Dynamics of the Interactions of Rotons with Quantized Vortices in Helium II

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We consider the theory of mutual friction in helium II which accounts for the coupling of the normal and superfluid flows by means of quantized vortices. Numerical simulations of roton-vortex interactions suggest a simple new expression which accounts for most mutual friction at low temperatures. By carefully separating that which can be known at the present time from that which cannot be known until the structure of quantized vortices is understood, we are able to account for mutual friction in a semiempirical way at all temperatures and pressures.

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The two-fluid theory of helium II envisions a mixture of a normal, viscous fluid and a superfluid which can move completely independently at sufficiently low velocities. At higher velocities, i.e., Reynolds numbers $\gtrsim 10$, the flow of the two fluids becomes locked together by a mechanism involving quantized vortices called mutual friction. The basis for understanding mutual friction was identified by Hall and Vinen in the 1950s as collisions between rotons and quantized vortices and characterized by macroscopic mutual friction parameters B and B' which enter the two-fluid equations of motion. Mutual friction is important to the theory of elementary excitations in superfluid helium, to superfluid turbulence, and to heat transfer in superconducting devices cooled by helium II. Hall and Vinen defined microscopic parameters D and D' describing the interaction of rotons with quantized vortices which need to be understood from theory. The connection between B and B' and D and D' is intricate and subject to significant sources of error.[1,2] There have been many attempts to calculate B and B' from various theories. None have succeeded in a convincing way. Using numerical simulations as a guide, we have developed an analytical expression (8) for the mutual friction caused by the long-range interactions of a roton with the velocity field of a vortex. Together with the scattering of rotons by the vortex core, described by Hillel and Vinen,[3] this expression allows us to calculate values for B and B' that are in substantial agreement with experiment for $T < 1.8$ K, with no adjustable parameters. Discussions and reviews of previous work are contained in Refs. 1 and 3–5.

The first objective of our investigation was to simulate roton-line collisions to gain insight into the interaction. We consider rotons to be point excitations obeying the dispersion curve for elementary excitations in helium II shown in Fig. 1. Rotons with $p > p_0$ have their group velocity $u = d\epsilon/dp$ parallel to their momentum and are called R^+ rotons; those for $p < p_0$ have their group velocity antiparallel to their momentum and are called R^- rotons.

The vortex line is considered to be rigid and aligned along the z axis with a circulation $\boldsymbol{\kappa} = (h/m)\hat{\mathbf{z}}$, where m is the mass of the helium atom. The velocity field of the vortex is azimuthal in the x-y plane and is given by $\mathbf{v}_s \geq (\kappa/2\pi r)\hat{\boldsymbol{\phi}}$, where the unit vector $\hat{\boldsymbol{\phi}}$ is in the azimuthal direction in the x-y plane and r is the radial distance from the core. When the roton enters into the velocity field of the vortex the $\mathbf{p}\cdot\mathbf{v}_s$ interaction modifies the energy from $\epsilon(p)$ to $\epsilon(p)+\mathbf{p}\cdot\mathbf{v}_s$ as discussed in detail by Rayfield and Reif.[6] Trajectories are obtained by integrating Hamilton's equations of motion as described by Roberts and Donnelly.[7]

We illustrate in Fig. 2(a) the trajectories of a single R^+ roton of momentum 2.1×10^{-19} g cm/sec incident upon a vortex line. This relatively slow roton (~ 72 m/sec) experiences minor deflections for positive impact parameters, but truly major deflections for negative impact parameters. These rotons make a transition from one side on the roton minimum to the other with no change in energy. These dramatic reversals of group velocity do not necessarily result in a large momentum exchange because the momentum vector may be deflected only a small amount. This type of "species change" may also be found in an early paper by Goodman (see espe-

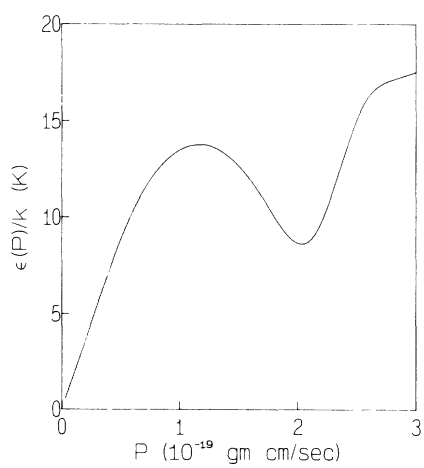

FIG. 1. Dispersion curve for elementary excitations in helium II. The minimum is described by an energy Δ and a momentum p_0. Rotons are excitations near this minimum.

