Evidence for Two-Dimensional Quantum Wigner Crystal

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We report observations of an electric-field threshold conduction and of related ac voltage (broad-band noise) generation in low-disorder two-dimensional electron systems in the extreme magnetic quantum limit. We interpret these phenomena as definitive evidence for formation of a pinned quantum Wigner crystal and determine its melting phase diagram from the disappearance of threshold and noise behavior at higher temperatures.

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The nature of the ground state of a two-dimensional electron system (2DES) subjected to an intense perpendicular magnetic field B has attracted considerable attention. At sufficiently small Landau-level filling factors ν ($\nu = nh/eB$, where n is the 2DES density) the ground state of a disorderless 2DES is expected to be a Wigner crystal (WC).[1] Other known 2DES ground states in this regime are the fractional quantum Hall effect (FQHE) states.[2] Both WC and FQHE states are induced by the interelectron Coulomb interaction $U \sim e^2/\varepsilon a$, where $a \sim n^{-1/2}$ is the WC lattice constant and ε is the host material dielectric constant. At temperature $T=0$, as ν decreases the ideal (disorderless) 2DES is expected to undergo at least one first-order phase transition between FQHE and WC states. If the temperature is raised at fixed ν, the WC state is expected to undergo a melting transition, while the FQHE states exhibit no phase transition but rather the electron correlations gradually and continuously become less important.

On the other hand, it is well known that disorder affects and can even destroy both the FQHE and WC. Phenomenologically, disorder leads to a magnetic-field-induced localization of 2DES at low ν: The diagonal resistivity (ρ_{xx}) diverges exponentially as ν is decreased at low T and is thermally activated.[3-5] The activated ρ_{xx} behavior can set in just below $\nu = 1$ in 2DES samples with *relatively* high disorder or at $\nu \sim 0.25$ in the currently lowest disorder samples. Following the classical $B=0$ 2DES analogy[6] it is often assumed that when the ratio of the strength of disorder potential $\langle V \rangle$ to the interelectron Coulomb energy $e\langle V \rangle/U \gg 1$, the single-electron localization is a good approximation; when $e\langle V \rangle/U \ll 1$, the ground state can be described as a finite correlation length WC, most likely pinned by the residual disorder; and in the fairly wide intermediate range $e\langle V \rangle/U \sim 1$, the ground state is a "Wigner glass."

In this paper we present our nonlinear transport measurements in a high-quality 2DES at low ν. The data, for the first time, show a very-low-electric-field threshold (~ 100 μV/cm) behavior, which we interpret as evidence for a WC state. At higher applied electric fields we also detect ac voltage ("noise") generated in the samples. We interpret this ac voltage as coming from the sliding lattice jerking against the pinning potential and take this phenomenon as additional evidence for WC. Somewhat unexpectedly, the pinned phase is reentrant; that is, it exists in a narrow range of ν greater than the FQHE state at $\nu = \frac{1}{5}$, as well as for $\nu < \frac{1}{5}$. The threshold behavior and accompanying noise disappear as the temperature is raised; we take this to manifest a melting transition and present a WC melting phase diagram.

Several samples cut from GaAs/AlGaAs wafers[7] show qualitatively similar behavior. A brief illumination by a red light-emitting diode at low T was used to prepare a 2DES. Usual magnetotransport measurements were used to determine n and to assess 2DES homogeneity and overall quality. Here, however, we present data obtained from only two of the samples: A (rectangle of 2.5×8 mm^2) and B (rectangle of 1.3×5 mm^2); four In-Sn alloyed contacts were located along the short sides of the rectangle as shown in the inset in Fig. 1. This geometry was selected since it provides a more or less uniform electric field in the sample (in the localized regime, when $\rho_{xx} \gg \rho_{xy}$) and also minimizes spreading and contact resistances.

