Nonlocal Phase Shifts Induced by Static Electric Fields in Neutron Interferometers when the Path-Enclosed Charge Vanishes

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An experiment, analogous to that of Cimmino *et al.*, is proposed to measure the phase shift induced by static electric fields on the passage of neutrons through an interferometer when the path-enclosed linear charge density is zero. A finite shift is predicted and discussed in light of the Aharonov-Casher result.

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Recently, Cimmino *et al.*[1] have measured the Aharonov-Casher[2] (AC) phase shift ϕ_{AC} via neutron interferometry. An expression for the electric-field-induced phase shift was first obtained by Anandan.[3] Derived originally from a duality principle,[2] the AC effect has also been discussed in terms of a spatially extended model of the neutron. Although interesting questions have been raised and answered,[4] physically, at the low momentum transfers inherent in such experiments,[1] the neutron can be treated as a point particle with a dipole moment $\boldsymbol{\mu}$ proportional to its intrinsic spin. In the present analysis ϕ_{AC} is obtained via the minimal prescription of electromagnetism, according to which the nonrelativistic (NR) Hamiltonian $H(\mathbf{P},\mathbf{x})$ follows from the NR classical energy functional $\mathcal{E}(\mathbf{p},\mathbf{x})$ by replacing the kinematic momentum $\mathbf{p}$ by $\mathbf{P}-e\mathbf{A}/c$, where $P_k = \partial L/\partial \dot{x}_k$ is the canonical momentum. ϕ_{AC} then follows directly from the Lorentz-invariant stationary-phase Feynman path integral $\oint dt\, L$ over the classical action, assuming the *same* axis of spin quantization can be maintained over the *entire* stationary-phase spatial trajectory (classical orbit). The concept of classical orbit, inherent in the Feynman approach to spatial quantum interference, is especially suited to consider nonlocal phase shifts induced by electric fields $\mathbf{E}$ on the quantized spin of the neutron in an interferometer under quite general conditions, including, but not limited to, those which mimic the geometric situation considered originally by AC. It is assumed that, in the absence of external magnetic fields, the spin nonetheless remains quantized along a fixed axis in three-space everywhere along the classical paths. Then $\mathbf{v}$ and $\mathbf{E}$ must be coplanar everywhere that $\mathbf{E}$ is not null, and to $O(\mathbf{E})$ *the local aspects (dynamics) are effectively one dimensional.* [To $O(\mathbf{E})$ the neutron experiences no acceleration (neglecting gravity) and its orbit differs from a single line only as a consequence of reflections from the interferometer crystal "mirrors."] Yet, the field-induced phase shift as the neutron amplitudes sample classically distinct paths in two dimensions results from *nonlocal* effects associated with the quantized spin. In the AC geometric arrangement the spin is quantized along an axis parallel to infinite line charges, i.e., along an axis normal to the plane within which $\mathbf{E}$ is confined. Then the phase shift is *topologically invariant;* i.e., it is proportional to the total charge contained in a right-cylindrical surface of unit height which projects onto the orbit but is otherwise independent of the detailed orbit, as AC showed. However, it is also possible for the electric field to induce nonlocal phase shifts via the interaction of the quantized spin of the neutron *when the net charge vanishes within such a surface.* This assertion, which is subject to experimental test, is not in conflict with the AC result, since now the neutron no longer orbits around a collection of "infinite" line charges, the situation to which AC limited their considerations.

To begin, I shall derive the canonical result under the AC conditions in a way that emphasizes the effective one-dimensional nature of the local aspects and then introduce the anomalous case. The latter will serve to point out a distinction between the AC and the Aharonov-Bohm[5] (AB) effects in a three-dimensional context, despite their equivalence in two dimensions. This distinction has been made before by Goldhaber[4] and again quite recently by Hagen[6] who emphasizes the scattering of polarized neutrons from regions where the local charge density does not vanish. The present approach differs from these and, in particular, applies when the neutron encounters no local charge density, as is essentially the case in the experiment of Ref. 1.

