Intensity-Interferometric Test of Nuclear Collision Geometries Obtained from the Boltzmann-Uehling-Uhlenbeck Equation

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Two-proton correlation functions measured for the ^{14}N + ^{27}Al reaction at $E/A = 75$ MeV are compared to correlation functions predicted for collision geometries obtained from numerical solutions of the Boltzmann-Uehling-Uhlenbeck (BUU) equation. The calculations are in rather good agreement with the experimental correlation function, indicating that the BUU equation gives a reasonable description of the space-time evolution of the reaction.

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Microscopic models of intermediate-energy nucleus-nucleus collisions have been successfully based on the semiclassical Boltzmann-Uehling-Uhlenbeck (BUU) equation[1] which describes the temporal evolution of the one-body density under the influence of the nuclear mean field and individual nucleon-nucleon collisions. In this paper we report the first quantitative test of the space-time geometry predicted by solutions of the BUU equation by using the technique of two-proton intensity interferometry[2] which utilizes the space-time sensitivity of the two-proton correlation function at small relative momenta.[2-11] For this purpose, we have measured two-proton correlation functions with high statistical accuracy for the relatively light projectile-target combination ^{14}N + ^{27}Al, at $E/A = 75$ MeV. For such a light system, numerical calculations can be performed with good accuracy and modest amounts of CPU time.

The experiment was performed with a ^{14}N beam of $E/A = 75$ MeV extracted from the K1200 cyclotron of the National Superconducting Cyclotron Laboratory at Michigan State University. An ^{27}Al target of 15 mg/cm^2 areal density was used. Protons were detected with two ΔE-E detector arrays consisting of 300-400-μm-thick silicon ΔE detectors and 10-cm-long CsI(Tl) or NaI(Tl) E detectors. An array consisting of 37 Si-CsI(Tl) telescopes[12] was centered at the polar and azimuthal angles of $\theta = 25°$ and $\phi = 0°$; each of its detectors had a solid angle of $\Delta\Omega = 0.37$ msr and a nearest-neighbor spacing of $\Delta\theta = 2.6°$. Another array consisting of 13 Si-NaI(Tl) telescopes was centered at $\theta = 25°$ and $\phi = 90°$; each of its detectors had a solid angle of $\Delta\Omega = 0.5$ msr and a nearest-neighbor spacing of $\Delta\theta = 4.4°$. Coincidence and downscaled singles data were taken

simultaneously. Energy calibrations are accurate to better than 2%. Typical detector energy resolutions were of the order of 2% and 1% for protons of 40 and 100 MeV, respectively. All the data were corrected for random coincidences and had a software energy threshold of 10 MeV.

The experimental two-proton correlation function $R(q)$ is defined in terms of the coincidence yield $Y(\mathbf{p}_1, \mathbf{p}_2)$ and the single-proton yields $Y(\mathbf{p}_1)$ and $Y(\mathbf{p}_2)$:

$$\sum Y(\mathbf{p}_1, \mathbf{p}_2) = C[1 + R(q)] \sum Y(\mathbf{p}_1) Y(\mathbf{p}_2). \tag{1}$$

Here, $\mathbf{p}_1$ and $\mathbf{p}_2$ are the laboratory momenta of the two protons, and $q = \frac{1}{2} |\mathbf{p}_1 - \mathbf{p}_2|$ is the relative momentum of the proton pair. For each experimental gating condition, the sums on both sides of Eq. (1) are extended over all energy and detector combinations corresponding to specific relative momentum bins. The normalization constant C is determined from the requirement $R(q) = 0$ at large q.

Our theoretical analysis is based upon the expression[2,10]

$$1 + R(\mathbf{P}, \mathbf{q}) = \int d^3r \, F_\mathbf{P}(\mathbf{r}) |\phi(\mathbf{q}, \mathbf{r})|^2. \tag{2}$$

Here, $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ is the total momentum of the proton pair, $\phi(\mathbf{q}, \mathbf{r})$ is the relative two-proton wave function, and $F_\mathbf{P}(\mathbf{r})$ is defined by

$$F_\mathbf{P}(\mathbf{r}) = \frac{\int d^3X f(\mathbf{P}/2, \mathbf{X} + \mathbf{r}/2, t_>) f(\mathbf{P}/2, \mathbf{X} - \mathbf{r}/2, t_>)}{|\int d^3X f(\mathbf{P}/2, \mathbf{X}, t_>)|^2}. \tag{3}$$

Here, the Wigner function $f(\mathbf{p}, \mathbf{x}, t_>)$ is the phase-space distribution of particles of momentum $\mathbf{p}$ at position $\mathbf{x}$ at

some time $t_>$ after the emission process. We will obtain $f(\mathbf{p},\mathbf{x},t_>)$ by solving the BUU equation.

