Stabilization of Relativistic Self-Focusing of Intense Subpicosecond Ultraviolet Pulses in Plasmas

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The characteristics of relativistic propagation in plasmas of subpicosecond ultraviolet (248 nm) radiation are studied for both spatially homogeneous plasmas and plasma columns. It is established for the first time that the defocusing properties of the interaction can represent a dynamical mechanism stabilizing the mode of propagation against radial oscillations. The dependence of both quasistabilized modes and pulsing waveguide regimes on the initial transverse intensity distribution is examined and, for the latter, the locus of the first focus produced in a homogeneous plasma is calculated.

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The dynamics of propagation of extremely intense subpicosecond pulses of radiation in self-generated plasmas is a rapidly developing area of study. In this Letter certain aspects of this question are treated theoretically for the following ranges of physical parameters: peak intensity $I \cong 10^{18}$–10^{21} W/cm^2, pulse duration $\tau \cong 0.5$–1.0 ps, initial radial aperture $r_0 \cong 1$–3 μm, wavelength $\lambda = 248$ nm, and plasma electron density $\sim 10^{20}$–10^{21} cm^{-3}. A key finding of these calculations is insight into the physical processes enabling the formation at a quasistable mode of propagation of such high-intensity radiation in plasmas.

For pulses having a peak intensity in the 10^{18}–10^{21}-W/cm^2 range, rapid ($\Delta\tau \cong 1$ fs) processes of multiphoton ionization[1,2] occurring on the leading edge ($I \geq 10^{16}$ W/cm^2) of the wave form will remove several ($\sim$6–10) atomic electrons. Thereby, for a focused beam with a given radial aperture r_0, a plasma column is rapidly formed in which the central temporal zone of the pulse propagates. The spatial and temporal dynamics governing the resulting propagation are determined by the competitive interaction of diffraction, defocusing, and self-focusing. We note that in the early portion of the pulse, some defocusing is expected to occur while the ionization is commencing, since the contribution to the refractive index from the free electrons tends to locally reduce its value in the central high-intensity region. In addition, as shown below, the process of defocusing can give rise to a previously unknown mechanism of stabilized propagation.

For subpicosecond pulses and the radial dimensions considered, gross plasma motion involving the ions is negligible.[3] Similarly, for full ionization, noninertial Kerr self-focusing[4] is insignificant. In the present case, both the relativistic increase in the electron mass arising from induced oscillations in the intense field[5,6] and the charge displacement resulting from expulsion of free electrons from the ionized column by the ponderomotive force[3,7,8] can produce strong self-focusing action.

We now describe the specific equations governing the propagation and present calculated solutions illustrating the development of self-focusing and channeled propagation. The origin of the stability exhibited by the propagating radiation is of primary importance. In considering the central temporal region of the pulse, we neglect the energy losses associated with further ionization and spatial broadening of the plasma column, since these effects can be quite small.[3] With that condition, the space-time behavior of the radiation field in the column, including relativistic-charge-displacement self-focusing, diffraction, and defocusing, is governed by the nonlinear Schrödinger equation

$$\left[\frac{1}{c_1}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right]E+\frac{i}{2k_0}\Delta_\perp E+\frac{ik_0}{2\varepsilon_{R_0}}\delta\varepsilon_R E+\frac{k_0}{2\varepsilon_{R_0}}\varepsilon_I E=0\,, \tag{1}$$

where $E(t,z,r)$ is the slowly varying (t,z) complex amplitude of the wave, r is the transverse coordinate, $\Delta_\perp=(\partial/r\partial r)r\,\partial/\partial r$, $k_0=2\pi/\lambda$, $c_1=c\varepsilon_{R_0}^{1/2}$ is the group velocity in the plasma, and c is the vacuum speed of light.