Consider a neutron with instantaneous velocity $\mathbf{v}=(v_x,0,0)$ traveling in an electric field $\mathbf{E}(x,y)=(E_x,E_y,0)$. In its instantaneous inertial rest frame, the four-momentum $(\mathcal{E}',\mathbf{p}')$ is given by ($c=1$, $\mu=-1.913\mu_N$)

$$\mathcal{E}' = m - \boldsymbol{\mu}\cdot\mathbf{B}' = m + \mu \sinh u\, \sigma_z E_y\,, \qquad (1)$$

$\mathbf{p}'=\mathbf{0}$, with $\tanh u = v_x$. In the frame of the interferometer, neglecting gravity and putting back c, $\mathbf{p}=(mv_x,0,0)$ and

$$\mathcal{E} = mc^2 + \tfrac{1}{2}mv_x^2 + \mu(v_x/c)\sigma_z E_y\,, \qquad (2)$$

to lowest order in v/c. Replacing the kinematic momentum $\mathbf{p}$ in the NR energy

$$\mathcal{E}_{NR} = p_x^2/2m + \mu p_x \sigma_z E_y/mc \qquad (3)$$

by letting $\mathbf{p} \to \mathbf{P} - e\mathbf{A}/c$, one obtains the NR (one-dimensional) Hamiltonian ($e=0$),

$$H = P_x^2/2m + \mu P_x \sigma_z E_y/mc \,, \tag{4}$$

where $P_x = \partial L/\partial \dot{x}$ is the canonical momentum, L is the one-dimensional Lagrangian, and $E_y = E_y(x,y)|_{y=\text{const}}$. By Hamilton's equation,

$$\dot{x} = \partial H/\partial P_x = (1/m)(P_x + \mu\sigma_z E_y/c) \,. \tag{5}$$

Taking the inverse Legendre transform, $L = \dot{x}P_x - H$, it follows that

$$L = m\dot{x}^2/2 - \mu\sigma_z E_y \dot{x}/c + O(E_y^2) \,. \tag{6}$$

From the Euler-Lagrange equation $\ddot{x} = O(E_y^2)$ and can be neglected. [In a full three-dimensional treatment $\ddot{x} = O(\mathbf{E}^2)$, $\ddot{y} = O(\mathbf{E}^2)$, $\ddot{z} = 0$. In any event the acceleration can be neglected in experiments such as that of Cimmino *et al.*[1,7]] Letting l_i be the length of a straight-line segment of path between mirrors (neglecting gravity), x_i the distance along the path, and ϕ_i the path-integral phase change due to the electric field, then to $O(\mathbf{E})$

$$\begin{aligned} \phi_i &= (1/\hbar)\int_0^{l_i} L_{i,\text{EM}}\, dt \\ &= (1/\hbar)\int_0^{l_i} (-\mu\sigma_z E_{yi}\dot{x}_i/c)\, dt \\ &= (1/\hbar)\int_0^{l_i} (-\mu\sigma_z E_{yi}/c)\, dx_i \,. \end{aligned} \tag{7}$$

At each reflection, we may rotate the coordinate frame about z (axis of quantization) such that the new x axis lies along the path. Then, in a circuit

$$\begin{aligned} \sum_i \phi_i &= (1/\hbar)\oint L_{\text{EM}}\, dt = -(\mu\sigma_z/\hbar c)\oint (d\mathbf{s}\times\mathbf{E})\cdot\hat{\mathbf{z}} \\ &= -4\pi\Lambda\mu\sigma_z/\hbar c \,, \end{aligned} \tag{8}$$

where Λ is the path-enclosed charge per unit length along $\hat{\mathbf{z}}$ and the sense of the contours is left-handed about $\hat{\mathbf{z}}$. Defining ϕ_{AC} right handedly, $\phi_{\text{AC}} = 4\pi\Lambda\mu\sigma_z/\hbar c$.[1,2]