Figure 1 shows measured two-proton correlation functions for representative ranges of the total momentum P of the proton pairs. In order to describe the dependence on P in terms of a simple variable, we have constructed experimental correlation functions for a number of narrow gates on P. Each such correlation function was characterized in terms of a simple Gaussian source of negligible lifetime,

$$f(\mathbf{P}/2,\mathbf{r},t) = \rho_0 \exp[-r^2/r_0^2(P)]\delta(t-t_0). \qquad (4)$$

The extracted source parameters $r_0(P)$ are shown by the solid circles in Fig. 2. The error bars indicate estimated systematic uncertainties. The dashed curves in Fig. 1 show examples of fits with Eq. (4) which are used to extract the momentum dependence of $r_0(P)$ shown in Fig. 2. In these calculations, appropriate averages over total momentum and the calculated resolution of the hodoscope were taken into account.[13]

In order to compare our results with microscopic dynamical calculations, we have calculated microscopic Wigner functions by solving the BUU transport equation

$$\partial_t f(\mathbf{p},\mathbf{r},t) + \frac{\mathbf{p}}{m} \cdot \nabla_r f(\mathbf{p},\mathbf{r},t) - \nabla_r U(\mathbf{r}) \cdot \nabla_p f(\mathbf{p},\mathbf{r},t)$$

$$= \frac{1}{2\pi^3 m^2} \int d^3 q_1' \, d^3 q_2 d^3 q_2' \, \delta \left[\frac{1}{2m}(p^2+q_2^2-q_1'^2-q_2'^2) \right] \delta^3(\mathbf{p}+\mathbf{q}_2-\mathbf{q}_1'-\mathbf{q}_2') \frac{d\sigma}{d\Omega}$$

$$\times \{ \hat{f}(\mathbf{q}_1',\mathbf{r},t)\hat{f}(\mathbf{q}_2',\mathbf{r},t)[1-\hat{f}(\mathbf{p},\mathbf{r},t)][1-\hat{f}(\mathbf{q}_2,\mathbf{r},t)]$$

$$- \hat{f}(\mathbf{p},\mathbf{r},t)\hat{f}(\mathbf{q}_2,\mathbf{r},t)[1-\hat{f}(\mathbf{q}_1',\mathbf{r},t)][1-\hat{f}(\mathbf{q}_2',\mathbf{r},t)] \}, \qquad (5)$$
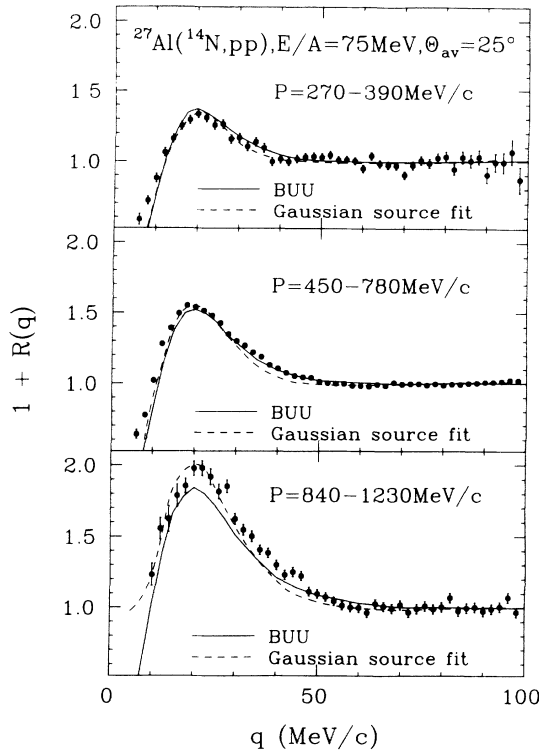
where $\hat{f}(\mathbf{p},\mathbf{r},t)$ is the phase-space density. The in-medium nucleon-nucleon scattering cross section $d\sigma/d\Omega$ was assumed to be proportional to the experimentally known (energy-dependent) cross section for free nucleons $d\sigma_{NN}/d\Omega$. The mean field $U(\mathbf{r})$ was approximated by density-dependent, simplified local Skyrme interactions:

