Route to Chaos and Competition between Relaxation Oscillations for a Semiconductor Laser with Optical Feedback

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We report the first experimental observation of the route to chaos for semiconductor lasers with weak optical feedback. For increasing feedback level the laser undergoes a quasiperiodic route interrupted by frequency locking, which is also predicted by a comprehensive numerical analysis. Experiments further demonstrate the occurrence of quantum-noise-induced transitions between two attractors with different relaxation oscillation frequencies. The attractors are created by two subsequent Hopf bifurcations of the same external cavity mode.

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It is by now well known that even minute fractions of optical feedback from an external reflector may cause a semiconductor laser to switch to a state of significantly increased phase and intensity noise, where the laser linewidth has broadened from a few MHz to several GHz. This so-called coherence-collapsed state[1] has been interpreted as a chaotic attractor, but even though this phenomenon has attracted a lot of interest due to its practical importance,[2-4] only little is known about the dynamics of the system. From the basic standpoint, however, the system serves as an important example of an infinite-dimensional delayed-feedback optical system (see also Ref. 5) with behavior quite different from that of the well-studied Ikeda system.[6]

For laser diodes with strong feedback it has been demonstrated that a chaotic state is reached through intermittency,[7] and subharmonic bifurcations have been reported for a system with a tilted external mirror.[8] For laser diodes with weak to moderate feedback we have recently shown theoretically[9] that the system undergoes a quasiperiodic route to chaos which can be interrupted by frequency locking. In the present Letter this behavior is verified experimentally, for the first time we believe. Previously, quasiperiodic phenomena and frequency locking have been observed in current-modulated laser diodes,[10,11] whereas the laser investigated here is biased with a constant current.

We also demonstrate that the two relaxation oscillation peaks often observed in time-averaged noise spectra may actually arise from noise-induced hopping between two attractors with different oscillation frequencies. The two attractors are created by subsequent Hopf bifurcations of the same external cavity mode (ECM). This behavior is quite unexpected and has not been predicted or observed previously.

The dynamics of a laser diode with weak to moderate feedback from an external mirror is governed by nonlinear rate equations for the complex electric field $E(t)$ and the carrier density $N(t)$; see, e.g., Ref. 4, Eqs. (4) and (5). A term proportional to $\kappa E(t-\tau)$ accounts for the delayed external reflection and renders the system infinite dimensional. τ is the round-trip time in the external cavity and κ^2 is the feedback power ratio. For fixed parameters κ, τ, and J/J_{th} (bias current relative to threshold current) the laser may operate in several ECM's and in the presence of spontaneous-emission quantum noise the laser jumps between the modes.[12] The ECM's are tuned (by small adjustments of τ) so that one of the frequencies coincides with the frequency of the solitary laser (i.e., without feedback). This is the dominant ECM (it also coincides with the minimum linewidth mode) and the laser very rarely jumps to other modes.

We have examined the system for increasing feedback level using the Poincaré-section technique. For a given value of κ, the rate equations are integrated numerically. The solution describes a trajectory in, e.g., (E_0,N,Δ) space, where $E_0=|E|$ is the electric-field amplitude and $\Delta=\arg\{E(t)\}-\arg\{E(t-\tau)\}$ is the phase delay. The trajectory obtained after transients have died away constitutes the attractor, which is characterized by its intersection with the plane $E_0=E_{\text{sol}}$. The index "sol" indicates the corresponding value for the solitary laser. A bifurcation diagram is now obtained by plotting the normalized carrier density $N/N_{\text{sol}}-1$ at the intersection points as a function of κ.