To discuss the "anomalous" case, first consider an experimental setup which is the same as that of Cimmino *et al.*[1] except that instead of placing the two sets of "upright" capacitor plates around contiguous legs of the interferometer paths, they are placed upright around opposite legs and the neutron mirrors are arranged such that the classical velocities through both sets of plates are parallel. (Here upright means normal to the orbital plane.) In principle, the phase shift is the same as that measured in Ref. 1 since the arrangement still mimics the AC conditions and for equal applied voltages the linear charge density within the loop would be the same. Now rotate the plates by $\pm\pi/2$ about the line of flight through the capacitors such that $\mathbf{E}$ is *normal* to the orbital plane along each of the opposite legs but points in opposite directions along the two, as shown in Fig. 1.

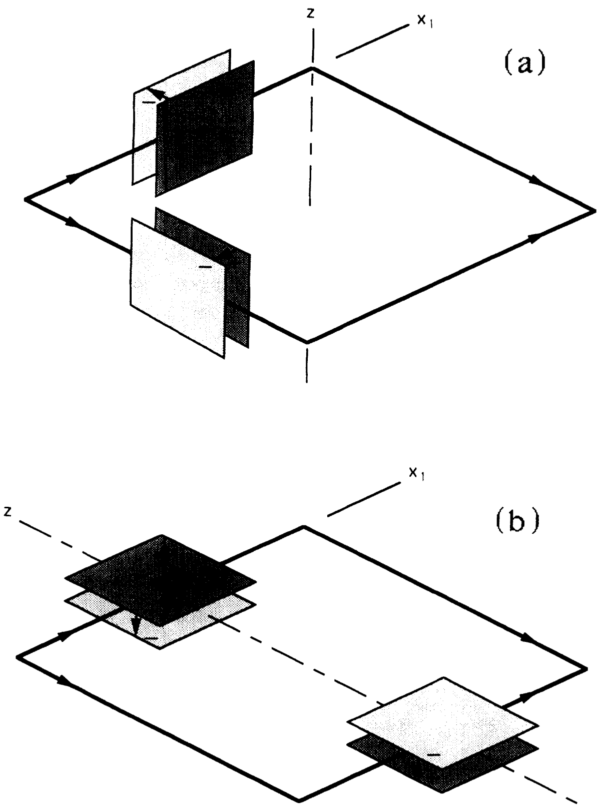

FIG. 1. Arrangement of the capacitor plates in a neutron interferometer (a) as in Ref. 1 where the axis of spin quantization is normal to the orbital plane and (b) in the proposed experimental setup where the axis lies in the orbital plane. In (a) the electric field $\mathbf{E}$ lies within the orbital plane whereas in (b) it does not. For simplicity, the crystal mirrors have been omitted from the diagrams. [Here, z labels the invariant axis of spin quantization, and x_1 the path variable along the first leg of the trajectory; y_1 (not shown) is such that (x_1, y_1, z) is a right-handed triad. In (a) the axes are rotated about z at each mirror such that x_i denotes the path variable (see text). In (b) x_i denotes the path variable along the active legs. Along the legs where $\mathbf{E}$ is null the path variable is $-z$, but the field-induced phase-change integrals vanish there. The physics, of course, is independent of the arbitrary labels and the quantization axis acquires a sense physically only in the presence of a polarizing biasing field.]

Along these legs $\mathbf{v}\times\mathbf{E}$ defines a unique axis of quantization in that only one component of spin enters the three-dimensional Hamiltonian

$$H = \mathbf{P}^2/2m + (1/mc)\boldsymbol{\mu}\cdot(\mathbf{P}\times\mathbf{E}) \,, \tag{9}$$

with $\boldsymbol{\mu} = \mu\boldsymbol{\sigma}$. The component of spin appearing in H is now *in the orbital plane* rather than normal to it. Since no other component enters H anywhere along the classical orbit ($\mathbf{E}$ is null along the other two legs), that component commutes with H *everywhere* along the orbit. It is therefore a constant throughout. Moreover, it is clear, based upon the above derivation of ϕ_{AC} in the AC configuration, that the dynamics are essentially one dimensional along each leg, once a unique axis of quantization has been arranged. Hence, in principle, the phase shift induced in the new situation is the same as in the case prior to rotating the capacitors. Yet, now the net charge contained in a right-cylindrical surface which projects onto the orbit vanishes, since equal amounts of positive and negative charge are contained within. In both cases the phase shift arises from the *nonlocal* aspect of a single quantized spin being transported along two classically distinct paths in which the local magnetic field $\mathbf{B}'$ in the neutron's rest frame points in opposite directions along the two paths, leading to quantum amplitudes of different field-induced phase changes, which add coherently.