The nonliner term $\delta\varepsilon_R(r,|E|^2)$ describing the combined relativistic-charge-displacement self-focusing and defocusing of the pulse is expressed as

$$\delta\varepsilon_R=\varepsilon_R-\varepsilon_{R_0},\quad \varepsilon_R=1-\omega_p^2/(\omega^2+\nu^2),\quad \varepsilon_{R_0}=1-\omega_{p_0}^2/(\omega^2+\nu_0^2)\,,$$

$$\omega_p^2=\{4\pi e^2N_e[I]/[m_e(1+3I/I_r)^{1/2}]\},\quad \omega_{p_0}^2=4\pi e^2N_{e,0}/m_e,$$

$$I_r=3m_e^2\omega^2c^3/4\pi e^2,\quad I=(c/2\pi)|E|^2\,.$$

In these expressions, ω_p is the relativistically shifted plasma frequency, ω is the laser angular frequency, v is the electron-ion collisional frequency, I_r is the relativistic intensity,[3,5] and $N_e[I]$ is the electron density arising from the quasistatic balance of the ponderomotive and the electrostatic forces densities.[3] We assume that $N_i(r,z)=N_{i,0}f(r,z)$ is the static ion density, in which $N_{i,0}$ is a constant denoting the maximal ion density and $f(r,z)$ describes the spatial distribution of the ions. Hence, $f(r,z)\geq 0$, $\max f(r)=1$ [$f(r,z)\equiv 1$ for homogeneous plasma]. For quasineutrality, this gives $N_{e,0}f(r,z)$ as the initial electron density with $N_{e,0}=ZN_{i,0}$ and Z the ionic charge.

The expression for $N_e[I]$, which describes the perturbation of the plasma from local neutrality by both the ponderomotive and relativistic mechanism, is given by

$$N_e[I]=N_{e,0}\max\left\{0,f(r,z)+\kappa\,\mathrm{div}\left[\frac{\mathrm{grad}\,I}{(1+3I/I_r)^{1/2}}\right]\right\}, \quad (2)$$

where

$$\kappa=(2c\omega^2 m_e N_{e,0})^{-1}, \quad (3)$$

and m_e denotes the electron mass. In deriving Eq. (2), the electronic pressure has been neglected, since it is small for the beam and plasma parameters examined here. The expression $\max\{0,-\}$ simply provides for $N_e[I]\geq 0$.

Calculations of the propagation have been performed corresponding to the following parameters for the radiation field and the plasma: $\lambda=248$ nm, $I_r=1.34\times10^{20}$ W/cm^2, $I_0=\frac{2}{9}I_r\cong 3.0\times10^{19}$ W/cm^2, $r_0=3$ μm, $N_{i,0}=7.5\times10^{19}$ cm^{-3}, and $Z=10$.

Normalization of r and I by the initial beam radius r_0 and the maximal value of the initial intensity I_0, respectively, gives $\kappa_1=\kappa I_0 r_0^{-2}\cong 1.4\times10^{-3}$, the dimensionless analog of the coefficient κ defined by Eq. (3). Therefore, for this set of conditions, the charge displacement[3] is expected to be relatively small so that its influence is confined to the structure of the focal regions which arise from the relativistic mechanism. Because of this, we will utilize the approximation that $\kappa=0$ in the current work. It is important to note, however, that other choices of the parameters can lead to a generally strong role for the charge-displacement process and future study is being directed to that case.

The collisional absorption of light in plasma is given by $\varepsilon_I=-(\omega_p^2/\omega^2)v=-\mu^-/k_0$, where μ^- is the absorption coefficient. Estimates[5] of μ^-, for the values of the parameters used in this work, show that the collisional absorption is extremely weak and does not influence the numerically obtained solutions. Therefore, the results presented below have been calculated with $\mu^-=0$.

The nonlinear Schrödinger equation stated in Eq. (1) is only applicable to the description of the dynamics of the field in a small focal zone, if its longitudinal scale l satisfies the condition $l\gg\lambda$. The numerical experiments showed, for the range of parameters studied, that $l>50\lambda$. In this connection, we note that the spatial limitations on the minimal dimensions of the focal zones, and consequently the bounds on the intensities developed in those regions, arise from saturation of the nonlinearity expressed by $\delta\varepsilon_R(r,|E|^2)$.