Figure 1 shows magnetoresistance of sample A at 40 mK. At lower B the trace exhibits the integer and FQHE. For $\nu > 0.25$ the magnetic-field-induced localization sets in although FQHE features are clearly visible at $\nu = \frac{2}{9}$ and $\frac{1}{5}$. We have measured resistance at zero dc bias at several ν in the localized region and plot it versus T^{-1} in Fig. 2. While for $\nu < 0.25$ resistance is *not* activated down to $T = 15$ mK, in the localized regime a weak temperature dependence at higher T is followed by an activated resistance ($\ln R \sim T^{-1}$). Interestingly, there is a second linear region of $\ln R \sim T^{-1}$ with a lower activation energy, after which the T dependence becomes weaker than T^{-1}. This behavior resembles the activa-

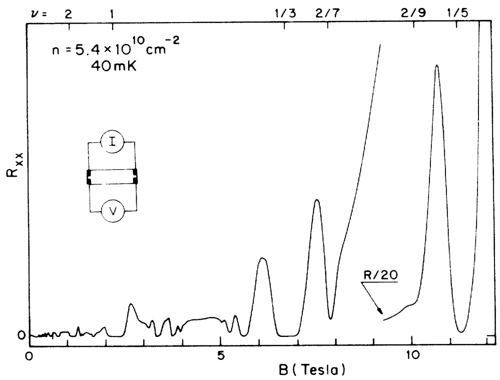

FIG. 1. Longitudinal resistance of sample A vs magnetic field. Inset: Measurement arrangement $(R=V/I)$.

tion across the gap, fixed-range, and then variable-range hopping.[8]

In the localized regime we have measured the differential resistance dV/dI versus applied dc voltage V_{dc}. Several representative traces exhibiting the bias threshold behavior are shown in Fig. 3. As V_{dc} is increased from zero the resistance remains constant until threshold V_{th} is exceeded, after which the resistance decreases monotonically. Figure 4(a) gives several dV/dI vs V_{dc} traces at various temperatures. It is evident that as T is raised the threshold behavior disappears, being replaced by a quadratic (near $V=0$) dependence. The upper trace in Fig. 4(a) shows a dV/dI trace up to $V_{dc}=12$ mV. It is noteworthy that at about 3 mV the decrease of dV/dI becomes much more gradual; in some other data (at different T,ν) dV/dI is nearly constant from ~ 3 to ~ 10 mV. This is a clear indication that the Joule heating of the 2DES is essentially negligible for biases up to a few mV under these experimental conditions. We note here that while Joule heating should yield a quadratic dependence (for small power inputs, where response is linear) $-\delta R/R \sim \delta T/T \sim V^2$, the re-

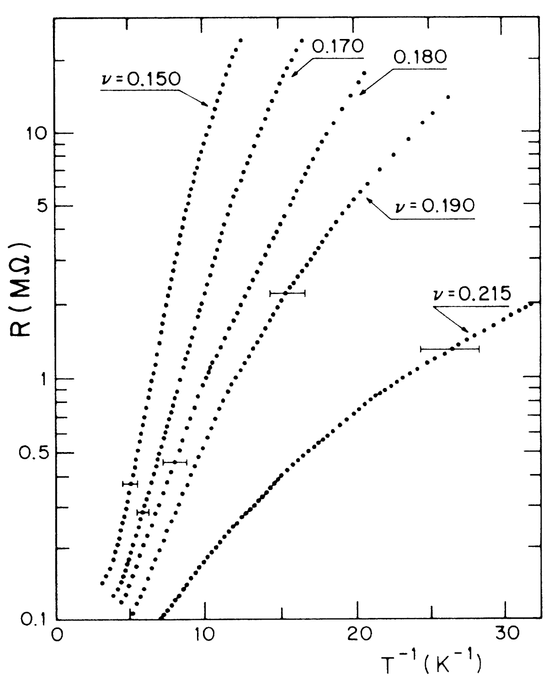

FIG. 2. Inverse temperature dependence of longitudinal resistance at several ν for sample A. Horizontal bars give the temperature range of the WC melting transition (see Fig. 5).

