Electric Dipole Moment of the Electron and of the Neutron

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It is shown that if Higgs-boson exchange mediates *CP* violation a significant electric dipole moment for the electron can result. Analogous effects can contribute to the neutron's electric dipole moment at a level competitive with Weinberg's three-gluon operator.

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Ever since the discovery of *CP* violation[1] a quarter of a century ago, experimentalists have steadily improved the bounds on the electron dipole moments of the electron[2,3] and of the neutron.[4] In the standard model, these moments are rather small and well below the present experimental bounds. Recently, Weinberg[5] pointed out that in models with *CP* or *T* violation carried by Higgs fields, a class of graphs neglected previously by theorists may contribute to the electric dipole moment of the neutron at a level just an order of magnitude below the present bound. Inspired by Weinberg's work, we have found a class of graphs, also neglected previously, which may contribute significantly to the electric dipole moment of the electron. Furthermore, a similar class of graphs competes with and may even dominate Weinberg's effect for the neutron electric dipole moment.

In the standard model, the electron electric dipole moment is predicted to be[6] $\lesssim 10^{-38} e\,\text{cm}$, which is much smaller than the present bound of[3] $(-1.5 \pm 5.5 \pm 1.5) \times 10^{-26}$ $e\,\text{cm}$, as to make experimentalists despair. Indeed, a survey of the literature[7] shows that even in nonstandard models the electron electric dipole moment tends to be very small, typically less than (and usually much less than) 10^{-26} cm. To cheer up the experimentalists, one of us has designed a model[8] specifically to produce a large electron dipole moment, perhaps as large as 10^{-25}–10^{-26} $e\,\text{cm}$. This model requires exotic scalar particles coupling the electron to the τ. It is also possible to have a large electron dipole moment in left-right-symmetric models.[9]

In contrast, in this Letter we will stick to the more conservative possibility envisioned by Weinberg[5,10] that there is some *CP* violation mediated by Higgs-boson exchange. This is possible in the two-Higgs-doublet standard model.[11] Encouragingly we find a typical value for d_e/e which is typically of order 10^{-26} cm. Experimentalists are currently working towards limits of a few $\times 10^{-27}$ cm, and hope to reach limits near 10^{-28} cm in the future.

One of our graphs involves a top quark just as does Weinberg's graph, except that we have pulled the Higgs field "out of the top-quark loop," which reduces what would otherwise have been at least a three-loop effect to a two-loop one. The point is that since a factor of m_e is required by chirality in any case, we may as well let an explicit Higgs-boson coupling flip the chirality. We can understand the behavior of this graph by considering the limit $m_t \gg m_H$, in which we have in the effective theory (for energies $\lesssim m_H$) the operators $v^{-1}\phi F_{\mu\nu}F^{\mu\nu}$ and $v^{-1}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$. By dimension counting, the coefficient of the operator is proportional to the Yukawa coupling of the top quark divided by its mass, $f_t/m_t \sim v^{-1}$. In the effective theory the logarithmic divergence of the relevant graph [Fig. 1(b)] is cut off by m_t. Thus, we have an electron's dipole moment of the order $(e^2/v)(m_e/v)(\ln m_t^2/m_H^2)\text{Im}$, where Im measures the *CP* violation carried by the Higgs field.

The exact two-loop calculation can actually be done. We need to keep terms only to first order in the external photon momentum and to leading order in m_e. The re-

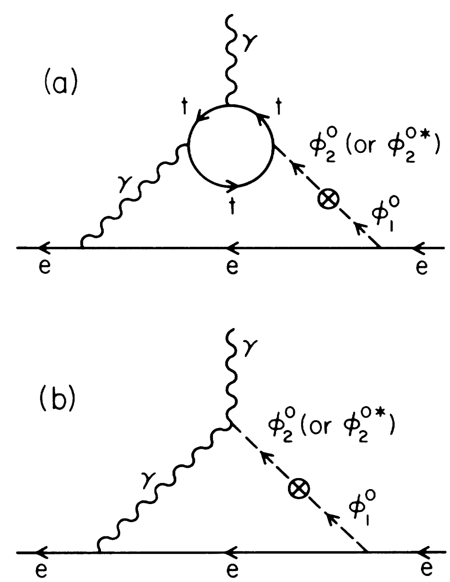

FIG. 1. (a) A graph contributing to the electron's electric dipole moment. *CP* violation indicated by the cross is assumed to exist in the propagators of the neutral Higgs bosons. (b) The same graph where the top-quark loop has been shrunk to a point.

