Broken Symmetries and Localization Lengths in Anderson Insulators: Theory and Experiment

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Extending a random-matrix theory developed earlier, we show that breaking a basic symmetry in an Anderson insulator (e.g., time-reversal symmetry or spin-rotation symmetry) generically yields a multiplication of the localization length ξ by universal factors. Numerical calculations and magnetoconductance measurements in the Mott variable-range-hopping regime confirm that the removal of time-reversal symmetry by a magnetic field yields $\xi \rightarrow 2\xi$ in the absence of spin-orbit scattering, and $\xi \rightarrow \xi/2$ in the presence of spin-orbit coupling.

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The classification of ensembles in quantum mechanics, first introduced by Dyson,[1] leads us to distinguish between systems which are not invariant under time-reversal symmetry (unitary case, e.g., when a sufficient magnetic field is applied) and systems having this invariance, for which two cases occur: the orthogonal case (invariant under spin rotation) and the symplectic case where this symmetry is broken, e.g., by sufficient spin-orbit scattering. This classification is familiar in transport theory, but until now it is mainly for the weak-localization corrections[2] that we have realized its importance. Magnetoconductance measurements have shown how weak localization and weak antilocalization are removed by a magnetic field. These results describe the effects of transitions between the different symmetry cases for disordered metals only, where the conductance g is measured in units of e^2/h is large.

With few exceptions,[3] these symmetry considerations have been ignored in the strongly localized regime ($g \ll 1$). However, a positive magnetoconductance, which is reminiscent of the suppression of weak localization by a magnetic field in disordered metals without spin-orbit scattering, has been observed in a number of studies[4] of Fermi glasses in the Mott hopping regime.

The nonperturbative description of a random-matrix theory developed earlier[5-8] allows us to understand in a unified manner the effects of symmetry breaking in conductors and insulators. The matrix X considered in this theory is related to the transfer matrix M of a N-channel elastic-scattering system of length L by $X=[(M^{\dagger}M)+(M^{\dagger}M)^{-1}-2\mathbf{1}]/4$. Its N real positive degenerate eigenvalues $\{\lambda_a\}$ are related to g by a two-probe Landauer formula: $g=2\sum_{a=1}^{N}[1/(1+\lambda_a)]$. The distribution of g is then given in terms of the joint probability distribution $P(\{\lambda_a\})$, which is obtained in two steps. First, we define for X three possible matrix spaces from symmetry considerations. We can visualize the matrix X as a point which diffuses in those spaces when, for instance, the microscopic locations of the scatterers are changed. When the system is time-reversal invariant (e.g., orthogonal case), the point is constrained to diffuse in a subspace embedded in a larger space which can be explored only when this symmetry is removed (unitary case). This yields three possible infinitesimal volume elements dX and associated invariant measures $\mu_\beta(dX)$. In eigenvalue-eigenvector coordinates,[5,6] one obtains $\mu_\beta(dX) \propto \prod_{a>b}^{N}|\lambda_a-\lambda_b|^{\beta}$, where β takes the values of 1, 2, and 4 characterizing the orthogonal, unitary, and symplectic cases, respectively. In the second step, we define a density on the X space which depends on the macroscopic physical parameters. The global model[5,6] assumes for this matrix density the most random one (maximum-entropy criterion) given an eigenvalue density $\rho_L(\lambda)$. After integration over the eigenvectors, which are randomized in their available space, one finds $P(\{\lambda_a\}) \equiv \exp(-\beta H)$, where

$$H(\{\lambda_a\}) = -\sum_{a<b}^{N}\ln|\lambda_a-\lambda_b| + \sum_{c=1}^{N}V(\lambda_c) \qquad (1)$$

and $V(\lambda)=\int_0^{\infty}d\lambda'\rho_L(\lambda')\ln|\lambda'-\lambda|$. Note the analogy to a Coulomb gas of N charged particles at an effective temperature β^{-1} which depends on the symmetries of the system. The effective Hamiltonian (1) has been tested by independent numerical calculations[5-7] on microscopic Anderson models. Moreover, if $\rho_L(\lambda)$ is the solution of a certain integro-differential equation,[9] this $P(\{\lambda_a\})$ satisfies the diffusion equation implied by the multiplicative composition law of M and a certain local model[10] of maximum information entropy given the elastic mean free path l. This local model yields quantitative agreement[11] with perturbative results in quasi-1D systems. Thus, taking the density implied by the local model restricts us to quasi-1D systems, and our knowledge of $\rho_L(\lambda)$ for higher dimensions is limited for the moment to numerical studies. Defining $2\lambda_a-1\equiv\cosh(\nu_a) \equiv\cosh(2L/\xi_a)$, Oseledec's theorem states that each inverse localization length ξ_a^{-1} self-averages[12] to a given Lyapunov exponent as $L\rightarrow\infty$. This implies that each ν_a grows roughly linearly with L, leading eventually to strong localization, where $1\ll\nu_1\ll\nu_2\ll\cdots\ll\nu_N$ and

