Comment on "Resistance Fluctuations in Narrow AlGaAs/GaAs Heterostructures: Direct Evidence of Fractional Charge in the Fractional Quantum Hall Effect"

In an interesting Letter, Simmons *et al.*[1] reported the observations of resistance fluctuations as a function of magnetic field B in the vicinity of the resistance minima between quantum Hall steps. They found that the average period ΔB increases by a factor of 3 between ν=integer and $\nu=\frac{1}{3}$ and cited this as evidence for fractional charge. We wish to make the following comments: (1) The existence of fractional-charge excitation actually implies that ΔB should not change, (2) the observed increase in ΔB is due to finite-temperature effects, and (3) Coulomb blockade may play an important role, which leads us to propose an alternate experiment that provides a direct measurement of fractional-charge excitations.

We accept the model employed by Simmons *et al.* where potential fluctuations give rise to islands in the conducting channel which bind discrete edge states. Peaks in the resistance occur when the internal edge states provide an intermediate step (real or virtual) in the scattering across the channel. A tripling of ΔB for $\nu=\frac{1}{3}$ was predicted by Kivelson and Pokrovsky,[2] who studied the binding of charge $e^*=e/3$ particles. Kivelson[3] has pointed out that this model does not describe the experiment because the fractional charge must be treated together with the fractional statistics, in which case the quantization condition at equilibrium involves only $\phi_0=hc/e$. This conclusion can be reached very simply based on Laughlin's concept of a fractionally charged quasiparticle. We can think of the island as a hole in the incompressible quantum liquid. As the magnetic field is increased, the radius r of the island shrinks until it is energetically favorable to increase the flux inside the hole by ϕ_0, so that the incompressible liquid expands outward, leaving a deficit of $\frac{1}{3}$ charge on the edge of the hole and an increase of $\frac{1}{3}$ charge in the edge of the sample. In terms of quasiparticles, this is a particle-hole excitation of $\frac{1}{3}$-charged edge states. It is worth emphasizing that a condition for the transport of fractional charge is that the initial and final quasiparticle states are edge states of the same incompressible liquid. Note the net increase in flux per cycle is always ϕ_0 and the equation used by Simmons *et al.* to determine ΔB, $\phi_0=\Delta B\, d(B\pi r^2)/dB$, is valid for both integer and fractional cases. In the simple case when dr/dB is negligible, one obtains $\Delta B\approx\phi_0/\pi r^2$, independent of ν.

At finite temperatures, the fluctuations in resistance will be smeared out when kT is comparable to the energy of the quasi-particle-hole excitation which can be estimated[1] by $\Delta E\approx(\phi_0/2\pi rB)e^*E_r$, where E_r is the local electric field. However, there is another energy scale in the problem, the Coulomb blockade energy $E_C=e^{*2}/2C$, where C is the capacitance corresponding to localizing a charge e^* on the edge of the island. The potential must increase by e^{*2}/C before it is energetically favorable to add a charge, and if $e^{*2}/2C>\Delta E$, Coulomb blockade dominates. We can estimate E_C by the Coulomb energy of a ring of charge, $E_C\approx e^{*2}\ln(r/r_0)/2\pi\varepsilon_0 r$, where r_0 is the transverse dimension of the ring. Assuming[1] $E_r=10^5$ V/m and $\varepsilon_0=13$, we find that $E_C\approx 6\Delta E$ for $\nu=1$.

Noting that E_C scales as e^{*2}, we conclude that if Coulomb blockade is important, the energy scale changes by a factor of 9 between ν=integer and $\nu=\frac{1}{3}$ so that the data for $\nu=\frac{1}{3}$ at 25 mK should be compared with the integer data at 225 mK. Indeed, Simmons *et al.* found that for $\nu=2$ the rapid oscillations with $\Delta B\sim 0.016$ T are washed out by 200 mK, and only a slower period of order 0.05–0.1 T remains, which is comparable to the period of 0.05 T $\pm$30% reported for the $\nu=\frac{1}{3}$ case at 25 mK. The slower period may correspond to islands of smaller size. If Coulomb blockade is not important, ΔE scales as e^*/B and the energy scale reduces even faster, by a factor of 18 between $\nu=2$ and $\nu=\frac{1}{3}$.

Recently, periodic structures in the conductance as a function of gate voltage were observed in tunneling through quasi-one-dimensional channels.[4] An interpretation in terms of Coulomb blockade was suggested.[5] The present setup is very similar except that a linear segment is replaced by a ring and conductance is replaced by resistance. We suggest that a back gate should be added to measure resistance as a function of gate voltage V_G. If Coulomb blockade dominates, the period in V_G will scale as e^*. *This will be clear evidence for transport by fractional charges, in that a fractionally charged channel must be open to permit the transport of charges on and off the* island. We also note that if Coulomb blockade is not important, V_G will scale as B^{-1} instead, so the importance of Coulomb blockade can be determined experimentally. If a back gate is not available, the fractional charge can also be determined by studying the threshold in the Hall voltage for the onset of non-Ohmic resistance, which should scale as e^* or B^{-1}, depending on the importance of the Coulomb blockade.

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