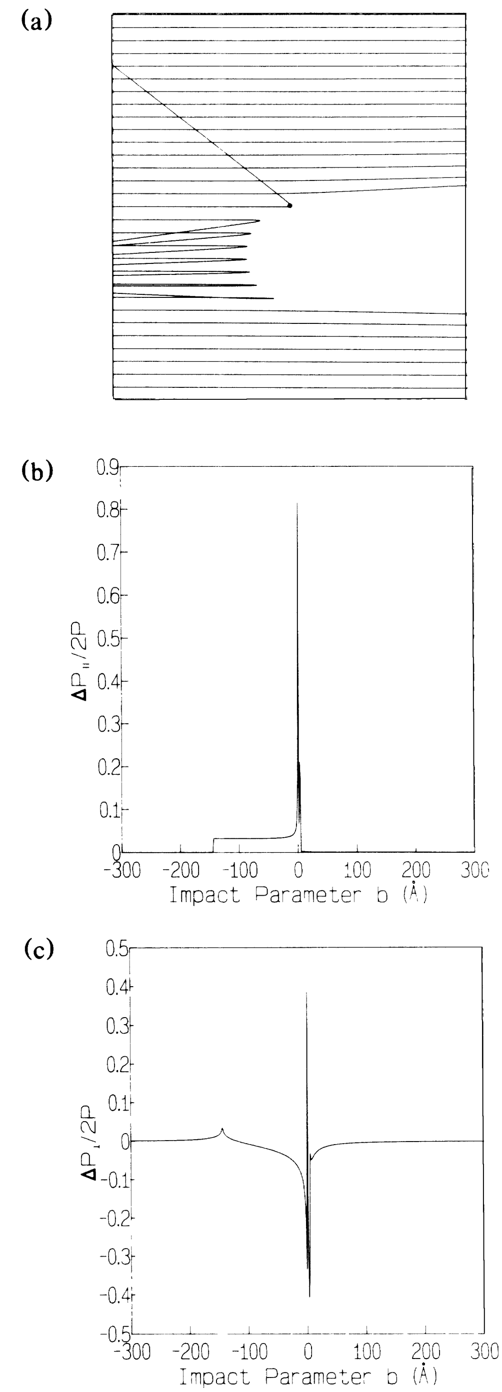

FIG. 2. (a) Trajectories of rotons incident upon a vortex line (indicated by a dot) with clockwise circulation. The momentum of the rotons is $p = 2.1 \times 10^{-19}$ gcm/sec and the impact parameter b is stepped in intervals of 20 Å. The almost total reversals of direction for negative impact parameters corresponds to the species change $R^+ \rightarrow R^-$. (b) Parallel momentum exchange for rotons with orbits such as in (a). This function can be approximated by the rectangular function in Eq. (4), ignoring close collisions. (c) Transverse momentum exchange from (a). Note that the total transverse momentum exchange, the integral under this curve, very nearly vanishes.

cially his Fig. 1).[8]

The corresponding momentum exchange parallel to the initial trajectory is shown in Fig. 2(b). One can see that while the maximum momentum exchange occurs for close collisions, features such as the reversal of the group velocity extend so far from the vortex line (~150 Å in this example) that they add up to a substantial net momentum transfer to the line.

Using the coordinate system described above and assuming a macroscopic drift of rotons with mean velocity in the $+x$ direction, we can show by methods developed by Rayfield and Rief[6] that

$$D = -2\pi \int_0^\infty dp\, p^4 \frac{df}{d\epsilon} u \int_0^\pi d\theta (\sin^3\theta)\sigma , \tag{1}$$

where the distribution function for the rotons at rest is

$$f(p) = [h^3(e^{\beta\epsilon} - 1)]^{-1} , \tag{2}$$

σ is the cross section,

$$\sigma = \int_0^\infty db\, \Delta p/2p , \tag{3}$$

θ is the polar angle, and Δp is the momentum transfer parallel to the original trajectory.