$g \propto \exp(-\nu_1) \propto \exp[-(2L/\xi)]$. The important result that the Lyapunov exponents have an essentially uniform density[11] arises from the multiplicative character of M and seems generic in that it is found in many other physical contexts, e.g., dynamical systems.[13] It has been shown analytically[9] for the local model that the ν variable is characterized by a uniform density $\sigma(\nu)=g_0/2$ for $0<\nu<2L/l$. Here $g_0=Nl/L$ is the classical Ohmic conductance. Quasi-1D systems are characterized by this density over the whole spectrum, but numerical studies[12] indicate very clearly that $\sigma(\nu)$ also remains uniform for the main part of the spectrum for 2D strips or 3D bars in the large-transverse-length limit, as well as in strongly disordered insulating squares.[7]

It is therefore more natural to consider the $\{\nu_a\}$ as the location of the charges, which simply spread themselves out more or less uniformly over an interval increasing linearly with L. It is then straightforward to show that the confining potential for the $\{\nu_a\}$ is quadratic[7] close to the origin: $U(\nu)=(A/2L)\nu^2$. The effective "Hamiltonian" for these variables thus takes the form

$$H(\{\nu_a\}) = -\sum_{a<b}^{N} \ln|\cosh(\nu_a)-\cosh(\nu_b)| + \sum_{c=1}^{N} -\frac{1}{\beta}\ln[2\sinh(\nu_c)] + \frac{A}{2L}\nu_c^2 , \qquad (2)$$

where $A=Nl/2$ if one takes for $\sigma(\nu)$ the uniform quasi-1D density. One of the assumptions underlying this result is that a symmetry-breaking perturbation alters only the statistics of the eigenvectors of X without significantly changing the eigenvalue density. In general, in random-matrix theory the eigenvectors are assumed to be fully randomized for the different symmetry cases. Thus, symmetry breaking changes only the value of β, and hence the β-dependent term in H, coming from the Jacobian of the change of variables. We underline that the presence of this β-dependent repulsion from the origin in (2) is a very important and quite general result. When time-reversal symmetry is removed by a magnetic field, the B independence of $\sigma(\nu)$ seems physically reasonable only when the magnetic length L_B is large compared to l. For instance, g is not changed in the local approach by B for a size $L \approx l \ll L_B$, and the resulting β-dependent diffusion equation for $P(\{\lambda_a\})$ gives[9] a β-independent equation for $\sigma(\nu)$ in the large-N limit. Thus the constant A in (2), which can be calculated once $\sigma(\nu)$ is given, does not depend on the symmetry-breaking perturbation in the local model. In higher fields (quantized Hall regime), the density of states and similarly $\sigma(\nu)$ are highly structured functions of B and our theory does not apply.

Expanding (2) in the strongly localized regime where $1 \ll \nu_1 \ll \cdots \ll \nu_N$, one finds that the ath level ν_a feels a potential $-(a-1)\nu_a-(1/\beta)\nu_a+(A/2L)\nu_a^2$, giving equilibrium lattice positions $\nu_a^0=(a-1+1/\beta)L/A$. This result was derived when $\beta=1$ in Ref. 7. Setting $a=1$ we obtain

$$\xi(\beta)=\beta\xi(\beta=1) , \qquad (3)$$

where $\xi(\beta=1)=Nl$ if we assume the uniform quasi-1D density.

Relation (3) agrees with the results[14] of a one-dimensional supermatrix σ model and was also derived in a study[15] of weakly coupled one-dimensional chains where predominant motion of carriers along the chains was assumed. Contrary to the β-dependent repulsion from the origin which is quite general, the presence of the quadratic potential in (2) depends on the additional assumption of a uniform ν density, only proved for quasi-1D systems. If 2D or 3D localization does not significantly change the uniform ν density close to the origin, as indicated by numerical studies for the main part of the spectrum, relation (3) is not limited solely to quasi-1D systems, but applies quite generally in higher dimensions. Since a mobility edge is shifted[16] by B, a larger effect on ξ is expected in its vicinity, and the corresponding potential felt here by ν_1 cannot be quadratic. But we point out that this shift of the mobility edge, as the weak-localization corrections in the metallic regime, must also be a consequence of the β-dependent term in (2), combined with the appropriate confining potential. A recent work[17] reporting a positive magnetoconductance for all strengths of spin-orbit scattering apparently contradicts our result. This disagreement can be due to the directed-path formalism used which neglects the "returning loops" within the localization domain, and leads us to make precise the range of B in which our results and their conclusions are valid.[18] It seems reasonable indeed that $B^*\xi^2(B^*)\approx\phi_0$ should be the natural criterion for characterizing the field B^* required to break time-reversal symmetry in our study. On the contrary, the conclusion of the MIT group could be correct at weaker fields where the flux quantum ϕ_0 is not applied through the localization domain, but only through a larger domain of the order of the Mott hopping length r_M.

