}} = 0.49\langle\phi\sigma\rangle_{\text{stnd}}, \quad E_e > 9.3 \text{ MeV} \tag{9}$$

for electrons with recoil energies above 9.3 MeV. Both of these results are in fortuitously good agreement with the observed rates;[1] the difference between calculated and observed rates is only $0.01\langle\phi\sigma\rangle_{\text{stnd}}$, which is much less than the quoted errors.

The Kamiokande II team stressed that their results are consistent with the spectrum of recoil electrons predicted by the standard (unmodified) neutrino spectrum.[1] Can our model be ruled out by these measurements?

Figure 1 compares the normalized energy distribution $P(T)$ of recoil electrons with kinetic energies T computed using the unmodified neutrino spectrum with the energy distribution obtained using the modified neutrino spectra given in Eqs. (4) and (5). The error bars are typical values taken from the experimental paper.[1] We see that the shape of the spectrum of recoil electron energies is practically the same for the MSW-modified flux and for the standard neutrino flux. The reason is that at the neutrino energies of interest (large compared to $m_e c^2$) the scattered neutrino and electron receive comparable amounts of the recoil energies.

We conclude that the nonadiabatic MSW solution is the most likely explanation of the solar-neutrino problem. The neutrino mass difference that corresponds to $C_{\text{jump}} = 10.5$ MeV is

$$\Delta m^2 = 1.0\times10^{-8}\sin^{-2}\Theta_V \text{ eV}^2\,. \tag{10}$$

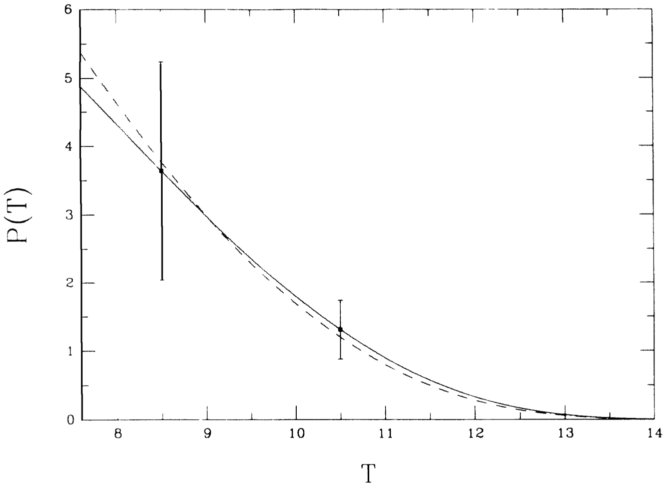

FIG. 1. Electron recoil spectrum. Here $P(T)$ is the probability that an electron has a recoil kinetic energy T (in MeV). The threshold energy is assumed to be 7.5 MeV. The solid curve refers to the unmodified recoil spectrum ($C_{\text{jump}} = 0.0$) and the dashed curve refers to the MSW-modified recoil spectrum. The error bars are taken from Ref. 1.

This result is essentially the same as that of Rosen and Gelb,[9] Eq. (10). In that important paper, Rosen and Gelb showed, using less precise data from Kamiokande, that the nonadiabatic solution was a viable possibility. If we arbitrarily assume Θ_V equals the Cabibbo angle, 13°, then $\sin^2\Theta_V = 0.05$ and $\Delta m^2 = 2\times10^{-7}$ eV^2. Since neutrinos of the nonelectron flavor, Eq. (5), contribute 20% of the calculated ν-e scattering rate for a threshold of 7.5 MeV (and 18% for a threshold of 9.3 MeV), we note that there is marginal experimental evidence that the second neutrino flavor ν_x is not sterile. Independent of solar models, the allowed range of ^{8}B neutrinos that are produced in the Sun is

$$0.35 \le \phi_{\text{prod}}/\phi_{\text{stnd}} \le 1.8\,. \tag{11}$$

The lower limit follows directly from the Kamiokande II measurements; the upper limit is obtained by considering the MSW solution for the lowest ratio of chlorine to e-ν reductions (relative to the expected value) that is consistent with both observations.

We consider unlikely the large-mixing-angle solution discussed by Rosen and Gelb. We expect that the mass matrix of neutrinos is similar to the Kobayashi-Maskawa matrix in which the nondiagonal elements, i.e., the mixing angles, are small. Bahcall and Haxton[10] have shown that the solution with large mixing angles is viable if the event rate in the gallium experiment exceeds 20 SNU.

The explanation discussed here has a number of experimental consequences. The best-estimate rate for the gallium experiments that are just getting started[11] is 5 SNU, much smaller than the standard-model prediction of 132 SNU.[4] This dramatic reduction occurs because essentially all of the lower-energy neutrinos that contribute almost all of the event rate in the standard model are converted to ν_x. The best-estimate rate of 5 SNU refers to the neutrinos as they come out of the Sun. Some of the neutrinos will be reconverted to ν_e in the vacuum between the Earth and the Sun and some will be reconverted at night when passing through the Earth to reach the detector.[12]

The expected event rates for the D_2O experiment[13] are not decreased dramatically from the standard model. The neutral-current disintegration rate will be unchanged and the charged-current absorption and the neutrino-electron scattering rates will each be decreased by factors of order 3 depending upon the precise detection thresholds.