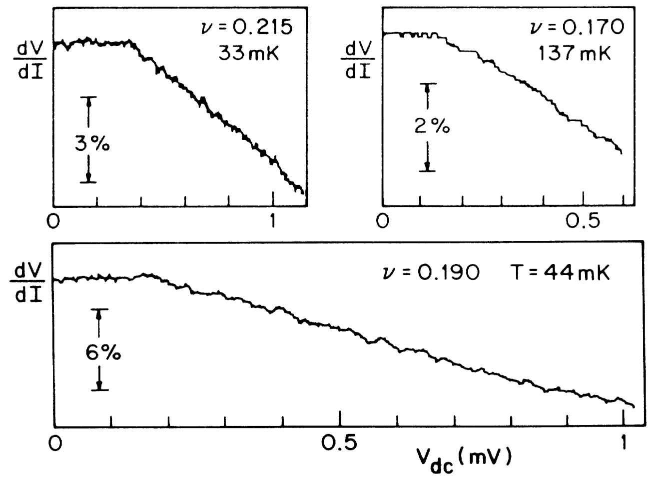

FIG. 3. True four-terminal differential resistance vs applied dc bias for sample A. Excitation voltage is $\sim 20\ \mu$V rms at 3.2 Hz.

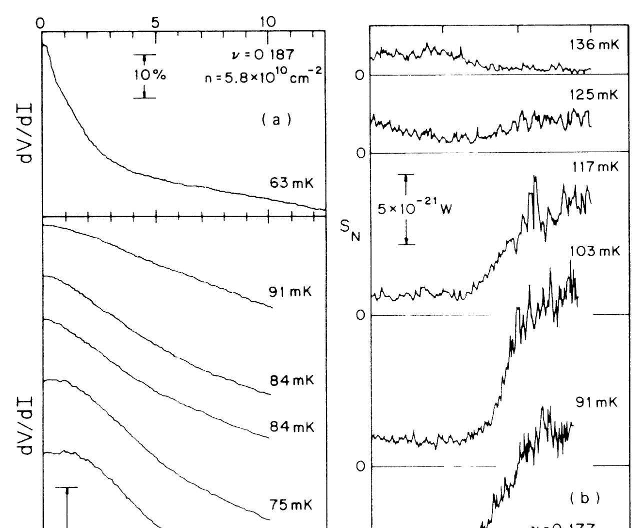

FIG. 4. (a) Differential resistance vs applied bias for sample A. Data at 84 mK are repeated to show reproducibility; $T_m \sim 84$ mK. Note different voltage scale for the upper trace. (b) Noise power measured at 1 kHz in 10-Hz bandwidth in sample B vs applied bias. The noise at low biases is instrumental and has an $1/f$ component. The high-bias broad-band "noise" is generated in the sample; $T_m \sim 125$ mK.

verse premise is not correct; that is, the quadratic dependence of dV/dI on bias at higher T [e.g., 91-mK data in Fig. 4(a)] occurs despite the fact that the Joule heating of 2DES is negligible for power inputs of 10^{-13} W at this T.

The dV/dI measurements were done starting the voltage sweep from $V_{dc}=0$, where the sample was allowed to equilibrate. In measurements with decreasing voltage sweeps, and upon reversal of the bias polarity, the samples often exhibit hysteretic, "glassy" behavior with long equilibration times (> 1 h) at lower temperatures.

We have also been able to detect the ac voltage generated in the samples under dc voltage bias. The experiment consisted of measuring the noise power S_N (the magnitude squared of ac voltage) generated in the bandwidth of 1 or 10 Hz at a frequency f. A rather abrupt increase of S_N above the instrumental noise has been observed at $3<f<1500$ Hz. In this range, the background level of the instrumental noise is higher at lower f ("$1/f$ noise"), so that the increase of S_N is *relatively* small; at higher f the capacitance of the wiring shorts out the sample. Figure 4(b) gives several traces of S_N vs V_{dc} measured in sample B with temperature as the parameter. At low biases the measurement gives instrumental noise; as bias is increased well above the threshold voltage, at low T the signal abruptly increases and then stays rather constant. This excess noise disappears at a fairly well defined temperature as T is increased. We note here that shot noise can be confidently ruled out as a source of the signal for several reasons. First, shot noise is not expected to be present in this kind of measurement; second, shot noise power is approximately linear in applied bias—in contrast to the data of Fig. 4(b); third, S_N for shot noise is some 3 orders of magnitude lower than the instrumental noise.