We have calculated the localization lengths of disordered microscopic models with an applied magnetic field in order to test (3). A Hofstadter model[19] with diagonal disorder (rectangular distribution of width W) was considered on a strip at the band center with periodic boundary conditions. When we increase B, we induce a transition from the orthogonal case to the unitary one, and we expect a doubling of the localization length. When W is small, the variation of ξ as a function of B is highly structured and dominated by the commensurability of the flux per lattice cell and the flux quantum ϕ_0. This yields for the spectrum of the Hamiltonian a complicated band structure (Hofstadter butterfly, Landau bands, gap opening effects, etc.) and a similarly complex behavior for $\sigma(\nu)$. Since we have assumed that $\sigma(\nu)$ is B independent, we do not find the simple doubling of ξ

for weak disorder. On the contrary, for large disorder, such magnetic structure is suppressed and the field independence of $\sigma(\nu)$ is a good approximation. Under these conditions we observe the predicted doubling of ξ (Fig. 1).

We have also been able to observe these universal factors experimentally in magnetotransport measurements. In the Mott hopping regime,[20] the activated conductivity $\sigma_e(T)$ varies as a function of the temperature T as $\exp[-(T_0/T)^{1/4}]$, where the slope $T_0 \propto 1/n(E_F)\xi^3$, $n(E_F)$ being the density of states at the Fermi level E_F. To observe the transition from the orthogonal ($\beta=1$) to unitary case ($\beta=2$), and the associated doubling of ξ, we have measured the hopping conductivity of a GaAs sample[21] Si doped at $\sim 10^{16}$ cm^{-3} in a temperature range between 20 mK and 4.2 K. Neglecting the dependence of $n(E_F)$ on B, related to our assumption of a B-independent $\sigma(\nu)$, we have a simple prediction for the behavior of the slope $T_0^{1/4}$ [universal factor $(\frac{1}{2})^{3/4}$]. The inset of Fig. 2(a) shows indeed the expected decrease of $T_0^{1/4}$ as a function of the applied field and a saturation above $B \approx 0.2$ T, although at a value 30% below that expected. The saturated behavior holds up to $B \approx 2$ T above which there is a huge increase attributable to the shrinkage of the orbital wave functions.[20] The second experiment [Fig. 2(b)] has been performed on an amorphous Y_x-Si_{1-x} alloy[22] ($x \approx 0.3$) in order to illustrate the transition from the sympletic ($\beta=4$) to unitary case ($\beta=2$). Metallic compounds of this series exhibit weak antilocalization which confirms the presence of strong spin-orbit scattering. In contrast with the experiment on GaAs, here we expect and observe a halving of ξ. $T_0^{1/4}$ increases as a function of B and saturates to within 10% of the expected value for $B^* > 3$ T. Below 100 mK we observe[23] in the Y_x-Si_{1-x} sample of size as large as 1 cm a dependence of $\sigma_e(T)$ on the measurement current below 10 nA, and reproducible fluctuations of the magnetoconductance at $T \approx 20$ mK.

We emphasize here that these samples are three dimensional, indicating the relevance of our theory outside purely quasi-1D systems. It would be of interest in future work to observe a transition $\beta=1 \rightarrow \beta=4$ by the introduction of spin-orbit scattering impurities, increasing ξ by a factor of 4 in the absence of a magnetic field. $B^* \approx 3\times10^{-3}$ T for the numerical calculations presented in Fig. 1, and using for ξ very crude estimates, $B^* \approx 0.002$ T for the GaAs sample and $B^* \approx 3$ T for the Y_x-Si_{1-x} alloy. We note an excellent agreement with our theory for the second sample where $r_M \approx 3\xi$. The agreement for the GaAs sample is not very good. This can be related to the weak value of T_0 indicating that we are too close for comfort to the mobility edge (formula[20] omitting numerical factors gives $r_M \approx \xi$). However, measurements were performed deeper in the localized regime on In_2O_{3-x} films[24] (no spin-orbit scattering) and were in