The rates for neutrino-electron scattering by ^{7}Be neutrinos (a feasible scintillator experiment approved for the Gran Sasso Laboratory[14]) is 0.21 of the standard prediction and the neutrino-electron scattering by *pp* neutrinos (perhaps observable with the liquid-helium detector[15] or with other bolometric detectors[16]) is 0.27 of the standard prediction. For both these experiments (*pp* neutrinos and ^{7}Be neutrinos), the entire event rate is due to the "other flavor" ν_x of neutrinos. The nonadiabatic MSW solution makes an observable prediction also for the altered shape of the spectrum of the recoil electrons for the low-energy ^{7}Be neutrinos: The recoil spectrum should turn up in the last hundred keV of recoil electron energy. This behavior is to be contrasted with the monotonically decreasing spectrum predicted if only electron-type neutrinos are present and can be seen clearly in Fig. 8.5 of Ref. 4. The day-to-night effect may be observable in ν-e scattering experiments[17] and would be very informative.

Coherent neutrino-nucleus scattering[18] should yield the rate calculated from the standard model since this process is independent of neutrino flavor.

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[1] K. S. Hirata *et al.*, Phys. Rev. Lett. **65**, 1297 (1990).

[2] R. Davis, Jr., in *Proceedings of the Seventh Workshop on Grand Unification, ICOBAN '86,* edited by J. Arafune (World Scientific, Singapore, 1987), p. 237; R. Davis, Jr., K. Lande, C. K. Lee, P. Wildenhain, A. Weinberger, T. Daily, B. Cleveland, and J. Ullman, in Proceedings of the Twenty-First International Cosmic Ray Conference, Adelaide, Australia (to be published); J. N. Bahcall and W. Press (to be published). The average event rate is 2.1 SNU in the entire data sample and also in the first third of the sample, 1970–1977, for which no significant time dependence is discernible. See also the many other references cited in these papers.

[3] J. K. Rowley, B. T. Cleveland, and R. Davis, Jr., in *Solar Neutrinos and Neutrino Astronomy,* edited by M. L. Cherry, W. A. Fowler, and K. Lande, AIP Conference Proceedings No. 126 (American Institute of Physics, New York, 1985), p. 1; see also Ref. 2.

[4] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge Univ. Press, Cambridge, England, 1989). The neutrino cross sections used in the present paper are calculated using computer codes described in Chap. 8 of this reference; the standard solar model is presented in Chap. 4; and the standard neutrino spectra are given in Chap. 6. See also J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. **60**, 297 (1988).

[5] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1986) [Sov. J. Nucl. Phys. **42**, 913 (1986)]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1979).

[6] H. A. Bethe, Phys. Rev. Lett. **56**, 1305 (1986).

[7] S. P. Rosen and J. M. Gelb, Phys. Rev. D **34**, 969 (1986); E. W. Kolb, M. S. Turner, and T. P. Walker, Phys. Lett. B **175**, 478 (1986).

[8] W. C. Haxton, Phys. Rev. Lett. **57**, 1271 (1986); S. J. Parke, Phys. Rev. Lett. **57**, 1275 (1986); A. Dar and A. Mann, Nature (London) **325**, 790 (1987); P. Pizzocher, Phys. Rev. D **36**, 2293 (1987). See pp. 270 and 271 of Ref. 4 for a concise summary of the nonadiabatic solution.

[9] S. P. Rosen and J. M. Gelb, Phys. Rev. D **39**, 3190 (1989).

[10] J. N. Bahcall and W. C. Haxton, Phys. Rev. D **40**, 931 (1989).

[11] I. Barabanov *et al.*, in *Solar Neutrinos and Neutrino Astronomy* (Ref. 3), p. 175; W. Hampel, *ibid.*, p. 162.

[12] M. Cribier, W. Hampel, J. Rich, and D. Vignaud, Phys. Lett. B **182**, 89 (1986); A. Baltz and J. Wenesser, Phys. Rev. D **35**, 528 (1987).

[13] G. Aardsma *et al.*, Phys. Lett. B **194**, 321 (1987).

[14] R. Raghavan (private communication).

[15] R. E. Lanou, H. J. Maris, and G. M. Seidel, Phys. Rev. Lett. **58**, 2498 (1987).

[16] See articles in *Low Temperature Detectors for Neutrinos and Dark Matter,* edited by K. Pretzl, N. Schmitz, and L. Stodolsky (Springer-Verlag, Heidelberg, 1987).

[17] J. N. Bahcall, J. M. Gelb, and S. P. Rosen, Phys. Rev. D **35**, 2976 (1987).

[18] A. Drukier and L. Stodolsky, Phys. Rev. D **30**, 2295 (1984).