# Critical Fluctuations in High-Temperature Superconductors and the Ettingshausen Effect

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We study the fluctuation Ettingshausen effect in a type-II superconductor using the time-dependent Ginzburg-Landau equation. The free-fluctuation theory predicts a divergence in the Ettingshausen coefficient at $T=T_{c2}(H)$, in conflict with the recent data of Palstra *et al.* [Phys. Rev. Lett. **64**, 3090 (1990)]. Including interactions through a self-consistent Hartree approximation eliminates the divergence and provides a quantitative explanation of the experiments.

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Recently, Palstra *et al.*[1] have made interesting observations of the Ettingshausen effect in single-crystal Y-Ba-Cu-O. The Ettingshausen effect is a transverse thermomagnetic effect in which a magnetic field is applied in the $z$ direction, a constant current supplied in the $x$ direction, and a temperature gradient is measured in the $y$ direction. This effect is typically quite small in the normal state. However, a substantial effect is produced in the mixed state of a type-II superconductor due to the transport of entropy by vortices, which move transverse to the applied current. The mean-field theory of this effect has been considered by Maki[2] who found that the Ettingshausen effect vanishes for $T>T_{c2}(H)$ due to an absence of vortices, and increases linearly with $T_{c2}(H)-T$ for $T<T_{c2}(H)$, where $T_{c2}(H)$ is the mean-field transition temperature. (At sufficiently low temperature the effect must also vanish due to the vanishing entropy.) However, Palstra *et al.*[1] find a large temperature gradient well above $T_{c2}(H)$, indicative of significant thermal fluctuations.[3]

In this paper we study the *fluctuation* Ettingshausen effect both above and below the mean-field transition temperature. We calculate the Ettingshausen coefficient $\alpha_{yx}$ using the Lawrence-Doniach model[4] of layered superconductors. If only *Gaussian* fluctuations are considered, then $\alpha_{yx}$ is predicted to diverge at the mean-field transition temperature, in conflict with the experimental results. One of our important conclusions is that *interactions* between the fluctuations must be considered in order to obtain even qualitative agreement with the experimental results. To do so, we apply the Hartree approximation to treat the quartic term in the Ginzburg-Landau Hamiltonian. In the limit of high magnetic fields, we find a smooth crossover from a regime dominated by two-dimensional Gaussian fluctuations for $T>T_{c2}(H)$, to the mean-field results for $T<T_{c2}(H)$, with no intervening divergence, in agreement with the experimental results. The absence of such a divergence is due to the one-dimensional character of the fluctuations—fluctuations transverse to the applied magnetic field are effectively "frozen out," as was first discussed by Lee and Shenoy.[5] Our approach is inspired by studies of the specific-heat transition in a magnetic field.[6-8]

In order to model the layered structure of Y-Ba-Cu-O, we use the Lawrence-Doniach model, which consists of superconducting sheets separated by a distance $s$, with a Josephson coupling between the sheets. The Hamiltonian is

$$H=\sum_i\int d^2x\left\{\frac{\hbar^2}{2m}\left|\left(\nabla-i\frac{2e}{\hbar c}\mathbf{A}\right)\psi_i\right|^2+\frac{\hbar^2}{2m_cs^2}|\psi_i-\psi_{i+1}|^2+a|\psi_i|^2+\tfrac{1}{2}b|\psi_i|^4\right\},\tag{1}$$

where $m\equiv m_{ab}$ and $m_c$ are the effective masses in the $a$-$b$ plane and along the $c$ axis, respectively, $a=a_0(T/T_0-1)$, with $T_0$ the bare transition temperature, all derivatives and coordinates are in the $a$-$b$ plane, and the applied field is assumed to be perpendicular to the $a$-$b$ planes. The terms for the field energy have not been displayed. We introduce relaxational dynamics[9] through the equation of motion

$$\Gamma_0^{-1}\left(\partial_t+i\frac{2e}{\hbar c}\Phi\right)\psi_i(\mathbf{x},t)=-\frac{\delta H}{\delta\psi_i^*(\mathbf{x},t)}+\zeta_i(\mathbf{x},t),\tag{2}$$

where $\Phi$ is the scalar potential and $\zeta$ is a noise term chosen to have Gaussian white-noise correlations: $\langle\zeta_i^*(\mathbf{x},t)\times\zeta_j(\mathbf{x}',t')\rangle=2k_BT\Gamma_0^{-1}\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')\delta_{ij}$. The heat current in the $y$ direction is related to the electric field in the $x$ direction via the transport coefficient $\alpha_{yx}=\langle J_y^h\rangle/E_x$. This coefficient can be related to the experimentally measured quantity $U_\phi$ by $\alpha_{yx}=K(dT/dy)/(dV/dx)=U_\phi/\phi_0$, where $\phi_0=hc/2e$ is the flux quantum; $U_\phi$ is plotted in Fig. 2 of Ref. 1. Therefore, we compute the heat current which is given by[10]

