Coulomb-Distortion Effects on Spectroscopic Strengths from the $(e,e'p)$ Reaction

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Quasielastic $(e,e'p)$ cross sections have been calculated with a full partial-wave expansion of both Coulomb-distorted electron waves and -distorted proton waves, in the framework of the relativistic σ-ω model. Results are compared with those from currently used nonrelativistic approximations and with data. Spectroscopic factors are derived for the valence-proton knockout of ^{40}Ca and ^{208}Pb and are compared with predictions from recent theoretical approaches.

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The quasielastic $(e,e'p)$ reaction has long been recognized as a preeminent tool for studying independent-particle aspects of the nuclear system.[1] Among the quantities basic to our understanding of the nuclear shell model that can be extracted from $(e,e'p)$ experiments are spectroscopic factors and occupation probabilities. A special case in point is the ^{208}Pb region where experimental information is now available[2] which could provide not only precise values of spectroscopic factors but also values of occupation probabilities by making use of ^{206}Pb-^{205}Tl charge-density measurements in a sum-rule approach.[3,4] This region gains particular importance from the availability of theoretical nuclear-matter calculations.[5-8] Unfortunately, the spectroscopic factors derived from $(e,e'p)$ experiments on ^{208}Pb and other heavy nuclei are strongly influenced by the Coulomb distortion of the electrons and, while calculations that include the strong proton-nucleus final-state interactions [distorted-wave impulse approximation (DWIA)] have long been available, the additional effects of electron-nucleus Coulomb distortions (CDWIA) have been handled, until now, only in an approximate manner and with standard nonrelativistic elements. These approximations[9] [hereafter referred to as (second-order) eikonal] indicate that the effects are large but do not in all cases produce good agreement with the data.[2] In addition, large differences between the results of first-order[10] and second-order[9] eikonal calculations lend further doubt as to whether this effect is as yet under control. Clearly, a more exact handling is called for.

According to the standard rules for evaluating the invariant amplitude associated with the Feynman diagram for single-photon exchange $(e,e'p)$, one must integrate over both the electron vertex $r_e=(t_e,\mathbf{r}_e)$ and proton vertex $r_p=(t_p,\mathbf{r}_p)$, as well as the momentum $q=(\omega,\mathbf{q})$ of the exchanged virtual photon. In DWIA, where the electron lines are taken to be plane waves, the integrations over r_e and q can be handled in a trivial analytical manner leaving a single (numerical) integration over $\mathbf{r}_p$ and a partial-wave expansion of only the outgoing continuum proton wave. In CDWIA this is possible only for the timelike components and it becomes necessary to numerically integrate over $\mathbf{r}_e$ with partial-wave expansions of both initial and final electron waves. This multifold explosion of programming complexity has blocked the development of an exact treatment of electron distortions in spite of its great usefulness.

In the calculations presented here, both electron and proton currents are treated relativistically. The bound-state proton wave functions are solutions of the Dirac equation in potential wells derived either from a relativistic mean-field Hartree calculation[11] or from fits to elastic-electron-scattering data.[12] The continuum proton waves are derived using relativistic optical potentials calculated from a Dirac phenomenological global fit[13,14] to elastic-proton-scattering observables. The proton current operator is of the usual form,

$$J^\mu=F_1(q^2)\gamma^\mu+F_2(q^2)(i\kappa/2m)\sigma^{\mu\nu}q_\nu, \tag{1}$$

where the F's are the free-proton form factors and κ its anomalous magnetic moment. The distorted electron wave functions are calculated from a variant of a widely distributed phase-shift analysis code.[15]

I have applied two minor approximations to make the problem more tractible. First, it is assumed that electron helicity is strictly conserved. Since, in all the cases to be considered, the electron energies are much greater than the electron mass and scattering angles are not in the vicinity of 180°, this assumption should not introduce appreciable error. Second, a "q-effective" method similar to that of Refs. 9 and 10 is applied but here it is merely used to convert the form factors in Eq. (1) from functions of q^2 to functions of $\mathbf{r}_e$. This ignores any angular dependence of q^2 on $\mathbf{r}_e$ but due to the slow and smooth variation of the proton electromagnetic form factors this approximation should likewise produce negligible error. The advantage gained is that it allows the remaining integral over the photon momentum $\mathbf{q}$ to be handled analytically using techniques developed for evaluating pion exchange contributions in (γ,p) (Ref. 16) and $(e,e'p)$ (Ref. 17) reactions.