The dynamics of propagation with relativistic self-focusing have been examined using model pulses $I(z,r,t)$ having spatial and temporal Gaussian or hyper-Gaussian incident beam profiles given by

$$I(0,r,t)\equiv I_0(r,t)=I_m\exp\{-(t/\tau)^{N_1}-(r/r_0)^{N_2}\}, \quad (4)$$
$$N_1\geq 2,\quad N_2\geq 2$$

for $I_m\sim I_r\gg I^*=10^{16}$ W/cm^2, the latter being the threshold intensity above which rapid multiphoton ionization occurs to the particles in the medium. Furthermore, we have concentrated on the central temporal zone of the pulse, $|t|\leq t_0$, where t_0 satisfies the expression

$$I_0(t_0)=I_0(0,t_0)=I_m\exp\{-(t_0/\tau)^{N_1}\}\gg I^*. \quad (5)$$

The transverse profile of the plasma column created by the front of the pulse is simulated by the hyper-Gaussian function

$$f(r)=\exp\{-(r/r_*)^{N_3}\},\quad N_3\geq 2, \quad (6)$$

with the aperture of the plasma column r_* defined by

$$I_0(r_*,t_0)\equiv I_0(t_0)\exp\{-(r_*/r_0)^{N_2}\}=I^*. \quad (7)$$

For a Gaussian transverse intensity distribution ($N_2=2$), an aperture defined with $I^*=10^{16}$ W/cm^2, and taking $I_0(t_0)=(0.1)I_r$, we find $r_*\cong 2.68r_0$. Consequently, the homogeneous plasma approximation $f(r)\equiv 1$ is valid.

The beam profiles of the solutions exhibit several salient features. Among them are the production of multiple foci and, under appropriate conditions, the development of a quasistable confined mode of propagation lacking the generation of sharply focused regions. Importantly for the latter, the calculations reveal the nature of the mechanism leading to the stabilization. The results of computations presented in this work are given in the frame (s,z,r), $s=t-(z/c_1)$, connected with the wave front of the beam. The data in the figures below illustrate the propagation of the radiation along the z axis for $s=$const.

Figure 1(a) illustrates the propagation that develops for the physical parameters stated above corresponding to a pulse with an entrance intensity $I_0(t)=\frac{2}{9}I_r$ having a transverse distribution given by $N_2=8$. In accord with an earlier study,[9] as the nonlinearity $\delta\varepsilon_R(|E|^2)$ is saturated at a sufficiently low level, the behavior of pulses propagating in a homogeneous plasma without absorption can generate a mode similar to a pulsing waveguide.[9,10] The result shown in Fig. 1(a) illustrates the formation of such a regime. Interestingly, the pulsing waveguide represents an alternation of ring struc-

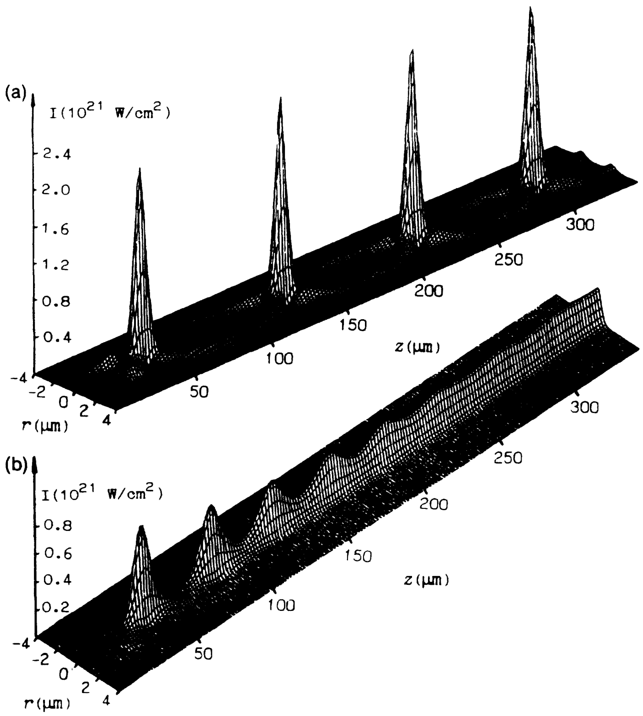

FIG. 1. The (r,z) distributions for relativistic self-focusing of the propagating intensity for a peak initial intensity $I_0=\frac{2}{9}I_r$ having a transverse distribution given by $N_2=8$ in Eq. (4) for the beam and plasma parameters defined in the text. (a) Spatially homogeneous fully stripped plasma. (b) Axially symmetric plasma column with $r_*=r_0$ whose transverse profile corresponds to $N_3=8$ in Eq. (6).

tures and focal regions along the axis of propagation. Significantly, the power trapped in it corresponds to approximately 90% of the total initial power of the beam.