We may now further clarify the distinction between the AC and AB effects in a three-dimensional context. To observe nonlocal effects due to the passage of a neutron through static electric fields, in the *absence* of an external magnetic field $\mathbf{B}_{ex}$ in the interferometer frame, the necessity of a unique axis of quantization over the entire classical orbit implies that $\mathbf{v}$ and $\mathbf{E}$ must be coplanar over the entire orbit in the original AC geometry. Thus, when $\mathbf{B}_{ex}=\mathbf{0}$, the orbital plane must be normal to the collection of line charges, whereas in the AB effect, the orbit can take on any configuration, provided only that the flux in the solenoid threads the orbit. [In practice, external magnetic fields cannot be wholly eliminated. Often, they must, in fact, be introduced along the desired direction of spin quantization in a neutron interferometer and then taken into account.[8] Just as an external magnetic field normal to the mean orbital plane is necessary in the setup of Ref. 1 to stabilize the quantized spin in the presence of ambient magnetic fields and of deviations of the orbital planes from that defined by $\mathbf{E}$,[8] so here a magnetic field applied along the intrinsic quantization axis in the orbital plane is likely necessary for stability. (In Ref. 1 the fringe field from the biasing magnetic field applied along the third leg provided the necesary stability.[8])]

All cases considered allow local *coupling* either of the charge e to the magnetic potential $\mathbf{A}$ or of the magnetic dipole moment $\boldsymbol{\mu}$ to the magnetic field $\mathbf{B}'$ in the rest frame of the neutron. But it is the latter that requires nonlocality in the quantized spin. The AB and AC effects relate the observed phase shifts to the total enclosed magnetic flux and the total enclosed linear charge density, respectively. The case considered here does not. Assuming that experiment bears out these ideas, what can we say about the topological nature of the effect? While clearly nonlocal in the quantized spin (as in the AC case), there is no barrier here such as an infinite line charge (finite charged plates in the effectively two-dimensional configuration in Ref. 1) nor, as in the AB case, infinite solenoid (or finite toroidal solenoid) around which the classical orbit must loop to produce the phase shift.[9] Whereas the local dynamics are effectively one dimensional, in the present example we have studied the nonlocal electric-field-induced phase shift on a neutron in a full three-dimensional context. In three dimensions the electric-field-induced phase shift of a spin-$\frac{1}{2}$ neutral particle such as the neutron is *not* topological although it remans so in two dimensions. [In neither context does the neutron experience acceleration to $O(\mathbf{E})$.] That is, as noted earlier, unlike the AB effect, the AC effect is *essentially* two dimensional. The present example allows the distinction to be drawn *in an experimentally verifiable way.*

I wish to thank S. A. Werner for helpful discussions.

