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Test of the Inverse-Square Law of Gravitation Using the 300-m Tower at Erie, Colorado

C. C. Speake,[a] T. M. Niebauer,[b] M. P. McHugh, P. T. Keyser, and J. E. Faller[c]
Joint Institute for Laboratory Astrophysics, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440

J. Y. Cruz and J. C. Harrison
Geodynamics Corporation, 5520 Ekwill Street, Suite A, Santa Barbara, California 93111

Jaako Mäkinen
Finnish Geodetic Institute, Ilmalankatu 1A, SF-00240 Helsinki, Finland

R. B. Beruff[d]
Defense Mapping Agency, F. E. Warren Air Force Base, Wyoming 82005
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Gravity was measured at eight different heights on a 300-m meteorological tower using LaCoste and Romberg gravimeters. The observed values were adjusted for tides, drift, and gravimeter screw errors, and tested for systematic effects due to tower motion. These results are compared with values predicted using Newton's inverse-square law from surface gravity. The differences exhibit no systematic trends and their rms value is only 10×10^{-8} m s^{-2}, well within the estimated errors of the experiment. This result places new constraints on the possible strength and range of any non-Newtonian force.

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Recent midrange tests of the inverse-square law have been prompted by the reported results of Stacey and co-workers[1] and Fischbach *et al.*[2] The first group measured the change in gravity down a mine shaft and found a value for G which differed from the accepted laboratory value by $\sim1\%$; the second group reanalyzed the results of the test of the weak equivalence principle of Eötvös, Pekar, and Fekete[3] and proposed a new midrange composition-dependent "fifth force." Although the hypothesized fifth force is composition dependent, for experiments sensitive to its range and strength only, we may write the potential energy of two point test masses due to the new force as

$$V_5 \equiv -GM_1M_2\alpha e^{-r/\lambda}/r\,, \qquad (1)$$

where λ is the range and α is the strength parameter of the interaction. The original results of Stacey and co-workers suggested $\alpha \simeq -0.007\pm0.004$ and $\lambda\sim200$ m.

Ander *et al.*[4] performed a measurement using a borehole in the Greenland ice cap. However, the advantages of the homogeneity of the ice were somewhat outweighted by the uncertainty in the density of the underlying bedrock. The results of this experiment, which at first appeared to give evidence of non-Newtonian gravity, now seem to be inconclusive.

Eckhardt *et al.*[5] avoided the problems associated with downward continuation by measuring gravity up a tower. In this scheme, data from a comprehensive gravity survey and knowledge of the topography around the tower are used to predict gravity at each level. They initially reported an anomaly which could have been attributed to a Yukawa term with $\alpha=+0.02$ and with a range $\lambda=300$ m, but also noted that their result is consistent with that of Stacey and co-workers if a model with two Yukawa terms is assumed.

Many[6] have questioned the results of Eckhardt *et al.* including Thomas *et al.*[7] who, in an independent tower experiment, found no evidence for non-Newtonian gravity. More recently, Eckhardt *et al.* have revised their analysis[8] and now their results appear consistent with Newtonian gravity. We decided that an independent experiment would help clarify the situation, and undertook to perform a tower test of gravity. The Erie tower, although shorter than the WTVD[5] and BREN[7] towers, is located near our laboratory, has comprehensive meteorological monitoring, is stable, and stands on nearly flat (out to 20 km) and easily surveyed terrain.

Because of concerns about possible systematics due to tower vibration, an investigation of the stability of the tower was undertaken first. Measurements of the motion of the tower at wind speeds of 2.4 $m\,s^{-1}$, and calculations, led us to assign a systematic uncertainty of 5×10^{-8} $m\,s^{-2}$ for the 295-m gravity value. All of the measurements were taken with wind speeds less than 5 $m\,s^{-1}$ during the hours between 10 p.m. and 6 a.m. Two LaCoste-Romberg G-type gravimeters (G115 and G139) with electrostatic feedback were employed. These meters were calibrated using seven gravity stations in Colorado which include the range of gravity values on the tower. The uncertainty in the calibration factors led to an uncertainty of 9×10^{-8} $m\,s^{-2}$ over the range of the tower. The precise heights of each tower (given in Table I) were determined using an electronic distance-measurement device (EDM) which we calibrated ourselves. The uncertainty is ±5 mm (equivalent to 1.5×10^{-8} $m\,s^{-2}$).

The raw gravity data were adjusted for Earth tides, and values of gravity for each tower level sampled within a measurement loop were calculated using a least-squares fit with a linear drift model. Corrections were made for the cyclical screw errors, temperature, and drift; together these amounted at most to 20×10^{-8} $m\,s^{-2}$.