sult is

$$\left(\frac{d_e}{e}\right)_{t\,\mathrm{loop}}=\left[\frac{16}{3}\frac{\alpha}{(4\pi)^3}\sqrt{2}G_F m_e\right]\left\{\left[f\left(\frac{m_t^2}{m_H^2}\right)+g\left(\frac{m_t^2}{m_H^2}\right)\right]\mathrm{Im}Z_0-\left[f\left(\frac{m_t^2}{m_H^2}\right)-g\left(\frac{m_t^2}{m_H^2}\right)\right]\mathrm{Im}\tilde{Z}_0\right\}, \tag{1}$$

where

$$f(z)\equiv\tfrac{1}{2}z\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-z}\ln\frac{x(1-x)}{z},$$

$$g(z)\equiv\tfrac{1}{2}z\int_0^1 dx\frac{1}{x(1-x)-z}\ln\frac{x(1-x)}{z}.$$

These functions are such that $f(1)\sim\frac{1}{2}$, $g(1)\sim 1$; for large z, $f(z)\sim\frac{1}{3}\ln z+\frac{13}{18}$, $g(z)\sim\frac{1}{2}\ln z+1$; and for small z, $f(z)\sim g(z)\sim(z/2)(\ln z)^2$. Thus, the effect of this graph is largest for a heavy top quark such that $m_t\gtrsim m_H$. The contributions of other quarks or leptons with $m_f\ll m_H$ are down by m_f^2/m_H^2 (logarithmic factor). In (1) we have calculated in the framework of the two-Higgs-doublet standard model with natural flavor conservation[11] in which one doublet, ϕ_1, gives mass to the down quarks and leptons, and the other, ϕ_2, gives mass to the up quarks. (The type of diagram we are considering should be interesting in an even wider class of models.)

We use Weinberg's conventions[5,12] for the names of the CP-violating phases that appear in the neutral-scalar propagators:

$$\langle\phi_2^0\phi_1^{0*}\rangle_q/v_1^*v_2\equiv\sum_n\sqrt{2}G_F Z_{0n}(q^2+m_{H_n}^2)^{-1},$$

and

$$\langle\phi_2^0\phi_1^0\rangle_q/v_1v_2\equiv\sum_n\sqrt{2}G_F\tilde{Z}_{0n}(q^2+m_{H_n}^2)^{-1},$$

where the index n refers to the neutral-scalar mass eigenstates. In (1) we have implicitly assumed that the lightest neutral Higgs boson dominates the sum. If not, one must replace Z_0, $\tilde{Z}_0$, and m_H^2 in (1) by Z_{0n}, $\tilde{Z}_{0n}$, and $m_{H_n}^2$ and sum over n. In the case where one Higgs boson dominates we may use some simple relations derived by Weinberg[12] to reexpress (1) in terms of Z_2 and Z_1 (defined analogously to Z_0 and $\tilde{Z}_0$ from the propagators $\langle\phi_2^0\phi_2^0\rangle_q$ and $\langle\phi_1^0\phi_1^0\rangle_q$),

$$\left(\frac{d_e}{e}\right)_{t\,\mathrm{loop}}=\left[\frac{16\alpha}{3(4\pi)^3}\sqrt{2}G_F m_e\right]\left\{f\left(\frac{m_t^2}{m_H^2}\right)\tan^2\beta\,\mathrm{Im}Z_2+g\left(\frac{m_t^2}{m_H^2}\right)\cot^2\beta\,\mathrm{Im}Z_1\right\}, \tag{2}$$

where $\tan\beta\equiv|v_2/v_1|$. Weinberg showed[12] that $|\mathrm{Im}Z_1|\le\frac{1}{2}|\tan\beta|(1+\tan^2\beta)^{1/2}$, and $|\mathrm{Im}Z_2|\le\frac{1}{2}|\cot\beta|\times(1+\cot^2/\beta)^{1/2}$. Thus, it is possible for the curly bracket in (2) to be large. Conservatively, we may take it to be order 1. The square bracket in (2) is about 3.4×10^{-27} e cm.

There are also graphs where the internal photon in Fig. 1 is replaced by a Z^0. These are naively of the same magnitude as the photon graphs since the momentum flowing through the γ or Z^0 is typically of order m_t^2 or m_H^2. It turns out that as a consequence of C invariance only the vector part of the Z^0 coupling to the electron and top quark contributes. As a result, the Z^0 diagrams are strongly suppressed relative to the γ diagrams by

$$(-\tfrac{2}{3}e^2)^{-1}\frac{e^2}{\sin^2\theta_W\cos^2\theta_W}(-\tfrac{1}{4}+\sin^2\theta_W)(\tfrac{1}{4}-\tfrac{2}{3}\sin^2\theta_W)\approx 1.5\%.$$

There is, of course, also some suppression due to the mass of the Z^0. We therefore do not bother to compute these graphs.