The problem of experimentally establishing a WC phase of a 2DES at low T,ν is very difficult. Direct observations of periodicity, e.g., by x-ray or neutron scattering, as have been accomplished for charge-density waves (CDW), appear to be nearly impossible at present for a number of reasons, one of which is that in heterostructure samples the 2DES is buried several thousand angstroms below the crystal surface. Therefore, less direct methods are called for. However, due to the lack of a comprehensive and detailed theory of the quantum WC, a less direct method must establish unambiguously: (1) The collective nature of the 2DES as may be evidenced by phenomena qualitatively different from those expected in an independent electron state, or a quantitative difference of at least several orders of magnitude, and (2) the rigidity of the 2DES—the property that distinguishes a WC from a collective yet fluid state.

Conventional magnetotransport measurements allow us only to establish that a magnetic-field-induced localization takes place, but do not provide information relevant to either of the points above. Localization is consistent with the following: a pinned WC, "Wigner glass," or noninteracting, single-electron Anderson localization. Radio-frequency experiments, whether attempting to determine the absorption of electromagnetic fields or surface acoustic waves by a 2DES, encounter the difficulty that the disorder potential $\langle V\rangle$ opens a gap of $\sim e\langle V\rangle$ for WC "gapless" acoustic magnetophonon collective excitations. The activation energies determined from transport measurements in the localized, low-ν regime of 2DES (this work, Ref. 3) give $e\langle V\rangle \sim 0.1$ meV corresponding to a gap of 25 GHz—an order of magnitude larger than maximum frequencies of the reported experiments. Therefore, these experiments cannot at present reach the collective modes and test the same 2DES conductivity as the low-frequency measurements, although often in an indirect way.

We argue below that our observations of threshold conduction and ac voltage generation provide unambiguous evidence for a WC state. Electric-field-threshold conduction is also observed in CDW (Ref. 9) and in the pinned classical 2D WC on liquid-helium films.[10] Using a two-fluid analogy, the conductivity of a pinned WC at finite T can be considered to consist of two contributions. At low electric fields the conductivity is due to electrons excited across the single-carrier disorder-induced gap. This component of the conductivity is activated and does not imply a WC state since it is a property of disorder and is observed also in noninteracting and interacting electron fluid states as, e.g., in bulk-doped semiconductors.[8] The other collective component of the conductivity sets in only for $V>V_{th}$ and is due to sliding of the depinned WC. The data of Figs. 1 and 2, therefore, give the single-electron component (in the localized regime), while the data of Figs. 3 and 4(a) show that an additional channel of conduction opens up when bias exceeds V_{th}.

The presence of an electric-field threshold conduction implies rigidity of the 2DES, since a fluid state cannot be pinned and would not display threshold conduction. Moreover, it can be shown that the very low value of the threshold electric field implies a correlation length $L \gg a$. In the absence of a more relevant theory we obtain a rough estimate of L following the argument developed for the weak-pinning limit in CDW.[11] An applied electric field E exerts a force $e(L/a)^2E$ on the correlated domain of the WC. By requiring the work done by the external E_{th} at threshold on sliding a correlated domain by one lattice constant to be equal to $e\langle V\rangle$, one obtains $(L/a)^2=a\langle V\rangle/E_{th}$. Taking $e\langle V\rangle$ to be equal to the activation energy, this estimate yields $L\sim 250a$. Note that the ratio $e\langle V\rangle/U\sim 10^{-2}$.