The next step is to predict Newtonian gravity values at the tower platform heights using surface measurements.[9] Gravity measurements, digital elevations, and the height of the sea-level surface (geoid) above the ellipsoidal reference surface are used in the modeling. The gravity data were obtained from the Defense Mapping Agency (DMA) gravity library but we supplemented these with a local survey of 265 stations within 8 km of the tower (191 within 800 m and 70 within 60 m). We also surveyed the positions and elevations of our stations within the 800-m radius. In total we used about 26000 stations in a $4°\times5°$ area, 2640 in a $1°\times1°$ area, and 402 in a $10'\times11'$ area centered on the tower. The digital elevations were generally DMA's Digital Terrain Elevation Data (DTED), but within 2.5 km of the tower these were replaced with $3''\times3''$ mean elevations read from the United States Geological Survey $7.5'\times7.5'$ quadrangle sheets contoured at 10-ft intervals. The geoid heights were generated from satellite-derived geopotential models and comparisons between leveled heights and those determined by satellite positioning.

In the computation of gravity values at the tower platforms, the Earth's field is treated as the sum of a global model and a residual. The global models—consisting of a satellite-derived spherical-harmonic potential function of degree and order eight, centrifugal acceleration, and five sets of point masses—were evaluated directly at the platforms (with a correction for the Earth's atmosphere) while the residual field is continued up to their levels by means of the Poisson integral.[10] The granularities of the finest mass sets (1' in model 1, 30" in model 2) were chosen after consideration of the behavior of the Poisson kernel, so that the residual field could be neglected beyond a radius of 20 km from the tower.

The observed gravity values are reduced to Bouguer anomalies by (1) subtraction of the value given by an ellipsoidal model on its surface at the latitude of the station, (2) a correction equal to the station elevation times the free air gradient of 0.3086×10^{-5} s^{-2}, and (3) subtraction of the attraction of the mass of the topography above sea level calculated at constant density. The latter is computed as the attraction of an infinite slab of thickness equal to the station elevation, minus the terrain correction which accounts for departures of the actual topography from a level surface. The terrain correction is computed on a regular grid using fast Fourier transforms[11] and interpolated to the station positions. The value of each gravity station is then checked by comparing the Bouguer anomaly with a value predicted from its neighbors using least-squares collocation,[12] and by comparing its elevation with a value interpolated from DTED. After elimination of suspect data, Bouguer anomalies are predicted at the centers of $2.5'\times2.5'$ elements, and these are averaged to $5'\times5'$ mean values.

TABLE I. Comparison between measurements and Newtonian predictions.

Observation height (m)	Measured Δg (10^{-5} $m\,s^{-2}$)	Predicted Δg Model 1 (10^{-5} $m\,s^{-2}$)	Predicted Δg Model 2 (10^{-5} $m\,s^{-2}$)	Measured − predicted average (10^{-5} $m\,s^{-2}$)
8.198	-2.556 ± 0.009	-2.550 ± 0.010	-2.553 ± 0.010	-0.005 ± 0.013
21.912	-6.789 ± 0.010	-6.782 ± 0.010	-6.783 ± 0.010	-0.007 ± 0.014
48.568	-14.986 ± 0.010	-14.994 ± 0.011	-14.992 ± 0.011	0.007 ± 0.015
97.323	-30.000 ± 0.010	-29.990 ± 0.012	-29.983 ± 0.012	-0.014 ± 0.016
149.136	-45.908 ± 0.011	-45.913 ± 0.015	-45.904 ± 0.015	0.001 ± 0.019
197.896	-60.892 ± 0.012	-60.888 ± 0.018	-60.876 ± 0.018	-0.010 ± 0.022
249.718	-76.800 ± 0.013	-76.800 ± 0.021	-76.785 ± 0.021	-0.008 ± 0.025
295.438	-90.816 ± 0.014	-90.835 ± 0.023	-90.817 ± 0.023	0.010 ± 0.027

The mean topographic attractions computed from the mean elevations and the mean terrain corrections are then added back to produce mean free air anomalies. Reduction to Bouguer anomalies improves the accuracy of the gravity field interpolation because most of the gravity variation over typical interstation distances in uneven terrain is directly due to the topography. These $5' \times 5'$ mean free anomalies are averaged further into $15' \times 15'$, $1° \times 1°$, and $5° \times 5°$ blocks. These are used to generate the point mass sets. In addition, mean Bouguer anomalies were computed for $30'' \times 30''$ and $3'' \times 3''$ elements within 20 and 2.5 km of the tower, respectively, and these were converted back to gravity values using the mean elevations and topographic attractions.