The basic principle underlying our discussion is clear. A heavy particle (with mass at least that of the Higgs particles) runs around in a loop and generates effective operators of the type ϕFF in the low-energy theory, thus inducing a substantial electric dipole moment. Our discussion may be extended by replacing the top quark by other heavy particles, such as the W boson. More exotic possibilities, for instance, supersymmetric particles or technifermions, may also be considered. There are numerous such two-loop diagrams with boson loops,[13] some of which are shown in Fig. 2. (There are other graphs of course related to these by gauge invariance.) Let us focus for a moment on Fig. 2(a). We can, as before, imagine shrinking the W-boson loop to a point if $m_H\ll M_W$. It is well known[14] that the W-boson-loop contribution to the on-shell $H\to\gamma\gamma$ amplitude is larger than the top-loop contribution by a factor of about 7. It is reasonable to expect that a similar ratio may hold for the W-loop and top-loop contributions to d_e/e. We can use the results of Ref. 14 to calculate the graph in Fig. 2(a) (plus those diagrams required to get a gauge-invariant result—we ignore for simplicity a set of graphs which is separately gauge invariant involving only scalars in the loop). The result for d_e/e is

$$\begin{aligned}\left(\frac{d_e}{e}\right)_{W\,\mathrm{loop}}&=\left[\frac{4\alpha}{(4\pi)^3}\sqrt{2}G_F m_e\right]\left[3f\left(\frac{M_W^2}{m_H^2}\right)+5g\left(\frac{M_W^2}{m_H^2}\right)\right]\sin^2\beta\,\mathrm{Im}\tilde{Z}_0\\&=\left[\frac{2\alpha}{(4\pi)^3}\sqrt{2}G_F m_e\right]\left[3f\left(\frac{M_W^2}{m_H^2}\right)+5g\left(\frac{M_W^2}{m_H^2}\right)\right](\sin^2\beta\tan^2\beta\,\mathrm{Im}Z_2+\cos^2\beta\,\mathrm{Im}Z_1).\end{aligned} \tag{3}$$

For large $\tan^2\beta$ and $M_W^2 \sim m_H^2$, this gives a result $(d_e/e)_{W\,\mathrm{loop}} \sim (9\times 10^{-27}\ \mathrm{cm})\tan^2\beta\,\mathrm{Im}Z_2$, which is indeed about a factor of 5 times the top-loop contribution given in (2). However, if $M_W \ll m_H, m_t$, then the W loop is suppressed relative to the top loop by $O(M_W^2/m_H^2)$. One expects the other graphs in Fig. 2 to contribute at a comparable level. For the boson-loop graphs where a Z^0 couples to the electron, as for the fermion-loop graphs of the same type, there is a suppression by $\frac{1}{4} - \sin^2\theta_W$ due to C invariance. In Ref. 15 it was noted that a W-boson electric dipole moment can induce, by a diagram like Fig. 3, a fermion electric dipole moment and that such an effect could arise in Higgs-boson exchange models of CP violation. The graph in Fig. 2(d) and related graphs are indeed a realization of this for the electron. [The blob in Fig. 3 is just the $SU(2)_L$ analog of the three-gluon operator of Weinberg.]

In this paper we restrict ourselves to CP violation mediated by neutral scalars. If there is CP violation mediated by charged Higgs bosons then there are many other possible graphs.

This discussion suggests to us that we should also consider the neutron's electric dipole moment arising from the same graphs as Fig. 1 with the electron replaced by a light quark. One finds for the analog of Fig. 1(a),

$$(d_d)_{t\,\mathrm{loop}} = e\left[\frac{16\sqrt{2}}{9(4\pi)^3}\alpha G_F m_d\right]\left\{f\left(\frac{m_t^2}{m_H^2}\right)\tan^2\beta\,\mathrm{Im}Z_2 + g\left(\frac{m_t^2}{m_H^2}\right)\cot^2\beta\,\mathrm{Im}Z_1\right\}, \tag{4a}$$

$$(d_u)_{t\,\mathrm{loop}} = e\left[-\frac{32\sqrt{2}}{9(4\pi)^3}\alpha G_F m_u\right]\left\{\left[f\left(\frac{m_t^2}{m_H^2}\right) + g\left(\frac{m_t^2}{m_H^2}\right)\right]\mathrm{Im}Z_2\right\}. \tag{4b}$$