$$\langle\mathbf{J}^h\rangle=\frac{\hbar^2}{2m}\left\{\left(\nabla-i\frac{2e}{\hbar c}\mathbf{A}(\mathbf{x})\right)\left(\partial_{t'}-i\frac{2e}{\hbar c}\Phi(\mathbf{x}')\right)+\left(\nabla'+i\frac{2e}{\hbar c}\mathbf{A}(\mathbf{x}')\right)\left(\partial_t+i\frac{2e}{\hbar c}\Phi(\mathbf{x})\right)\right\}C_{ii}(\mathbf{x},t;\mathbf{x}',t')|_{(\mathbf{x}',t')=(\mathbf{x},t)},\tag{3}$$

 

where $C_{ij}(\mathbf{x},t;\mathbf{x}',t') = \langle \psi_i(\mathbf{x},t)\psi_j^*(\mathbf{x}',t')\rangle$ is the full nonequilibrium correlation function. The Ettingshausen coefficient is obtained by expanding the correlation function to linear order in the electric field. We choose the electric field to be in the $x$ direction: $\Phi = -E_x x$; and the vector potential in the Landau gauge is $\mathbf{A} = Hx\hat{\mathbf{y}}$.

We first consider the Gaussian fluctuations, neglecting the quartic term in the Hamiltonian. The Hamiltonian is diagonalized by eigenstates consisting of Bloch waves in the $z$ direction and Landau levels in the $x$-$y$ plane. We then solve Eq. (2) using a standard Green's-function approach, hence determining the correlation function to linear order in the electric field. The correlation function is used to calculate the heat current via Eq. (3), and we find for $T > T_{c2}(H)$ the fluctuation Ettingshausen coefficient[11]

$$\tilde{\alpha}_{yx}^{+} = \frac{2k_B T}{\phi_0 s} dh \sum_{n=0}^{N} [f_{n+1}(\epsilon,h) - f_{n+1/2}(\epsilon,h)] , \quad (4)$$

$$f_n(\epsilon,h) = \frac{n}{\{[\epsilon+2hn][1+d^2(\epsilon+2hn)]\}^{1/2}} , \quad (5)$$

where $N$ is a cutoff, $\xi_{ab}(0) = (\hbar^2/2ma_0)^{1/2}$ and $\xi_c(0) = (\hbar^2/2m_c a_0)^{1/2}$ are the zero-temperature correlation lengths in the $a$-$b$ plane and along the $c$ axis, respectively, $\epsilon = T/T_0 - 1$ is the reduced temperature, $h = H/H_{c2}^{ab}(0)$ is a dimensionless magnetic field, $H_{c2}^{ab}(0) = \phi_0/2\pi\xi_{ab}^2(0)$ is the zero-temperature critical field, and $d = s/2\xi_c(0)$ is a dimensionless interplanar spacing (the three-dimensional limit being obtained by taking $d\epsilon \to 0$, and the two-dimensional limit being $d\epsilon \gg 1$). Below $T_{c2}(H)$ the Ettingshausen coefficient is computed by linearizing the equation of motion about the mean-field solution. The appropriate mean-field solution is the Abrikosov flux lattice; however, this solution is unwieldy so we resort to the approximate uniform order-parameter solution $|\psi|^2 = -a/b$.[12] It follows that the fluctuation Ettingshausen coefficient below $T_{c2}(H)$, $\tilde{\alpha}_{yx}^{-}$, is given by Eq. (4) with $a \to -2a$. Therefore, for $T < T_{c2}(H)$,

$$\alpha_{yx}^{-} = \alpha_{yx}^{\mathrm{MF}} + \tilde{\alpha}_{yx}^{-} , \quad (6)$$

where the first term is the mean-field result due to Maki,[2]

$$\alpha_{yx}^{\mathrm{MF}} = \frac{L_D}{4\pi(1.16)(2\kappa^2-1)} \frac{dH_{c2}}{dT} [T - T_{c2}(H)] , \quad (7)$$

where $L_D = 1$ in the dirty limit, and where $\kappa$ is the Ginzburg-Landau parameter.