Figure 1 shows the result for $J/J_{\text{th}}=2.0$, $\tau=2$ ns, and parameters typical for a long-wavelength laser diode. The upper part of Fig. 1 is for increasing κ and is discussed first. No intersections exist when the solution is a stable fixed point, but for $\kappa=6.0\times10^{-3}$ relaxation oscillations become undamped and give rise to a self-sustained limit-cycle solution with oscillation frequency 4.24 GHz. This solution leads to a single intersection point ("mirror" points are rejected). The relaxation oscillation corresponds to a modulation of the optical signal with the relaxation frequency, and it leaves the mean optical frequency $\langle d\arg\{E(t)\}/dt\rangle$ almost unaffected. For $\kappa=8.6\times10^{-3}$ an apparent broadening sets in which reflects the creation of a quasiperiodic solution with two incommensurate frequencies; i.e., the trajectory lies on a torus.[9] The second frequency that comes into play is related to but slightly lower than τ^{-1}. At $\kappa\simeq1.15\times10^{-2}$ the torus "explodes" due to the creation of a new quasi-

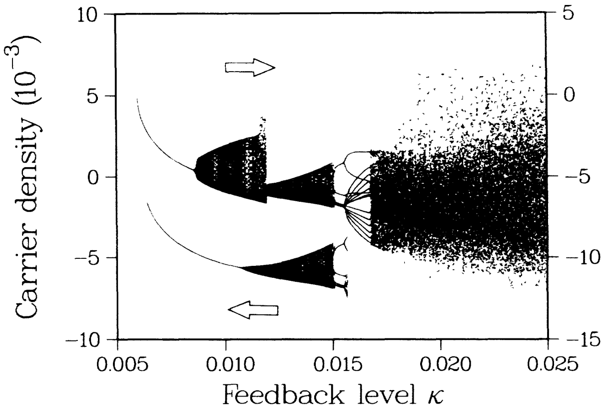

FIG. 1. Calculated bifurcation diagram for $J/J_{th}=2.0$ and $\tau=2$ ns. The upper part (right axis) is obtained for increasing κ and the lower part (left axis) is obtained for decreasing κ. The traces are separated vertically for clarity.

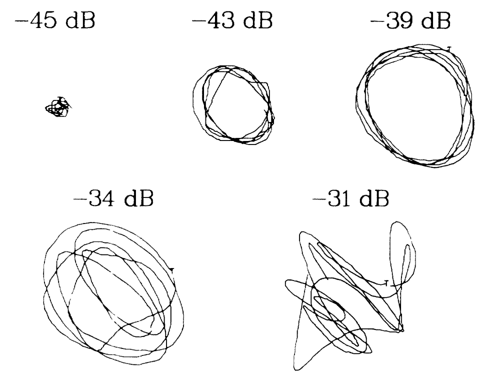

FIG. 2. Experimental phase portraits for various feedback levels. $J/J_{th}=1.83$, $\tau=1.07$ ns.

periodic solution with *three* incommensurate frequencies. This solution forms a three-dimensional torus, as verified by the closed curve obtained by performing two consecutive transverse cuts through the attractor. The three-frequency attractor quickly evolves into a chaotic attractor and at $\kappa \simeq 1.20\times10^{-2}$ we observe a transition to another attractor. The "old" chaotic attractor probably coexists with this new attractor, but due to the self-generated noisiness it is numerically very hard to avoid switching to a nearby solution with more "damped" trajectories. The "new" attractor is traced back in the lower part of Fig. 1, and is created as a stable limit cycle with frequency 3.83 GHz at $\kappa \simeq 6.4\times10^{-3}$. The limit cycle bifurcates to a torus followed by a frequency-locked solution of order eight that period doubles and finally becomes chaotic.