The five mass sets of the global model are located, at depths equal to their mean horizontal spacing, directly beneath the centers of areas bounded by lines of constant latitude and longitude spaced at 5°, 1°, 15′, 5′, and either 1′ (model 1) or 30″ (model 2), respectively. All sets are centered on the tower and they are of $60° \times 60°$, $26° \times 26°$, $7.5° \times 9°$, $4° \times 5°$, and $25' \times 40'$ extent, respectively.

In flat areas the actual vertical gradient of gravity is close to that of the ellipsoidal model, so the gradient of the free air anomaly is small and the mean anomalies on the geoid can be taken to be the same as those on the topographic surface. In rough terrain the vertical gradient departs significantly from the ellipsoidal model and this departure is correlated with elevation in a manner that causes mean free air anomalies on the geoid to differ systematically from those on the topographic surface. These systematic differences are removed by application of a reduction correction.[13] Representative geoidal gravity values for the 5°, 1°, 15′, and 5′ elements are then computed by adding back the attraction of the ellipsoidal model.

Mean attractions computed from the spherical-harmonic potential function and centrifugal acceleration are then subtracted from the mean gravity values representing the 5° elements, and the masses in the 5° set adjusted to make the attraction of this set, averaged over the 5° elements, match these differences. The average attractions of the spherical-harmonic model, centrifugal force, and the 5° mass set are then evaluated at the 1° elements, subtracted from the representative 1° values, and so on down to the finest granularity. For the first four sets the matching is done at the geoid; the fifth set may lie above the geoid and the matching is done at the topographic surface. Finally, the attraction derived from the spherical-harmonic potential function, centrifugal acceleration, and all five mass sets is evaluated at the tower platforms and at the centers of the 30″ and 3″ elements near the tower (within 20 km). These latter attractions are subtracted from the estimated representative gravity values to determine the residual that is to be continued upward by Poisson integration. The gravity residuals are continued analytically from the topographic surface into a plane through the base of the tower using Fourier-series techniques[14] prior to the integration. For proper discretization of the Poisson integral the 6×8 array of 3″ values around the tower were splined to $0.3'' \times 0.3''$ resolution: The splined values at the inner 20×30 points of the 0.3″ grid were improved by use of the dense local data.

Our estimates of gravity values on the surface, made by interpolating Bouguer anomalies and then removing mean Bouguer reductions computed from the elevation data, will produce errors in these estimates if there are inconsistencies between the station elevations and the digital databases. Normally these errors are small compared with those that would occur if gravity were interpolated directly from the station values. Near the tower, however, they are significant and were corrected; within 800 m we had a sufficient density of surveyed stations so that we could determine the inconsistencies and correct the gravity estimates in the individual 3″ blocks. There are too few stations for us to estimate corrections between 800 m and 2.5 km. The estimated mean height inconsistency there corresponds to $(-23 \pm 28) \times 10^{-8}$ $\mathrm{m\,s^{-2}}$ in surface gravity; a constant value of -23×10^{-8} $\mathrm{m\,s^{-2}}$ would propagate to -5×10^{-8} $\mathrm{m\,s^{-2}}$ over the height of the tower. No correction was made but the variance and correlation length of the height inconsistencies were estimated and their influence propagated statistically[15] through the Poisson integral to estimate their contribution to the error of the prediction ($\pm 6.5 \times 10^{-8}$ $\mathrm{m\,s^{-2}}$ at the top). The mean inconsistency between the elevations at 361 gravity stations and the DTED between 2.5 and 25 km from the tower is 1.77 ± 0.57 m, corresponding to $(163 \pm 53) \times 10^{-8}$ $\mathrm{m\,s^{-2}}$ in our estimate of the difference to be propagated in the Poisson integration. This difference is statistically significant and a correction was made, amounting to 15×10^{-8} $\mathrm{m\,s^{-2}}$ at the top of the tower. Also, statistical propagation of the height uncertainties in this area contributed $\pm 16 \times 10^{-8}$ $\mathrm{m\,s^{-2}}$ to our estimate of the prediction error at the top of the tower.

It was not possible to make a realistic statistical estimate of the errors in interpolating the Bouguer anomalies near the tower but the anomalies vary smoothly and the deviations of the estimated values from a quadratic trend were taken as an estimate of the error and propagated to the platforms; this was done separately for the 3″ and 30″ areas. In addition, we estimated that there could be a 10^{-7} $\mathrm{m\,s^{-2}}$ inconsistency between the modeled field and the base station at the base of the tower. The final error estimates are the root square sum of all the individual errors. No account was taken of errors in the global model outside the 30″ area as their contribution to the gradients up the tower is very small.