There are several points to be made about the above results. First, note that the order-parameter relaxation time $\Gamma_0^{-1}$ is absent from the expression for $\alpha_{yx}$. Second, in the mean-field case the transverse heat current may be expressed in terms of the flow of entropy:

$$\langle J_y^h \rangle = T S_V^{\mathrm{MF}} \left[ L_D \frac{H}{T} \left( \frac{dH_{c2}}{dT} \right)^{-1} \right] \left( \frac{B}{\phi_0} \right) u , \quad (8)$$

where

$$S_V^{\mathrm{MF}} = (\phi_0/\beta_A)(1/H)(dH_{c2}/dT)^2 [T - T_{c2}(H)]$$

is the *equilibrium* mean-field entropy per vortex line (per unit length)[13] and $u = cE_x/B$ is the vortex velocity.[14] These two observations suggest that the *fluctuation* Ettingshausen coefficient (a kinetic quantity) may also be written in terms of the equilibrium fluctuation entropy (a thermodynamic quantity). In order to verify this conclusion, we have calculated the fluctuation entropy using standard methods,[15] and find that the fluctuation heat current [for $T > T_{c2}(H)$] may indeed be written as

$$\langle J_y^h \rangle = T S_V^{+} \left[ 2\frac{H}{T} \left( \frac{dH_{c2}}{dT} \right)^{-1} \right] \left( \frac{B}{\phi_0} \right) u , \quad (9)$$

where $S_V^+$ is the fluctuation entropy per vortex in either the low- or high-field limits. It is interesting to note that the fluctuation specific heat, $c_p = -(B/\phi_0) T \partial S_V^+/\partial T$, may be deduced from a measurement of $\tilde{\alpha}_{yx}$.

Although the expression for $\alpha_{yx}$ given by Eq. (4) is rather cumbersome, it is easily evaluated in the high-field limit, or for temperatures close to $T_{c2}(H)$. In this limit the sum is dominated by the contribution from the lowest Landau level, which corresponds to keeping the most divergent term in the sum in Eq. (4). Thus, we have

$$\alpha_{yx} = \frac{k_B T}{\phi_0 s} \frac{dh}{[\epsilon_H(1+d^2\epsilon_H)]^{1/2}} , \quad (10)$$

where we have defined $\epsilon_H \equiv \epsilon + h$. The mean-field transition temperature $T_{c2}(H)$ is defined by $\epsilon_H(T_{c2}) = 0$, so that close to the transition we have

$$\epsilon_H \approx \frac{1}{H_{c2}(0)} \frac{dH_{c2}}{dT} [T - T_{c2}(H)] . \quad (11)$$

Therefore $\alpha_{yx}$ diverges at the mean-field transition temperature; in two dimensions the divergence is of the form $(T - T_{c2})^{-1}$, while in three dimensions it is $(T - T_{c2})^{-1/2}$. Also note that the size of the fluctuation scales with magnetic field. There is *no* evidence for such a divergence in the data of Ref. 1.

Given the discrepancy between the Gaussian calculation and the data, we are led to consider the effect of interactions between the fluctuations. The simplest approximation is to treat the quartic term using a self-consistent Hartree approximation,[6-8] and this is implemented by replacing the quartic term $|\psi_i|^4$ by $2\langle|\psi_i|^2\rangle|\psi_i|^2$. The energy of the magnetic field can be absorbed into a renormalized coupling[8] $b_\kappa = b(1 - 1/2\kappa^2)$. This leads to a self-consistent equation for the coefficient of the quadratic term, $\tilde{a} = a + b_\kappa\langle|\psi_i|^2\rangle$. We consider only the lowest Landau level, so that the self-consistent equation in our dimensionless units becomes

$$\epsilon_H = \tilde{\epsilon}_H - \frac{2\kappa^2-1}{4\gamma^2 d} \frac{s}{\Lambda_T} \frac{h}{[\tilde{\epsilon}_H(1+d^2\tilde{\epsilon}_H)]^{1/2}} , \quad (12)$$

where $\gamma = \xi_c(0)/\xi_{ab}(0)$ is the anisotropy parameter, $\Lambda_T = \phi_0^2/16\pi^2 k_B T$ is a thermal length, and we have used the fact that in mean-field theory $b = 2\pi\kappa^2(2\hbar e/mc)^2$.[13] In this approximation $\alpha_{yx}$ is obtained by replacing $\epsilon_H$ by $\tilde{\epsilon}_H$ in Eq. (10):

$$\alpha_{yx}=\frac{k_BT}{\phi_0 s}\frac{dh}{[\tilde{\epsilon}_H(1+d^2\tilde{\epsilon}_H)]^{1/2}}. \tag{13}$$

The Ettingshausen coefficient is thus obtained by solving Eq. (12) for $\tilde{\epsilon}_H$ as a function of $\epsilon_H$, and substituting the results into Eq. (13). Equations (12) and (13) are the main results of this paper.