Note added.—Since submitting this Letter, I have learned that Anandan has independently considered the implications of rotating the electrodes.[10] He does so in a general framework by invoking a gauge field with non-Abelian qualities arising from the quantum noncommutivity of the neutron spin components.[10] In the present work the configuration of the non-AC case has been designed to maintain the constancy of one component of the quantized spin everywhere along the classical orbit, at least in principle. Hence, the distinction drawn here between the AC and AB effects is not due to possible non-Abelian effects (which Anandan has shown to exist), but rather to the effective number of spatial dimensions. It is also important to note that the pairs of plates are positioned around opposite parallel legs when rotated by $\pm\pi/2$ (cf. Fig. 1). Were one to rotate them about contiguous legs, then the phase shift would *not* be the same as in the original AC-like configuration. Put another way, if only one pair of plates is involved, positioned one above and one below the orbital plane, then the phase shift will remain the same as in the AC-like configuration *only* if the plates do not enclose nonparallel segments of the orbit. In summary, in agreement with Anandan, I find that phase shifts induced by electromagnetic fields in neutron interferometers need not, in general, be topological in origin. However, I conclude that, *interpreted in a two-dimensional context,* Cimmino *et al.* have indeed confirmed the topological AC effect (when one accounts for the polarizing fringe field of the biasing magnet[8] and ignores the 1σ discrepancy between theory and experiment[11]). I thank J. Anandan for calling my attention to this work.

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[1]A. Cimmino, G. Opat, A. Klein, H. Kaiser, S. Werner, M. Arif, and R. Clothier, Phys. Rev. Lett. **63**, 380 (1989).

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[5]Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).

[6]C. R. Hagen, Phys. Rev. Lett. **64**, 2347 (1990).

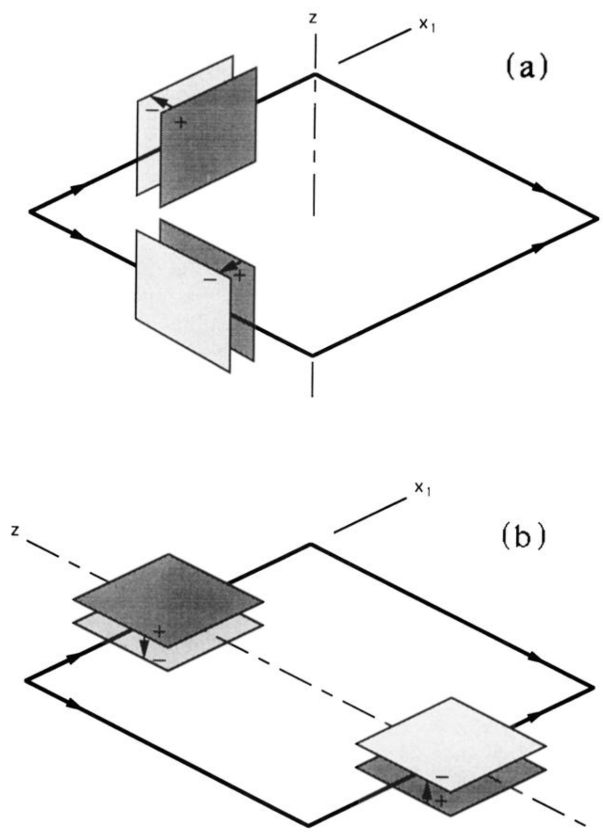
[7]Assuming, for example, that at the entrance to the field-free regions of the apparatus of Ref. 1, E_y changes linearly from 0 to 1000 statvolts/cm over a distance $\Delta x \gtrsim 1$ mm, then $\ddot{x} \lesssim 4 \times 10^{-13}$ cm/s^2, wholly negligible compared with the acceleration of gravity.

[8]S. A. Werner (private communication).

[9]Such a physical barrier may be thought of as producing a hole in a mathematical noncompact space. While there is no such barrier in the example given in Fig. 1(b), one cannot shorten the active legs shown there such that (in principle) mirror reflections occur within the plates and still maintain a unique axis of spin quantization. In this sense, there is a hole there also. Nevertheless, in Fig. 1(b), the field-induced neutron phase shift ϕ is not proportional to a path-enclosed topological charge which preserves ϕ under arbitrary deformations of the surrounding orbit.

[10]J. Anandan, in *Proceedings of the Third Symposium on Foundations of Quantum Mechanics, Tokyo, 1989* (Physical Society of Japan, Tokyo, 1990), p. 98.

[11]H. Kaiser, S. A. Werner, R. Clothier, M. Arif, A. G. Klein, G. I. Opat, and A. Cimmino, University of Missouri–Columbia report, 1990 (to be published).



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