Predicted gravity intervals for both models are given in Table I. Agreement of the measured values with the Newtonian predictions is clearly excellent and the validity of the inverse-square law under the conditions of the

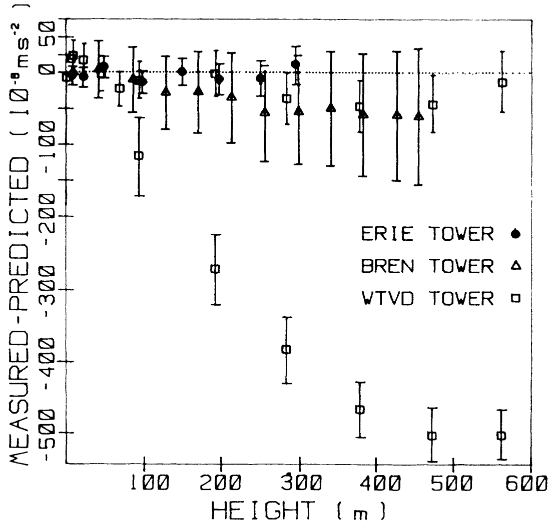

FIG. 1. The dotted line corresponds to the Newtonian prediction. The BREN-tower result is from data in Ref. 7. The WTVD-tower points shown are from the initial report (Ref. 5) that motivated this work, and from their latest work (Ref. 8).

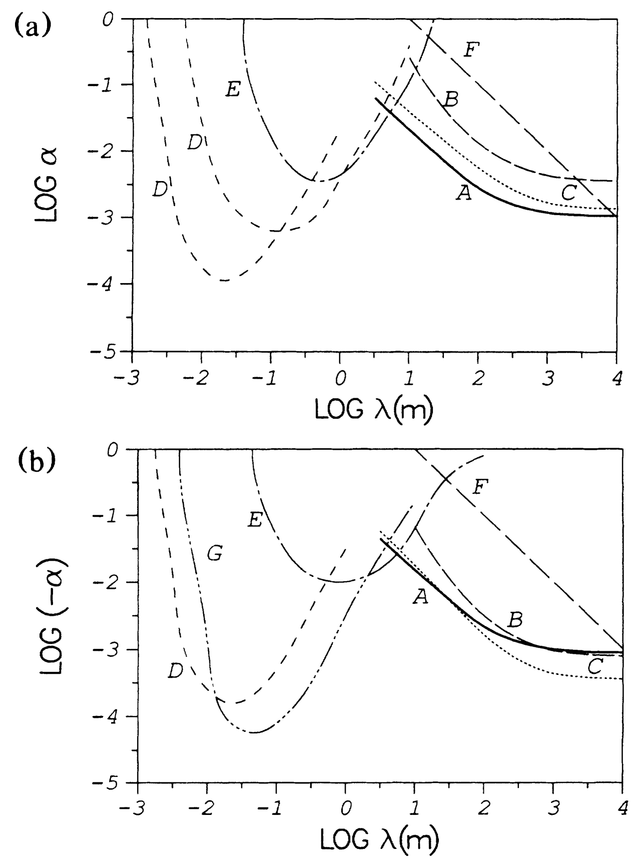

FIG. 2. Excluded strengths (α) and ranges (λ) for a single Yukawa model are in the area above the curves at the 1σ level. (a) The $\log_{10}(\alpha)$ graph is for an attractive force; (b) the $\log_{10}(-\alpha)$ graph is for a repulsive force. Solid curves A represent this work; curves B are from data in Ref. 7 (note that Thomas *et al.* use the opposite-sign convention for α in their paper); curves C are from data in Ref. 8; curves D–G are from Refs. 16–19, respectively. The tower-experiment curves are calculated by fitting the Yukawa potential with the residuals plus or minus the 1σ uncertainties.

experiment is confirmed. The residuals of the present experiment are shown together with those of other tower experiments in Fig. 1.

A Yukawa potential of the type in Eq. (1) would predict a difference in acceleration between a point at height z and a point on the ground equal to $2\pi G\rho\alpha\lambda(e^{-z/\lambda}-1)$, for $\lambda \ll$ (radius of the Earth), where ρ is the density of the Earth's surface. Figure 2 illustrates the constraints placed by this experiment on such an interaction. Also shown are the constraints placed by other recent experiments.

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[a]Visiting from Bureau International des Poids et Mesures, Pavillion de Breteuil, 92312 Sèvres, CEDEX, France; now at School of Physics and Space Science, University of Birmingham, Birmingham B15 2TT, United Kingdom.

[b]Present address: Max Planck Institut für Quanten Optik, Ludwig-Prandtl Strasse 10, D-8046 Garching-bei-München, Federal Republic of Germany.

[c]Quantum Physics Division, National Institute of Standards and Technology.

[d]Retired.

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