Possible Solution to the Cosmological-Constant Problem

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Classically the unimodular theory of gravity with a constrained determinant $g_{\mu\nu}$ is equivalent to general relativity augmented by an arbitrary cosmological constant Λ which arises as an integration constant. At the quantum level we argue that an integration over Λ should be included in the Euclidean path integral for the vacuum functional. The fully renormalized $\Lambda=0$ overwhelmingly dominates all other contributions yielding a zero observed cosmological constant. While the technical part of our argument is similar to that of the "wormhole" argument, the two approaches are logically different.

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The cosmological constant Λ is a macroscopic parameter that controls the large-scale behavior of the Universe. All cosmic observations to date have shown that the observed cosmological constant is vanishingly small. From the viewpoint of microscopic physics, Λ (in the conventional theory of general relativity) is just like any other coupling parameter. Since anything that contributes to the vacuum energy density acts just like a cosmological constant, modern microscopic theory of particle physics and gravity would lead us to believe that Λ is very large. Indeed, if one believes general relativity up to the Planck scale, one will conclude that the observed cosmological constant is smaller than theoretical expectations by about 120 orders of magnitude. Of course, since the vacuum energy density can assume any value positive or negative, in principle, such a small (possibly zero) observed macroscopic cosmological constant may be due to some miraculous cancellations among the different contributions to the vacuum energy in microscopic physics. But such extreme fine tunings are hard to believe. The physics question is well posed: Why is there nothing rather than something? In this Letter we will attempt to provide a new answer to this question. There may be gaps in our argument, but we believe we have the right physics.

We start not with the usual Einstein-Hilbert theory of classical general relativity but with the unimodular theory of gravity proposed by van der Bij, van Dam, and Ng,[1] and also by others.[2,3] (This theory is actually well motivated from the viewpoint of the little group in the description of massless spin-two particles; see Ref. 1.) In this classical unimodular theory the determinant of the metric is not a dynamical field; its value is fixed: $-\det g_{\mu\nu}\equiv g=1$,[4] hence the name "unimodular theory." The main difference between the Einstein-Hilbert theory and the unimodular theory is that for the former the cosmological constant is a coupling parameter, whereas for the latter it arises as an integration constant[5] unrelated to any coupling parameters in the action.

For the unimodular theory, since the cosmological constant Λ' is an arbitrary integration constant (we have attached a prime to Λ for this discussion for a reason that will be clear below), in the corresponding *quantum* theory one expects[2] the state vector of the Universe to be given by a superposition of states with different values of Λ'. Similarly we believe that one should sum over the different contributions of Λ' to the vacuum functional, weighted by an action which, upon variation, gives the Einstein field equations with that cosmological constant Λ'. The action given in Refs. 1-4 is not suitable for this purpose. But a sensible generally covariant quantum version of the classical theory described in Refs. 1-4 has been found by Henneaux and Teitelboim.[6] For this Letter we will use this version of the unimodular theory. In the absence of matter fields ϕ the action takes the form[6]

$$S=-\frac{1}{16\pi G}\int dx[\sqrt{g}(R+2\Lambda')-2\Lambda'\partial_\mu\mathcal{T}^\mu]\,, \qquad (1)$$

which, upon variation with respect to $g_{\mu\nu}$, Λ', and the new vector field $\mathcal{T}^\mu$, yields Einstein's equations (with cosmological constant Λ' appearing as an integration constant) and the constraint $\sqrt{g}=\partial_\mu\mathcal{T}^\mu$, a generalized version of the unimodular condition. It is found[6] that the only part of $\mathcal{T}^\mu$ that is not pure gauge is the zero-mode cosmic time conjugate to the constant of motion $2\Lambda'$. To distinguish this type of theory from the conventional theory we will (still) refer to it as the unimodular theory.

Armed with the new action we propose that the Euclidean version of the vacuum functional for the unimodular theory is given in terms of path integrations over ϕ, $g_{\mu\nu}$, Λ', and $\mathcal{T}^\mu$,

$$Z=\int d\mu''(\Lambda')\int d[g_{\mu\nu}]d[\phi]\int d[\mathcal{T}^\mu]\exp\left(-\frac{1}{16\pi G}\int dx[\sqrt{g}(R+2\Lambda')-2\Lambda'\partial_\nu\mathcal{T}^\nu+\cdots]\right), \qquad (2)$$

where the action is understood to include the matter part $S_M(\phi,g_{\mu\nu})$. The integration over $\mathcal{T}^\mu$ changes the measure of the Λ' integration: $d\mu''(\Lambda')\rightarrow d\mu'(\Lambda')$ such that now Λ' is independent of spacetime and takes on only numerical values[7] (as expected on physical grounds since it is known[6] that the theory has only one additional overall degree of freedom). Next, following Ref. 8 we integrate over the metric and the matter fields to give

$$\int d[g_{\mu\nu}]d[\phi]\exp\left[-\frac{1}{16\pi G}\int dx\sqrt{g}(R+2\Lambda')+\cdots\right]=\exp[-S_{\Lambda'}(\bar{g}_{\mu\nu},\bar{\phi})]\,, \tag{3}$$

where $\bar{g}_{\mu\nu}$ and $\bar{\phi}$ are the background fields that minimize the effective action $S_{\Lambda'}$. A curvature expansion for $S_{\Lambda'}$ yields

$$S_{\Lambda'}(g_{\mu\nu},\phi)=\frac{1}{16\pi G}\int dx\sqrt{g}(R+2\Lambda)+\cdots\,, \tag{4}$$