$$U(\rho) = -124\rho/\rho_0 + 70.5(\rho/\rho_0)^2 \text{ MeV}, \qquad (6a)$$

$$U(\rho) = -356\rho/\rho_0 + 303(\rho/\rho_0)^{7/6} \text{ MeV}. \qquad (6b)$$

FIG. 1. Two-proton correlation functions measured for the reaction $^{14}\mathrm{N}+^{27}\mathrm{Al}$ at $E/A=75$ MeV. The gates placed on the total momenta P of the coincident particle pairs are indicated. The dashed curves represent calculations for Gaussian sources of negligible lifetime, assuming the momentum dependence of the radius parameter $r_0(P)$ shown by the solid circles in Fig. 2. The solid curves correspond to solutions of the BUU equation using the stiff equation of state, Eq. (6a).
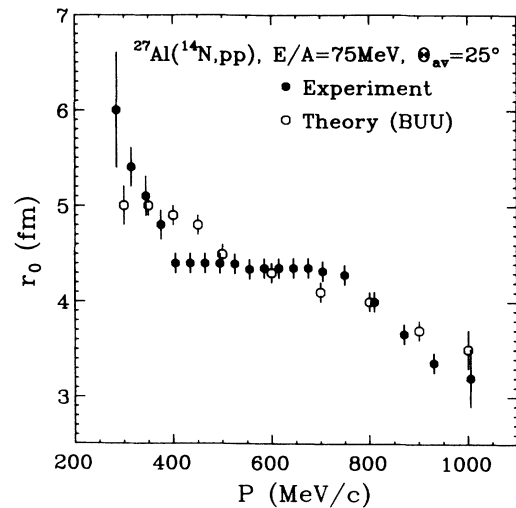

FIG. 2. Radius parameters $r_0(P)$ for Gaussian sources of negligible lifetime extracted from two-proton correlation functions gated by different total momenta P. Solid and open circles represent experimental and theoretical correlation functions, respectively.

Here, ρ_0 denotes the equilibrium density of normal nu-
clear matter. Equations (6a) and (6b) represent a stiff
and a soft equation of state with compressibility coef-
ficients $K = 380$ and 200 MeV, respectively. The left-
hand side of Eq. (5) is the Vlasov term describing the in-
teraction of the nucleons with the mean field; the right-
hand side is the collision integral which includes a semi-
classical Pauli-blocking factor. We solved this equation
by numerical methods which are similar to those de-
scribed in Ref. 14. By explicitly storing $\hat{f}(\mathbf{p},\mathbf{r},t)$ on a
six-dimensional lattice in every time step we were able to
greatly speed up the computer program without relaxing
the accuracy of the treatment of the Pauli exclusion prin-
ciple.[15] Nucleon emission was calculated for a time in-
terval of $\Delta t_e = 140$ fm/c following initial contact of the
colliding nuclei. Nucleons were considered as emitted
when the surrounding density fell below $\rho_e = \rho_0/8$ and
when subsequent interaction with the mean field did not
cause recapture into regions of higher density.[16] For the
calculation of a correlation function, we generated
phase-space points from a total of 5250 events with im-
pact parameters distributed according to their geometri-
cal weights; appropriate averages over impact parameter,
orientation of the reaction plane, and momenta of the
outgoing particles were taken into account.

Correlation functions calculated from Wigner func-
tions predicted by the BUU equation are shown by the
solid lines in Fig. 1. For these calculations, we used the
stiff equation of state, Eq. (6a), and the free nucleon-
nucleon scattering cross section, $d\sigma/d\Omega = d\sigma_{NN}/d\Omega$. In
order to facilitate a more detailed comparison with the
experimental data, the predicted correlation functions
were characterized in terms of equivalent Gaussian
sources, Eq. (4), following the procedure outlined above.
The corresponding equivalent radius parameters $r_0(P)$
are shown by the open circles in Fig. 2. The errors rep-
resent estimates for the numerical accuracy of our calcu-
lations. The predicted correlation functions are in good
agreement with the data, indicating the BUU equation
gives a good overall description of the average space-
time evolution of the reaction.