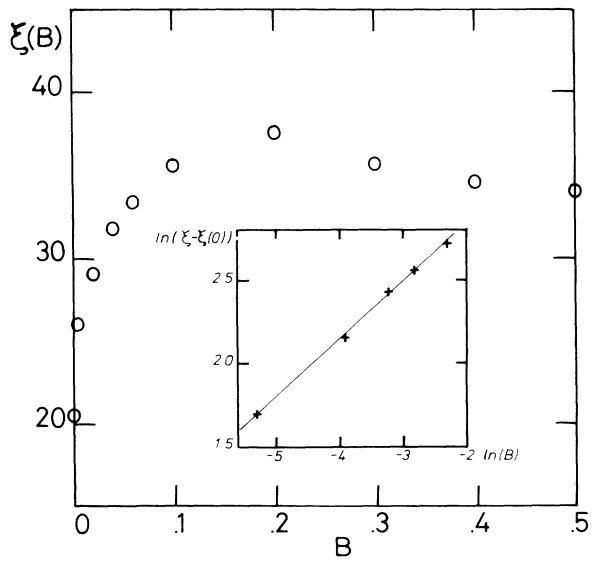

FIG. 1. Localization length ξ vs B in units of flux quanta per lattice cell for a strip of width 20 and site disorder width $W=5$. Inset: Power-law dependence of ξ at low field. The exponent ($\frac{1}{3}$ here) appears to vary with W.

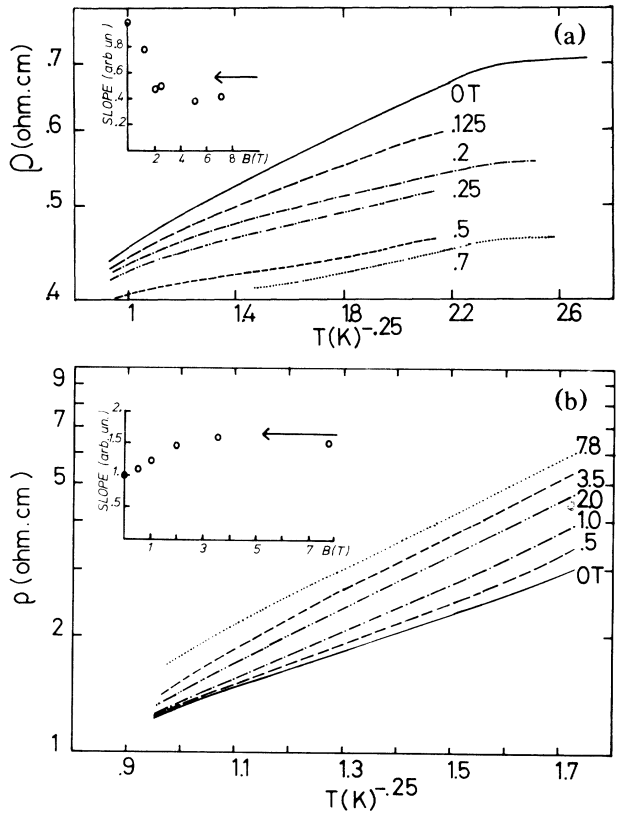

FIG. 2. Logarithim of the resistivity as a function of $T^{-1/4}$. Inset: Variation of the slopes $T_0^{1/4}$ as a function of the applied field B. The arrow indicates the expected saturation value. (a) Si-doped GaAs sample. The saturation observed below 35 mK is due to heating of the electron gas. (b) A Y_x-Si_{1-x} alloy.

agreement with a doubling of ξ.

Before concluding, we would like to underline the deep relation between our general result [Eq. (3)] and the weak-localization corrections in the metallic regime. The symmetry-breaking perturbation (magnetic field or spin-orbit scattering) is assumed to modify the available matrix space which can be explored by X, without changing significantly $\sigma(\nu)$. We find that the ν levels are subject to a β-dependent repulsion from the origin and to a quadratic confining potential. Then we perform an expansion valid deep in the localized regime when $1 \ll \nu_1 \ll \cdots \ll \nu_N$. As a consequence, when β is changed there is a uniform translation of the level equilibrium positions of the order of their uniform spacing. This modifies the decay length $\xi_a = A/(a-1+1/\beta)$ of the ath channel. Deep in the metallic regime, using a simplified picture due to Imry,[25] g measures the number N_{eff} of open transmission channels for which $\xi_a \gg L$. It is easy to see that the change of the decay lengths derived above in the localized regime will change N_{eff} by an amount of order 1 in the metallic regime. The doubling or halving of ξ is therefore the counterpart in the localized regime of the suppression of weak localization or antilocalization in the metallic regime.

We note that changes of ξ can also be probed by measuring the dielectric susceptibility ϵ which is proportional[26] to ξ^2. The dependence of ϵ and related optical properties on an applied magnetic field is therefore a question which deserves to be reexamined.

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