Figures 1 and 2 shows results for the present method plotted against NIKHEF data.[2,18] Calculations and data are presented as reduced cross sections ρ versus "missing momentum" p_m, where ρ is the $(e,e'p)$ cross section divided by kinematical factors and the off-shell electron-proton cross section σ_{ep}^{cc1} of de Forest[19] and $\mathbf{p}_m$ is the difference between the momentum of the ejected proton $\mathbf{p}'$ and that of the virtual photon corresponding to undistorted electron waves $\mathbf{q}_0$. In the plane-wave impulse approximation (PWIA) ρ and p_m reduce to the momentum distribution and momentum, respectively, for the bound state of the ejected proton. Figure 1 shows a $1d_{3/2}$ transition in proton knockout from ^{40}Ca, while Fig. 2 shows the $3s_{1/2}$ transition in ^{208}Pb, both in parallel kinematics ($\mathbf{p}'\|\mathbf{q}_0$). The dashed curve in each figure represents a calculation with full distortion of the ejected proton but no electron distortion, i.e., DWIA. The solid curve is the result of the CDWIA calculation. The dotted curves in Figs. 1 and 2 are the result of what I refer to as an "undistorted-wave" (UDWIA) calculation. In it the full relativistic CDWIA formalism presented here is used but the Coulomb distortion is turned off by using partial-wave expansions of plane waves (i.e., Bessel functions) for the electron waves. Thus, the UDWIA (dotted curves) and the CDWIA (solid curves) represent *identical* calculations *except for the numerical representation of the electron wave functions.*

While the UDWIA calculations are formally the same as the CDWIA, they are nonetheless without Coulomb distortions and should therefore ideally agree with the DWIA (dashed curves). The agreement between these calculations constitutes a very rigorous check of the accuracy of the numerical techniques and approximations employed here. As can be seen in Figs. 1 and 2 the agreement is very good. What little difference there is between the dashed and dotted curves shows up primarily at higher missing momenta as would be expected since the UDWIA calculation represents a numerical integral of a Dirac δ function with cutoffs in radius as well as partial-wave expansions. In the calculations presented here the same cutoffs were used for the electron vertex as were used for the proton vertex. If desired, improved agreement can be attained at the expense of running time as can be seen in Fig. 1 for ^{40}Ca where (relatively) higher cutoffs were used.

One sees clearly in the figures the characteristic effect of Coulomb distortions on $(e,e'p)$, a shift in the reduced cross section to higher values of missing momentum. As would be expected, the effect is considerably smaller for calcium than for lead. For comparison, the reduced cross section for the $3s_{1/2}$ transition in ^{208}Pb was also calculated with the nonrelativistic program[9] DWEEPY using the second-order eikonal approximation. The results are presented in Fig. 3. While there are some differences between the present and eikonal calculations as to the amount of the shift as well as the shape of the reduced cross section due to their differing elements, by far the most significant difference between the two methods is in their effect on the magnitude of the reduced cross sec-

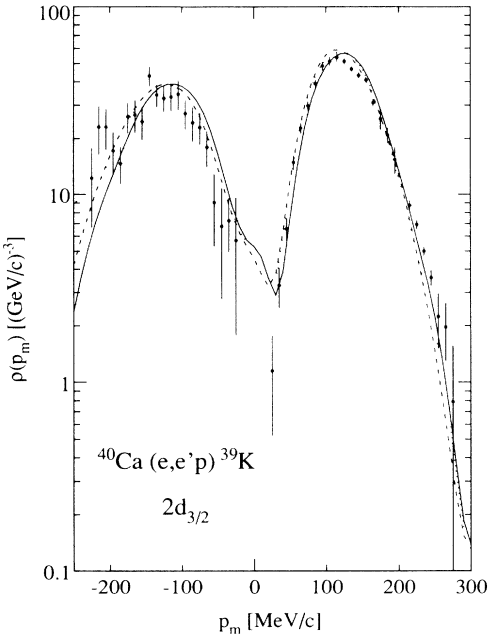

FIG. 1. Calculations using the methods presented here vs data for the knockout of a $1d_{3/2}$ proton from ^{40}Ca. The solid (dashed) curve is with (without) electron distortion. The dotted curve is the UDWIA calculation as defined in the text. All curves have a spectroscopic factor of $S=2.47$.