We have also explored the sensitivity of the calculated
correlation functions to the nuclear equation of state and
the magnitude of the in-medium nucleon-nucleon cross
section. Figure 3 shows correlation functions predicted
for intermediate-momentum proton pairs, $P = 500$
MeV/c. The solid and dotted curves show correlation
functions predicted for the stiff and soft equations of
state, using $d\sigma/d\Omega = d\sigma_{NN}/d\Omega$. The two calculations
are very similar, indicating little sensitivity to the nuclear
equation of state.[17] The solid, dashed, dot-dashed, and
dot-dot-dashed curves represent calculations with the
stiff equation of state performed with the assumption
that the in-medium nucleon-nucleon cross section is
equal to 1.0, 0.8, and 0.5, and 0.0 times the free
nucleon-nucleon cross section. The predicted correlation
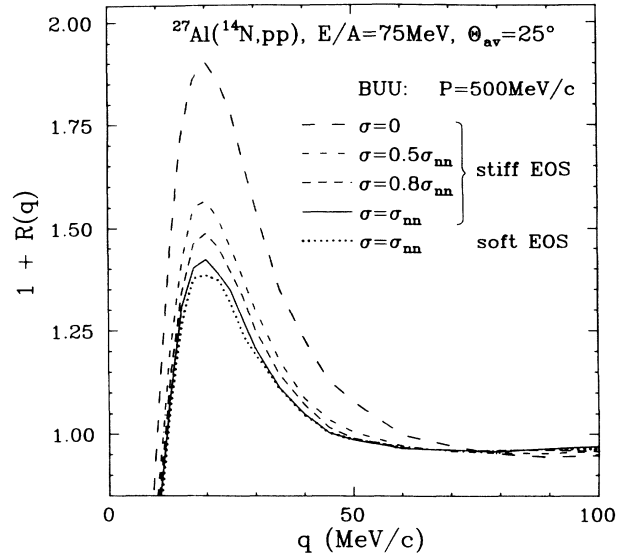functions become more pronounced for decreasing values

FIG. 3. Sensitivity of two-proton correlation functions at in-
termediate momentum to the nuclear equation of state and the
in-medium nucleon-nucleon cross section.

of $d\sigma/d\Omega$, indicating that increased nucleon-nucleon
scattering leads to slower emission time scales. For in-
termediate total momenta of the proton pairs, $P \approx 500$
MeV/c, the calculated correlation functions are rather
sensitive to the magnitude of the in-medium nucleon-
nucleon cross section. For the emission of very energetic
particles, $P \gtrsim 800$ MeV/c, this sensitivity is reduced.
The measured correlation functions are rather well
reproduced by calculations using free nucleon-nucleon
cross sections; see Figs. 1 and 2. The agreement is
significantly worse when in-medium cross sections are
used which are smaller than one-half of the free
nucleon-nucleon cross section.

In summary, we have investigated two-proton correla-
tion functions for $^{14}N + ^{27}Al$ collisions at $E/A = 75$
MeV. Microscopic calculations based upon solutions of
the BUU transport equation are in good agreement with
the measured correlation functions, indicating that the
theory predicts reasonable space-time geometries for
nonequilibrium light-particle emission. The predicted
correlation functions are rather insensitive to the stiffness
of the equation of state, but exhibit considerable sensi-
tivity to the magnitude of the in-medium nucleon-
nucleon cross section.[17] The measured correlation func-
tions are rather well reproduced by calculations in which
the in-medium cross section is approximated by the
energy-dependent free nucleon-nucleon cross section.

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[13]The finite resolution of the hodoscope is only important at small relative momenta, $q \lesssim 10$ MeV/c; since this region of momenta is of little interest for the present study, finite-resolution effects were not incorporated into our BUU calculations.

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[15]A detailed account of the numerical method is given in W. Bauer, Michigan State University Report No. MSUCL-699 (unpublished).

[16]The calculated correlation functions depend on the specific choice of Δt_e and ρ_e. At present, these parameters must be treated as unknown model parameters which introduce uncertainties of the order of 5%–10% into the magnitude of the predicted correlation functions.

[17]Sensitivities of the predicted two-proton correlations functions to in-medium cross sections and equation of state can be expected to depend on incident energy and the size of projectile and target. In particular, the lack of sensitivity of the present calculations to the stiffness of the equation of state may be related to the small size of projectile and target investigated in this paper. Calculations for heavier systems and different incident energies would clearly be valuable.