While the relaxation frequencies of the two attractors in Fig. 1 clearly differ, their mean optical frequencies are very close. This indicates that they are created by bifurcations from the same ECM. A small-signal stability analysis[9] of the dominant ECM shows that the "first" limit-cycle solution is created when a pair of complex-conjugate zeros of the system determinant $D(s)$ moves into the right half of the complex s plane at $s/2\pi = \pm j(4.24\text{ GHz})$ for $\kappa=6.0\times10^{-3}$. This corresponds to a normal Hopf bifurcation where the stable-fixed-point solution (ECM) bifurcates into an unstable fixed point and a stable limit cycle with oscillation frequency 4.24 GHz. For $\kappa=6.3\times10^{-3}$, another pair of zeros moves into the right half s plane at $s/2\pi=\pm j(3.83$ GHz). This indicates a Hopf bifurcation from the now unstable fixed point (ECM) which, in general, leads to the creation of an unstable limit cycle. For $\kappa > 6.4\times10^{-3}$, however, direct numerical integration shows the presence of a stable limit cycle with a frequency of 3.83 GHz that is centered around the unstable-fixed-point solution. Thus, for $\kappa=6.45\times10^{-3}$ the mean optical frequency deviates by less than 2 MHz (0.4% of the mode spacing) from the optical frequency of the unstable ECM. This strongly suggests that the stable limit cycle bifurcates off the unstable limit cycle for $\kappa\simeq6.4\times10^{-3}$. We found similar behavior possible in a model system of two coupled oscillators.[13] However, the details of the formation of the second limit cycle needs further investigation. The birth of a second attractor is clearly observed in cases like this where J/J_{th} and τ are chosen so that two sets of zeros of $D(s)$ move into the right half s plane for almost the same κ.

For the experiments, a 1.3-μm temperature-stablized distributed-feedback laser diode ($J_{th}=15$ mA) was exposed to optical feedback from a 16-cm-long external cavity with mirror reflector ($\tau=1.07$ ns). The optical spectrum was analyzed with a confocal Fabry-Pérot interferometer [2-GHz free spectral range (FSR)] and the intensity was monitored with a fast photodiode.

Because of the very short time scale ($\simeq 0.25$ ns) on which the basic relaxation oscillations occur, it is difficult to measure the time-dependent behavior directly. Instead, we have employed an rf-heterodyne technique where the photodiode current is mixed with a strong local oscillator signal with frequency f_{LO} in a nonlinear mixer. The difference signal is then studied directly in the time domain with a (500-MHz) real-time digital storage oscilloscope. Phase portraits were obtained by splitting the signal after the mixer in two arms and delaying one of these electrically by a quarter of the oscillation period. The resulting single-shot x-y traces for various feedback levels are shown in Fig. 2. The frequency of the local oscillator was fixed at $f_{LO}=4.088$ GHz, and change of this (or, equivalently, the electric delay time) only led to attractors of more elliptical shape. The first trace in Fig. 2 is for a low feedback level where the laser operates stably in the minimum-linewidth mode. Though the linewidth is narrowed considerably in comparison with the solitary laser, the intensity noise level (dominated by quantum noise) is somewhat larger due to less damping of the relaxation oscillations. The noise level of our detection system, including amplifiers, is much lower than the inherent level of quantum noise in the laser. At $\kappa = -43$ dB a noisy limit cycle has evolved which then grows in size. The basic frequency of the self-sustained relaxation oscillations is 4.2 GHz, which means that the difference signal has a frequency of $\simeq 110$ MHz. For $\kappa = -34$ dB the limit cycle is distorted to a degree that cannot just be explained as due to quantum noise, and for $\kappa = -31$ dB the trajectory has a character typical of chaotic behavior. The heterodyne technique used in obtaining these results, of course, in some sense, obscures the picture when more than one frequency is present in the basic signal. However, the results of Fig. 2 are consistent with the quasiperiodic route to chaos observed in Fig. 1, and further evidence is provided in Fig. 3. In these experiments we carefully avoided the presence of another attractor, cf. Fig. 1, by choosing a short cavity length and an appropriate bias current.