Of course the quark electric dipole moment operator must be scaled from m_t or m_H down to the nucleon state. (Its anomalous dimension is $g_s^2/6\pi^2$.) One may then use either the nonrelativistic-quark-model result $d_n = \frac{4}{3}d_d - \frac{1}{3}d_u$, or "naive dimensional analysis"[16] which gives $d_n \approx d_q$. Either way one finds that the top-quark-loop graphs give $d_n/e \approx (6\times 10^{-27}\ \mathrm{cm})F(m_t^2/m_H^2, \beta, \mathrm{Im}Z_{1,2})$, where we take F to be the factor in curly brackets in (4a). The quark analogs of Fig. 2(a) should give an even larger contribution of order $(2\times 10^{-26}\ \mathrm{cm}) \times [\frac{3}{5}f(M_W^2/m_H^2) + g(M_W^2/m_H^2)]\sin^2\beta\,\mathrm{Im}\tilde{Z}_0$. Weinberg's three-gluon operator[5] gives an effect typically of order

$$(d_n/e)_{3G} \approx (2\times 10^{-25}\ \mathrm{cm})h(m_t^2/m_H^2)\mathrm{Im}Z_2, \tag{5}$$

where $h(z)$ is a function, given in Ref. 5, having the limits $\lim_{z\to 0}h(z) = \frac{1}{2}z\ln z^{-1}$, $h(\infty) = \frac{1}{8}$. It is apparent that the electroweak diagrams we have been discussing are competitive and can even dominate the three-gluon operator. This may be surprising especially considering that the former effect is proportional to m_q while the latter is proportional to m_N. However, e^3 and $eg_s^3/4\pi$ are nearly equal at the scale m_t, the three-gluon operator has a larger anomalous dimension, and the three-gluon diagram has a smaller parameter integral. Also of interest are the chromo-electric-dipole moments of the light quarks that arise from the same diagrams but with the photons replaced by gluons. However, these are probably smaller than the contributions of the electric dipole moment operators of the quarks.

A few remarks are in order. First, the operators we have identified are effectively of dimension 6 in Weinberg's sense. [The operator has dimension D if its coefficient is of order $(m_{\mathrm{small}}/m_{\mathrm{large}})^{D-4}$, where $m_{\mathrm{small}} = m_u$, m_d, or $4\pi f\pi$, and $m_{\mathrm{large}} = M_W$, m_t, m_H, etc.] In Ref. 4, Weinberg assumes that such operators will be further suppressed by two powers of a light-quark mass (or small Kobayashi-Maskawa angles) and hence be of effective dimension 8. Indeed, this is so of the one-loop contributions. The main point of our paper is that this is

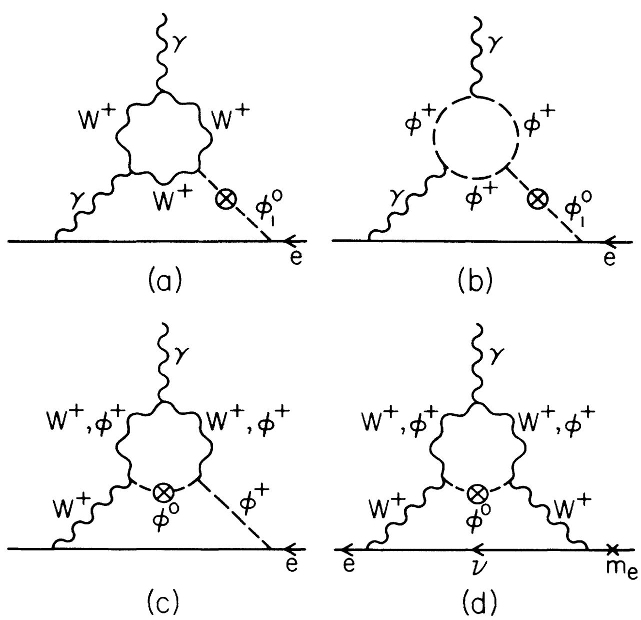

FIG. 2. Diagrams involving boson loops contributing to d_e.

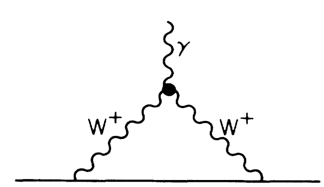

FIG. 3. The electric dipole moment of the $W^{\pm}$ can induce a fermion electric dipole moment via this diagram. Compare to the diagram in Fig. 2(d).

not so of the two-loop contributions. Second, heretofore in the literature[17] it has been assumed that the electric dipole moment of the electron in models where Higgs-boson exchange mediates CP violation (as in the Weinberg model) is of order $(1/16\pi^2)G_F m_e^3/m_H^2 \approx 10^{-35}$ e cm. We have shown that the two-loop contribution dominates this one-loop contribution by 8 orders of magnitude. Finally, it should be emphasized that the electron electric dipole moment is a particularly clean test of these models of CP violation, since for the neutron there are soft QCD effects which are at present uncalculable.

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