The simulation results of Fig. 2(b) suggest that the integrand of σ is very simple:

$$\Delta p = \begin{cases} 0, & b < b_{\text{crit}}, \\ |p - p'|, & b_{\text{crit}} < b < 0, \\ 0, & b > 0, \end{cases} \tag{4}$$

where b is the impact parameter of the roton, b_{crit} is the critical impact parameter dividing scattered from unscattered rotons, and p' is the momentum of a roton on the opposite side of the roton minimum with the same energy as the original roton. The critical impact parameter is simply the distance from the vortex where the $\mathbf{p} \cdot \mathbf{v}_s$ potential reduces the roton's kinetic energy to Δ. The roton's group velocity then reverses direction and it backs out of the vortex potential. Therefore,

$$\sigma \approx \frac{p - p'}{2p} b_{\text{crit}} = \frac{|p - p'| \kappa \sin\theta}{4\pi(\epsilon - \Delta)} \tag{5}$$

and

$$D_1 = -2\pi \int_0^\infty dp\, p^4 \frac{df}{d\epsilon} u \frac{3}{32} \frac{|p - p'| \kappa}{\epsilon - \Delta} , \tag{6}$$

where we define D_1 as the part of D that is due to scattering from the velocity field of the vortex.

To find D_1, we can perform the integration in (6) numerically, or (at low temperatures) use the Landau parabolic approximation

$$\epsilon \cong \Delta + (p - p_0)^2/2\mu , \tag{7}$$

where μ is the roton effective mass, in which case

$$D_1 \cong \frac{3\pi}{4} \kappa \left(\frac{2\pi\mu}{kT} \right)^{1/2} \frac{p_0^4 e^{-\Delta/kT}}{h^3} . \tag{8}$$

Equation (8) is a new result in the theory of mutual friction.

For close collisions (<10 Å) we cannot expect reasonable physical results from the simulation for at least three reasons. First, the vortex may deform in a close collision and send waves along the core. Second, there are localized roton states near the line which are not taken into account in a single excitation model.[9] Finally, the (still unknown) quantum-mechanical structure of the vortex lines and the roton must be involved in these collisions.

Hillel and Vinen[3] have produced an ingenious account of the consequences of close collisions. They assume roton interactions between the incoming roton and the localized rotons near the core will result in the absorption of the roton. Taking into account this absorption and subsequent (nonisotropic) reemission of rotons, they find the corresponding microscopic friction parameters

$$D_2 = 2\rho_n v_G \sigma_{\text{core}}, \tag{9}$$

$$D' = \rho_n \kappa / 2, \tag{10}$$

where $v_G = \sqrt{2kT/\pi\mu}$ is the average group velocity of rotons and σ_{core} is the core cross section which we take to be the core diameter. Taking $D = D_1 + D_2$ from (8) and (9), D' from (10), and using the formulas of Barenghi, Donnelly, and Vinen[1] gives a reasonable account of B and B' at low to moderate temperatures (i.e., $T < 1.8$ K).

We can test our theory below 1 K by comparing with the drag on vortex rings measured by Rayfield and Reif.[6] They determine a drag coefficient α which, after allowance for drag by phonons and ^{3}He, is interpreted in terms of a roton-line cross section:

$$\alpha = \frac{3\pi^2}{8} \frac{\kappa}{h^3} p_0^4 e^{-\beta\Delta} \bar{\sigma}_0 \tag{11}$$

and quote the surprisingly large result $\bar{\sigma}_0 = 9.5 \pm 0.7$ Å. Vortex-ring drag formulas of Sec. 3 of Ref. 1 can be shown to yield $D \cong 2\alpha/\kappa$. Interpreting our result (8) in the same way as (11) gives the contribution to the cross section from distance collisions

$$\bar{\sigma}_0 = \kappa \sqrt{2\mu / kT\pi}, \tag{12}$$

which yields $\bar{\sigma}_0 = 8.5$ Å at $T = 0.67$ K. The contribution from the core in D_2 is about 2 Å and raises $\bar{\sigma}_0$ to ~ 10.5 Å in substantial agreement with experiment. This result clears up a long-standing puzzle.