Figure 3(a) shows an intensity spectrum obtained by feeding the output of the photodiode to a spectrum analyzer. The two basic peaks C and R are situated at frequencies f_c and f_r, where $f_c \simeq \tau^{-1}$ and f_r is close to the relaxation frequency of the solitary laser. The example demonstrates a frequency-locked state where all peak frequencies are multiples of f_c, and $f_r/f_c = 5.0$. Halfway between the strong peaks there are weak bumps which might be attributed to a period doubling of the frequency-locked solution, cf. Fig. 1. In Fig. 3(b) we plot the frequency ratio f_r/f_c vs κ and four different regimes are identified. Regime I, which starts at $\kappa \simeq -44$ dB, is the limit-cycle regime characterized by a strong relaxation peak at R and a weak peak at C. In regime II the amplitude of C increases strongly which indicates that the system is quasiperiodic. Regime III is characterized by a plateau in f_r/f_c at the integer value 5. It is therefore a regime of frequency locking. In the chaotic regime IV the peaks get very broad and are hardly resolvable. The example shows a typical scenario. However, we find both numerically and experimentally that the range of frequency locking depends strongly on the laser parameters, and in some cases the quasiperiodic attractor evolves directly into the chaotic state. The uncertainties in the knowledge and control of the laser parameters do not permit a detailed quantitative comparison between theory and experiment.

Finally, Fig. 4 demonstrates the experimental observation of the two coexisting attractors predicted in Fig. 1. For this experiment, the external cavity length was increased to 30 cm. Figure 4(a) shows the time-averaged spectrum observed with a planar Fabry-Pérot interferometer with 12-GHz FSR. In addition to the central peak A of the oscillating mode, there are two relaxation oscillation sidebands B and C, which are displaced from the central peak by 4.36 and 4.79 GHz. The identification of B and C as relaxation oscillation sidebands of A was confirmed by their frequency shift with bias current; see Ref. 9, Eq. (5). The width of all the peaks is

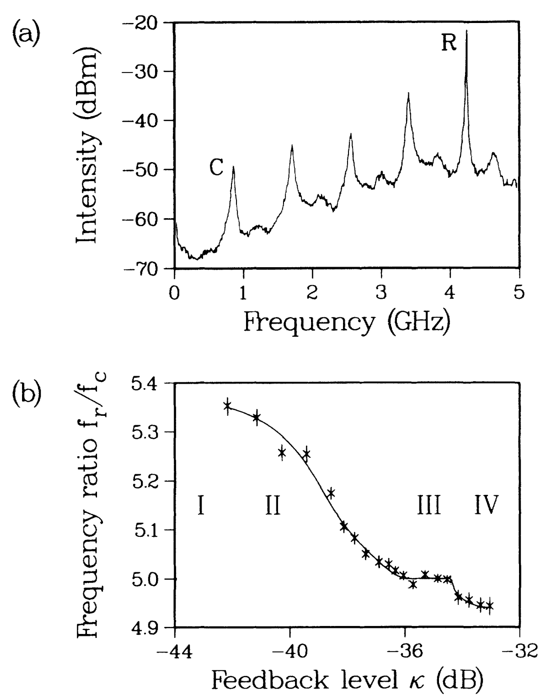

FIG. 3. (a) Measured intensity spectrum and (b) frequency ratio f_r/f_c vs feedback level with identification of regimes I (limit cycle), II (quasiperiodic), III (frequency locked), and IV (chaotic). $J/J_{th} = 1.84$, $\tau = 1.07$ ns.

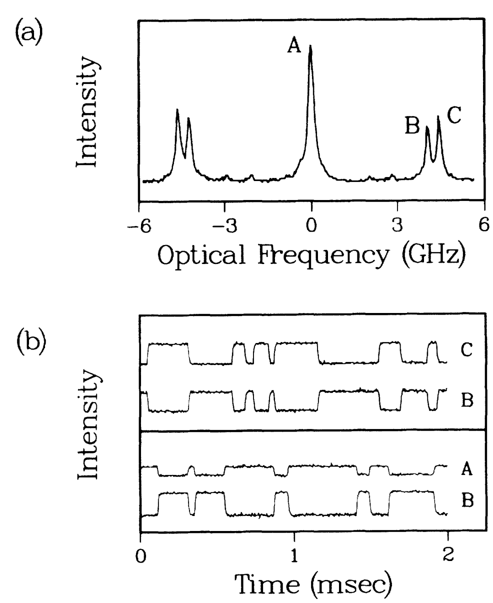

FIG. 4. (a) Time-averaged optical spectrum. The frequency of the solitary laser corresponds to 0 GHz. (b) Time dependence of peaks B and C (upper two traces) and A and B. Dotted lines indicate zero intensity level (common for two lower traces). $J/J_{th} = 2.01$, $\tau = 1.97$ ns, $\kappa^2 = -37$ dB.