Before comparing our approximation to the experimental data, several comments are in order. (1) In this high-field Hartree approximation there is no finite-temperature transition to a state with $\langle\psi_i\rangle\neq 0$, which would be signaled by $\tilde{\epsilon}_H=0$.[16] Hence, there is only a crossover behavior from a regime dominated by two-dimensional Gaussian fluctuations for temperatures far above the mean field $T_{c2}(H)$, to three-dimensional Gaussian fluctuations at temperatures $T\sim T_{c2}(H)$, and finally to the mean-field regime which consists of well-defined vortices for temperatures below $T_{c2}(H)$. (2) Note that $\alpha_{yx}$ exhibits scaling behavior in either the two-dimensional or three-dimensional limits.[5,7,8] In two dimensions,

$$\alpha_{yx}^{2D}=\frac{k_BT}{\phi_0 s}h\left[\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right]^{1/2}F_{2D}\left[\left(\frac{s\Lambda_T}{(2\kappa^2-1)\xi_{ab}^2(0)h}\right)^{1/2}\epsilon_H\right], \tag{14}$$

while in three dimensions,

$$\alpha_{yx}^{3D}=\frac{k_BT}{\phi_0 s}dh\left[\frac{4\gamma^2 d\Lambda_T}{(2\kappa^2-1)sh}\right]^{1/3}F_{3D}\left[\left(\frac{4\gamma^2 d\Lambda_T}{(2\kappa^2-1)sh}\right)^{2/3}\epsilon_H\right]. \tag{15}$$

The scaling functions $F_{2D}(x)$ and $F_{3D}(x)$ have the asymptotic forms $F_{2D}(x),F_{3D}(x)\sim -x$ for large negative values of $x$; reinstating the units, this leads to $\alpha_{yx}$ in the regime well below the mean field $T_{c2}(H)$,

$$\alpha_{yx}^{\text{Hartree}}=\frac{1}{8\pi(2\kappa^2-1)}\frac{dH_{c2}}{dT}[T_{c2}(H)-T]. \tag{16}$$

This expression differs from the correct mean-field result Eq. (7) by a factor of $2L_D/1.16$; the Hartree approximation does not incorporate the correct vortex lattice structure below $T_{c2}(H)$. For large positive values of $x$, $F_{2D}(x)\sim x^{-1}$ and $F_{3D}(x)\sim x^{-1/2}$, which reproduces the Gaussian results. Even though the Hartree approximation does not produce the correct mean-field result, we expect that the above scaling forms would still hold in a higher-order approximation.[8] However, we note that for intermediate values of the dimensionless interplanar separation $d$, there is no simple scaling form for $\alpha_{yx}$. (3) The reader should compare the calculation outlined above with recent attempts to understand the "magnetic-field-induced broadening" of the resistive transition in Y-Ba-Cu-O (Ref. 17) using extensions of the Hartree approximation.[18] Our calculation could also be extended to higher order; however, a consistent calculation beyond the Hartree approximation requires consideration of vertex corrections in evaluating $\alpha_{yx}$.[11] (4) We have so far neglected vortex pinning, which will be important in the vicinity of the vortex-glass transition.[19]

Finally, we compare our theory with the data of Palstra *et al.*[1] First, we attempt to fit the data by the scaling forms given in Eqs. (14) and (15). We perform a linear fit to the data in the region where we expect the mean-field theory to apply, and thereby obtain the mean field $T_{c2}(H)$. We plot $U_\phi$ vs $\epsilon_H=[1/H_{c2}^{ab}(0)](dH_{c2}/dT)(T-T_{c2})$, where we take $dH_{c2}/dT=7$ T/K and $H_{c2}^{ab}(0)=400$ T from Ref. 1. Next, the resulting curves are scaled according to Eq. (14) or (15); we find a much better fit with the two-dimensional scaling form of Eq. (14). The result of this fit is shown in Fig. 1. While the data appear to collapse onto a single curve for $\epsilon_H>0$, the scaling form does not hold for negative values of $\epsilon_H$. This breakdown of scaling is due to the magnetic-field dependence of the mean-field slopes of the $U_\phi$-vs-$T$ curves; this field dependence is not yet understood, but is most likely related to the field-induced broadening.[17] Second, we have solved explicitly the Hartree equations (12) and (13). We have chosen a magnetic field of 7.5 T; an effective value of $\kappa$ is obtained by equating the