The observed quantities B and B' have been reported only above 1.3 K.[1] They depend on the frequency of second sound used in their observation in a complicated way and the means to get accurate values are tedious.[2] It would be better to have the microscopic parameters D and D' available at all temperatures and pressures. Unfortunately, our single excitation model cannot be extended much above 1.8 K.

The critical-region investigations of Pitaevskii[10] and Onuki[11] show that the limit of D as $T \rightarrow T_\lambda$ is zero, the corresponding limit for D' is $\rho_n \kappa$ (see Hillel[5] for a discussion). With this information we can construct a semiempirical model for D and D' which is superior to (6) and (10) at temperatures greater than 1.8 K, but which adds terms motivated solely by forcing D and D' to approach their critical values. Thus,

$$D = (D_1 + D_2)(1 - e^{-90t^2}), \tag{13}$$

$$D' = \kappa \rho_n [(1-t) + 20t^{4/3} e^{-200t^3}], \tag{14}$$

where $t = (T_\lambda - T)/T_\lambda$. At temperatures high enough to require (13) and (14), the parabolic approximation (7) will not work, and we must find the value of D_1 in (6) numerically using the observed roton dispersion curve.[12] Fortunately, our numerical investigations show that D and D' are surprisingly independent of the temperature dependence of the dispersion curve and it appears sufficiently accurate to use the low-temperature values of Δ, p_0, and μ of Ref. 13 and the dispersion curve of Ref. 12. The results of this procedure are shown in Fig. 3.

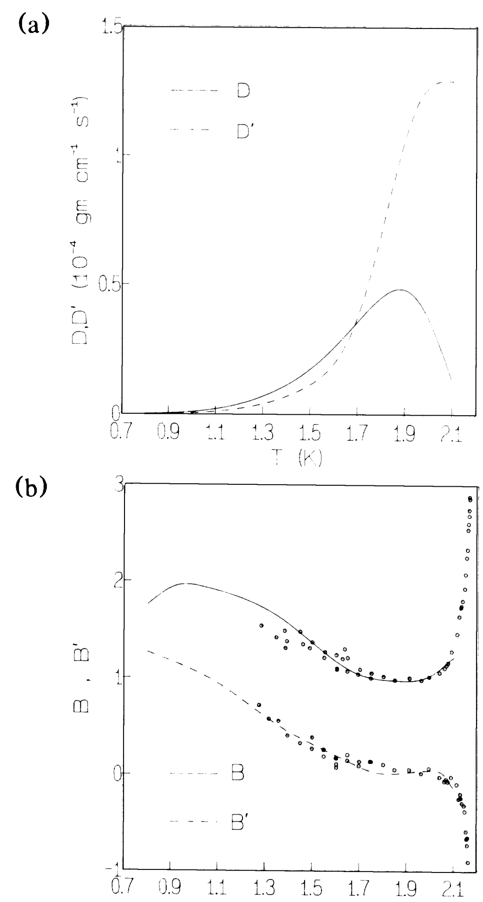

FIG. 3. (a) Values of D and D' from the semiempirical model. (b) Values of B and B' deduced from D and D' compared to experimental values taken from Ref. 1.

Data available today limit the calculation of B and B' at higher pressures. No reliable tabulation of viscosity as a function of temperature and pressure exists. If, however, we use the roton viscosity formula of Ref. 7 and the low-temperature core parameter of Ref. 1, we can compute B and B' at $T=1.4$ K from $P=0$ to 24 bars. We find B is nearly independent of pressure, falling about 25% over the pressure range from a value of ~ 1.4 at $P=0$. The data of Mathieu, Marechal, and Simon[14] are roughly constant at ~ 1.5 over the same range in pressure. This result is surprising since D and D' are functions of ρ_n which increases by a factor of 3 over this pressure range at $T=1.4$ K.[13]

This research gives, we believe, the basis for understanding the mutual friction coefficients D and D', and B and B', at all temperatures and pressures. A number of important challenges remain: (I) We need accurate viscosity data under pressure; (II) we need to measure vortex-ring drag under pressure and compare the data to the predictions of (6) and (10); (III) B and B' are poorly known below 1.6 K and need to be measured with accuracy down to ~ 0.8 K and as a function of pressure; and (IV) a theoretical explanation of the correction terms in (13) and (14) would be helpful, but probably very difficult as these terms are necessary in the temperature range above the limit of the single excitation model but below the critical temperature region.

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