limited by the $\simeq$250-MHz resolution of the Fabry-Pérot interferometer. The time-resolved behavior of the system was analyzed by using two planar Fabry-Pérot interferometers (FSR=12 GHz) as frequency discriminators, i.e., as filters with transmission around a specific center frequency only. The upper two traces in Fig. 4(b) result when the Fabry-Pérot interferometers are set at the frequencies of the two sidebands, B and C. The anticorrelated traces clearly show that the laser jumps randomly between the two bands, thus corresponding to random jumps between two periodic (or possibly quasiperiodic) solutions with different oscillation frequencies. The two lower traces show that the central peak also changes in response to the jumps between sidebands. These changes are, however, very small compared to the case of mode jumps which would result in a nearly 100% modulation of the signal and also give rise to two central peaks in the time-averaged spectrum. This proves that the two attractors belong to the same external cavity mode.

In conclusion, we have demonstrated that semiconductor lasers with weak feedback undergo a quasiperiodic route to chaos which may be interrupted by frequency locking. The laser dynamics occur on a very short time scale ($\simeq$0.25 ns), but the use of a heterodyne (down-conversion) technique facilitated recordings of experimental phase portraits. It was also demonstrated that the laser shows quantum-noise-induced hopping between attractors arising from the same external cavity mode but with different relaxation oscillation frequencies. The origin of the new dynamical solution was identified as a secondary Hopf bifurcation from an *unstable* external cavity mode.

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[1]D. Lenstra, B. H. Verbeek, and A. J. den Boef, IEEE J. Quantum Electron. **21**, 674 (1985).

[2]N. Schunk and K. Petermann, IEEE J. Quantum Electron. **24**, 1242 (1988).

[3]J. S. Cohen and D. Lenstra, IEEE J. Quantum Electron. **25**, 1143 (1989).

[4]B. Tromborg and J. Mørk, IEEE J. Quantum Electron. **26**, 642 (1990).

[5]F. T. Arecchi, G. Giacomelli, A. Lapucci, and R. Meucci, Technical Digest Nonlinear Dynamics in Optical Systems, Oklahoma, 4–8 June 1990 (unpublished), paper No. TuA4.

[6]K. Ikeda, H. Daido, and O. Akimoto, Phys. Rev. Lett. **45**, 709 (1980); M. Le Berre, E. Ressayre, A. Tallet, and H. M. Gibbs, Phys. Rev. Lett. **56**, 274 (1986).

[7]J. Sacher, W. Elsässer, and E. O. Göbel, Phys. Rev. Lett. **63**, 2224 (1989); J. Mørk and K. Kikuchi, Technical Digest, Conference on Lasers and Electro-Optics, Anaheim, California, 25–29 April 1988 (unpublished), paper No. WM-14.

[8]T. Mukai and K. Otsuka, Phys. Rev. Lett. **55**, 1711 (1985).

[9]B. Tromborg and J. Mørk, Photonics Technol. Lett. **2**, 549 (1990).

[10]H. G. Winful, Y. C. Chen, and J. M. Liu, Appl. Phys. Lett. **48**, 616 (1986).

[11]D. Baums, W. Elsässer, and E. O. Göbel, Phys. Rev. Lett. **63**, 155 (1989).

[12]J. Mørk, M. Semkow, and B. Tromborg, Electron. Lett. **26**, 609 (1990).

[13]J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields* (Springer-Verlag, New York, 1983), p. 398.