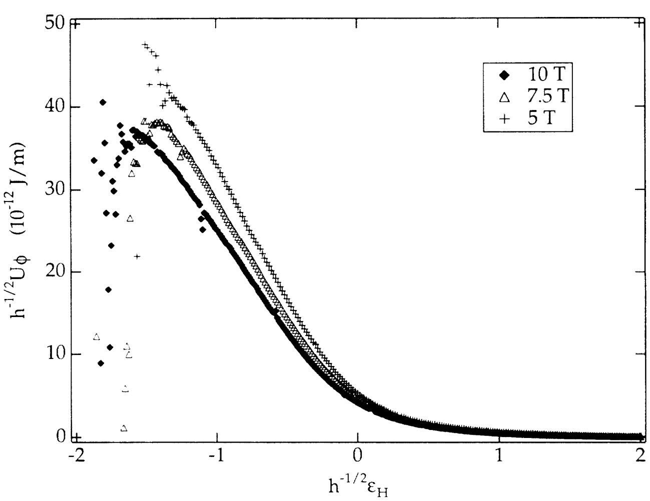


FIG. 1. The experimental data of Ref. 1 of the transport energy $U_\phi$ as a function of the reduced temperature $\epsilon_H\equiv[1/H_{c2}^{ab}(0)](dH_{c2}/dT)[T-T_{c2}(H)]$ for different magnetic fields. As suggested by the two-dimensional scaling form of Eq. (14), both axes are scaled by $h^{-1/2}$, where $h\equiv H/H_{c2}^{ab}(0)$ is the reduced magnetic field. The fitting procedure is discussed in the text.

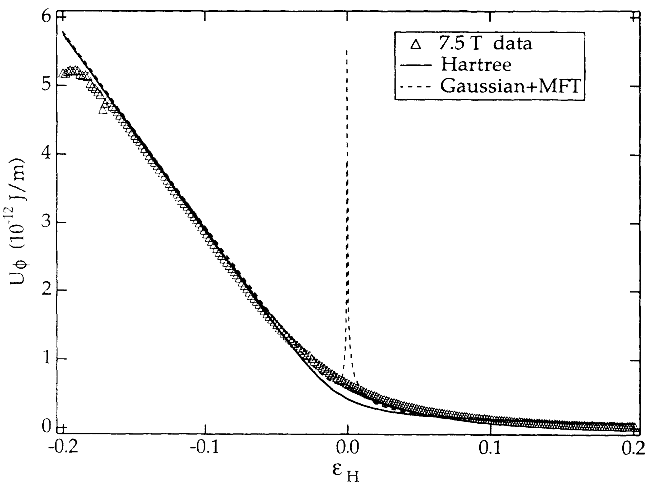


FIG. 2. The transport energy $U_\phi$ in the Hartree approximation (solid line) calculated from the solution of Eqs. (12) and (13) in the text, and compared to the data of Ref. 1 for a field of 7.5 T (open triangles). The parameters are $H_{c2}^{ab}(0)=400$ T, $s=12$ Å, and $\xi_c(0)=2$ Å. Also shown are the results of the Gaussian calculation (dashed line), Eqs. (6) and (10). The fitting procedure is discussed in the text.

slope of $U_\phi$ vs $\epsilon_H$ in the mean-field regime, to the corresponding slope given by the Hartree result, Eq. (16). We take the following parameter values to be representative of Y-Ba-Cu-O: $s=12$ Å, $\xi_c(0)=2$ Å, $\xi_{ab}(0)=9$ Å. The result of the calculation is shown in Fig. 2, along with the results of the Gaussian calculation, Eqs. (6) and (10), and the 7.5-T data taken from Ref. 1. We note that the agreement of the Hartree result with the data in the regime $\epsilon_H \sim -0.1$ is guaranteed by our fitting procedure. The Hartree approximation interpolates smoothly between the Gaussian result for $\epsilon_H > 0$ and the mean-field behavior for $\epsilon_H < 0$, in agreement with the data.

In summary, we have calculated the fluctuation Ettingshausen effect in type-II superconductors, using a Hartree approximation to treat the quartic term in the Hamiltonian, and have obtained quantitative agreement with the recent experiments of Palstra *et al.*[1]

We thank T. Palstra for providing us with the experimental data and for useful discussions, and D. Huse for bringing this problem to our attention. After this work was completed we became aware of a recent calculation by K. Maki who has independently obtained the Gaussian results for $T > T_{c2}(H)$. This work was supported by NSF Grant No. DMR 89-14051.

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