with $\Lambda(\Lambda')$ the fully renormalized cosmological constant. Changing the integration variable from Λ' to Λ so that

$$d\mu'(\Lambda')=d\mu(\Lambda)\,,\quad S_{\Lambda'(\Lambda)}(\bar{g}_{\mu\nu},\bar{\phi})=S_\Lambda(\bar{g}_{\mu\nu},\bar{\phi}) \tag{5}$$

we have

$$Z=\int d\mu(\Lambda)\exp[-S_\Lambda(\bar{g}_{\mu\nu},\bar{\phi})]\,. \tag{6}$$

A priori we do not know the measure for the Λ integration; but we expect that generically, $d\mu(\Lambda)$ is smooth and nonvanishing at $\Lambda=0$. [Actually the exact form of $d\mu(\Lambda)$ is not crucial to our argument to follow due to the essential singularity of S_Λ at $\Lambda=0$ to be given below.] Neglecting the effects of $\bar{\phi}$ we can follow the arguments of Baum[9] and Hawking[9] to evaluate the vacuum functional. For negative Λ, $S_\Lambda(\bar{g}_{\mu\nu},0)$ is positive; for positive Λ, the four-sphere of radius $(3/\Lambda)^{1/2}$ is the metric that minimizes $S_\Lambda(\bar{g}_{\mu\nu},0)$ yielding

$$S_\Lambda(\bar{g}_{\mu\nu},0)=-3\pi/\Lambda G \tag{7}$$

so that

$$Z=\int d\mu(\Lambda)\exp(3\pi/\Lambda G)\,. \tag{8}$$

The essential singularity of the integrand at $\Lambda=0+$ implies that $\Lambda=0+$ overwhelmingly dominates all other contributions in the integration over Λ. (The alert readers must have noticed that the technical part of our argument is similar to that of the "wormhole" argument.[10] But the two approaches are logically very different.) Therefore if this scenario of the unimodular theory of gravity is correct,[11] the observed cosmological constant is zero. This is the main result of this Letter.

Note that the integrand in Eq. (8) has an essential singularity also at $G=0$. But, unlike Λ, the Newton's constant G is a coupling parameter in the action. Accordingly, there is no integration over G and the essential singularity at $G=0$ is physically irrelevant.

On the other hand, in the unimodular theory, the classical constraint on $\det g_{\mu\nu}$, which frees the cosmological constant, makes it necessary to include an integration over Λ for the vacuum functional. While classically there is no preferred value for Λ, at the quantum level the overwhelmingly preferred value is $\Lambda=0$.

It is curious that the explanation for the vanishing of a macroscopic parameter (that controls the cosmic evolution) lies in quantum mechanics (which describes microscopic physics). It is perhaps even more surprising that what looks classically like a slight change of emphasis can have such a drastic consequence at the quantum level.

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[4] See also A. Einstein, Ber. Preuss. Akad. Wissl **142** (1917) [English translation: *The Principle of Relativity* (Methuer, 1923, reprinted by Dover, New York, 1952), p. 177]. Unknown to van der Bij, van Dam, and Ng, Einstein had proposed a set of field equations similar to that found in Ref. 1. However, Einstein took for the stress tensor only the traceless tensor of radiation alone and not of the full energy-momentum tensor.

[5] For the classical unimodular theory with constraint $-\det g_{\mu\nu}=1$, the gravitational field equations are $R^{\mu\nu}-\frac{1}{4}G^{\mu\nu}R=-8\pi G(T^{\mu\nu}-\frac{1}{4}g^{\mu\nu}T^\lambda{}_\lambda)$ which, with the aid of the Bianchi identities $D_\mu(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R)=0$ and the conservation law $D_\mu T^{\mu\nu}=0$, imply that $R-8\pi GT^\lambda{}_\lambda$ is a constant. Denoting that (integration) constant by $-4\Lambda'$ we can rewrite the field equations in the form of the Einstein equations with cosmological constant Λ'. To the best of our knowledge, that Λ' arises as an integration constant in the unimodular theory was first explicitly shown in Ref. 1.

[6] M. Henneaux and C. Teitelboim, Phys. Lett. B **222**, 195 (1989). See also D. B. Brown and J. W. York, Jr., Phys. Rev. D **40**, 3312 (1989); R. D. Sorkin, in *Proceedings of the Conference on the History of Modern Gauge Theories*, edited by M. Dresden and A. Rosenblum (Plenum, New York, 1989).

[7] In the Minkowskian version, the integration over $\mathcal{T}^\mu$ gives $\delta(\partial_\mu\Lambda')$ (aside from a multiplicative factor of infinity coming from overcounting gauge copies of the $\mathcal{T}^\mu$ field). We can do this integration before the Wick rotation to the Euclidean formulation.

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[10]For the wormhole approach to the cosmological constant problem, see, e.g., S. Coleman, Nucl. Phys. **B310**, 643 (1988); T. Banks, *ibid.* **B309**, 493 (1988); S. B. Giddings and A. Strominger, *ibid.* **B321**, 481 (1989); S. Adler, Phys. Rev. Lett. **62**, 373 (1989); see also I. Klebanov, L. Susskind, and T. Banks, Nucl. Phys. **B317**, 665 (1989).

[11]One may, e.g., question the validity of the Euclidean formulation for theories of gravity. Also the functional integration over $\mathcal{T}^\mu$ in Eq. (2) merits a closer scrutiny.