The observation of the ac voltage generation in the depinned state provides additional, independent evidence for a WC state. This evidence is of a qualitative nature since no ac voltage generation under moderate dc bias is expected or known for single-electron localization. This phenomenon is also called "broad-band noise" and is related to the "narrow-band nose," or voltage oscillations observed in the MHz frequency range in CDW samples with L comparable to the sample dimensions.[9] The noise is thought to be generated by the electron lattice sliding by pinning centers (disorder potential) in external electric fields strong enough to depin the WC. The ac power generated should be inversely proportional to the area of the sample since the noise voltage is generated incoherently by individual pinning centers. While no quantitative study of the sample-size dependence has been done yet, the generated ac power is greater in our smaller samples.

As is shown in Fig. 4 the electric-field threshold behavior and ac voltage generation both disappear at higher temperatures. This can be interpreted as a consequence of WC melting. In Fig. 5 we plot thus determined transition temperatures, where t is the melting temperature T_m normalized to the classical $B=0$ WC melting temperature $T_{mc}=e^2(\pi n)^{1/2}/\varepsilon\Gamma$, where $\Gamma \cong 127$.[12] Noteworthy is the very good agreement of T_m determined from the noise data with that determined from the threshold data.

It is well known that disorder broadens melting phase transitions in ordinary matter, so that experimentally there is always a transition *region* of T, with width depending on the degree of disorder. Equally well known is the fact that in binary alloys (such as Pb-Sn) there is always a mixed-phase region of T where solid crystallites coexist with the liquid phase of eutectic composition. Effects such as these can be expected for the WC melting transition in our samples and may account for the nonthreshold parabolic behavior of dV/dI at low bias, such as seen in Fig. 4(a). This parabolic behavior practically disappears, however, at T some 20% higher than T_m [e.g., 103 mK for the data of Fig. 4(a)], while the activated behavior of resistance at $V=0$ persists into the fluid phase to much higher T as can be seen in Fig. 2.

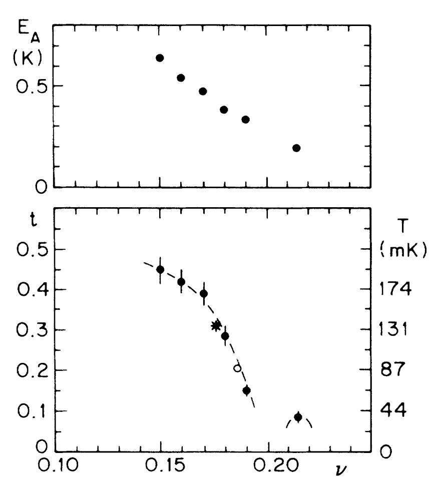

FIG. 5. The WC melting transition phase diagram determined from disappearance of the threshold conduction behavior [circles, open circle from a different run, Fig. 4(a)] and noise generation (asterisk). $\nu=0.20$ corresponds to the FQHE state $\nu=\frac{1}{5}$. The temperature scale on the right is for sample A. The dashed line is a guide to the eye only. The activation energies E_A of the data of Fig. 2 are plotted in the upper panel.

Our present results show that the pinned phase (WC according to the evidence presented above) is reentrant; that is, it exists in a narrow region within $\frac{2}{9} > \nu > \frac{1}{5}$ as well as for $\nu < \frac{1}{5}$.[13] Therefore, at low temperatures as ν is decreased at least *three* FQHE WC quantum phase transitions occur. In Ref. 4 we have reported transport anomalies at $\nu=\frac{1}{7}$ and have interpreted them as evidence for the FQHE state at this ν. We have subsequently observed a several times stronger dip in ρ_{xx} at a lower temperature (95 mK, $t=0.22$). However, it is not clear at present whether these anomalies result from a developing FQHE state, which would imply two more FQHE WC phase transitions, or are manifestation of an effect such as the cooperative ring exchange[14] in the WC state.

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[13] The FQHE state at $\nu=\frac{1}{5}$ is well established by now. In Ref. 4 we have reported the observation of a well developed plateau in ρ_{xy} at $5h/e^2 \pm 0.3\%$; see also Ref. 5.

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