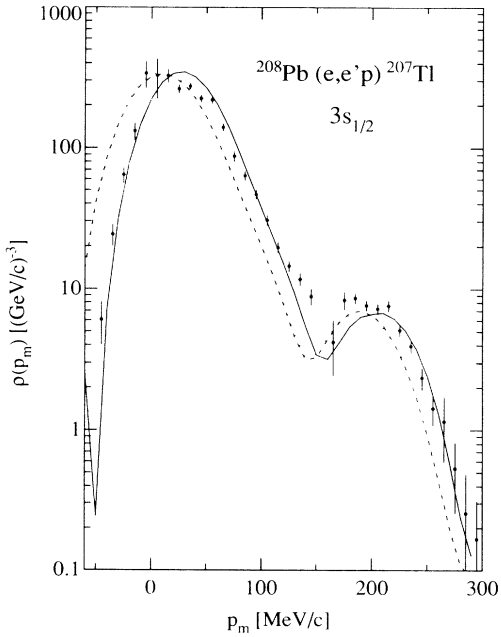

FIG. 2. The same as Fig. 1 except that the data and calculations are for the $3s_{1/2}$ transition in ^{208}Pb. $S=1.30$.

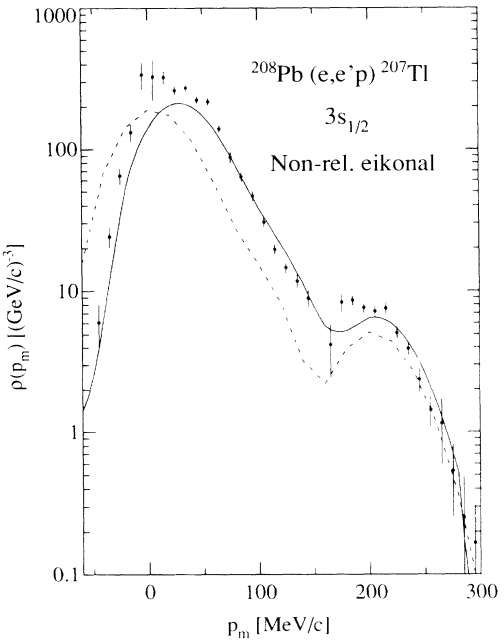

FIG. 3. The same as Fig. 2 except that the calculations were performed using the program and methods discussed in Ref. 9. $S=1.03$.

tion. Primarily due to its "focusing" effect,[9,10] the eikonal method can give a significant change in the magnitude of reduced cross section in addition to the shift. This is usually (but not always, depending on the shell) an increase, as in Fig. 3. In contrast, the present method has, in all calculations performed so far, produced no significant overall shift in the vertical direction. The result is that the two methods can, for high-Z nuclei, yield *significantly different spectroscopic factors* to fit the data.

For the $1d_{3/2}$ transition in ^{40}Ca, for which the effects of electron distortions are small, the eikonal and present calculations produce similar spectroscopic strengths, 2.58 or just under 65% of the sum-rule limit versus 2.47 or 62%, respectively. For the $3s_{1/2}$ transition in ^{208}Pb the values are very different. The eikonal calculations give 1.03, or almost 52% of sum rule, while the present calculations give 1.30, or 65%.

A long-standing problem in the analysis of $(e,e'p)$ data has been that they produced unexpectedly low spectroscopic factors.[1] Theoretical random-phase-approximation calculations[20,21] predict a depletion of valence-shell occupation to particle-hole states above the Fermi energy resulting in occupation numbers of about 85%–90% of full value.[5] This corresponds to spectroscopic factors in the 70%–80% range,[6] well above the experimental values. A solution of this problem has been proposed[18,22] in light of more recent theoretical work.[5–8] In this approach the disparity is to be explained by the effects of short-range hole-hole-particle correlations which would lead not only to still higher occupation above the Fermi energy but also to fragmentation of the shell strength for the bound proton, spreading it over a very broad range of the excitation-energy spectrum. Experimental support for this view is given by the appearance of strength when detection is extended to very high missing energy.[23] The result of the fragmentation would be that some of the hole strength is pushed outside the range of the usual experimental cutoffs. The theoretical calculations predict that a total of around 10% of the hole strength is thereby "lost." The result is values of spectroscopic factors in the 60%–70% range. This is in agreement with the experimental value for ^{40}Ca. In light of the present calculations, it seems that there is also agreement between theoretical calculations and experiment for ^{208}Pb.

The results claimed in this Letter can be experimentally checked by performing the experiment with a higher electron-beam energy where Coulomb distortion effects are smaller. But assuming that these results hold out and pending further experimental and theoretical scrutiny on lighter nuclei, it appears that a consistent and coherent picture for nuclear shell occupation may have emerged. It should be added that since the conclusions drawn here show consistency with theoretical calculations that predict significant effects from short-range correlations, it follows that these results thereby show promise that the upcoming generation of experiments designed to probe two-particle correlations should prove to be quite fruitful.

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