From Eq. (7), it follows that, for plateaulike initial intensity distributions, the aperture r_* of the plasma column becomes comparable with the beam aperture r_0. For example, in the case treated above, we have $r_*\cong 1.28r_0$. As the apertures of the laser beam and the plasma column tend toward coincidence, however, defocusing becomes relatively more significant. Therefore, this aspect of the propagation must be carefully taken into account when the evolution of pulses having plateaulike initial intensity distributions was studied.

Figure 1(b) shows the propagation of a pulse in a plasma column for $N_2=8$ corresponding to the same physical parameters as those pertaining to the illustration in Fig. 1(a). In this case, the transverse profile of the electron density in the plasma column is described by $f(r)$ in Eq. (6) for $N_3=8$ and $r_*=r_0$. The comparison of Figs. 1(a) and 1(b) demonstrates the strong effect of defocusing on the spatial dynamics of propagation when the column has a radial dimension close to that of the beam. Defocusing causes a fraction of the beam to spread away from the column while the remaining energy adjusts to balance the relativistic self-focusing, defocusing, and

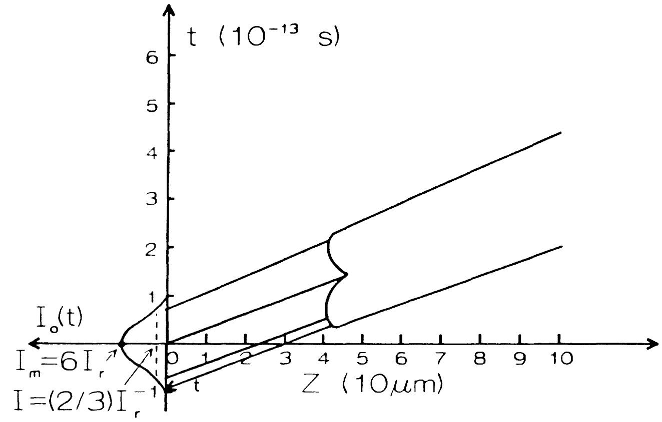

FIG. 2. The locus (t,z) of the first focal zone for a pulse with a Gaussian initial spatial and temporal intensity distribution for the parameters defined in the text. The solution pertains to relativistic self-focusing in a spatially homogeneous fully stripped plasma.

diffraction. In this way, the dynamics of the interaction tend toward the formation of a spatially stable mode of propagation. In addition, as evident from Fig. 1(b), a substantial fraction, nominally 25% of total initial beam power, can be confined in such a mode. Importantly, the further study of such cases has also indicated that such stable patterns can be maintained for a length of at least twenty normal diffraction lengths.

The solutions of Eq. (1), with the initial conditions given by Eq. (4), satisfy important similarity laws. Specifically, the results shown in Figs. 1(a) and 1(b) are valid for any other set of physical parameters satisfying the relationships $a_1\equiv[(k_0r_0)^2/\varepsilon R_0]\omega_{p0}^2/\omega^2=2.486\times 10^2$ and $a_2\equiv I_0/I_r=2/9$. Furthermore, if full account of both the relativistic and charge-displacement mechanisms ($\kappa_1\neq 0$) is made, these similarity statements are preserved and $\kappa_1=a_2/4a_1$.

Following earlier studies[10,11] we have analyzed the evolution of plane waves having small perturbations in order to determine the dependence of the self-focusing length on the initial pulse parameters. The results of this analysis showed that the largest growth rate of the perturbations and, consequently, the minimum self-focusing length occur at an entrance intensity on the beam axis of $I_0(t)=\frac{2}{3}I_r$. Specifically, the computations have shown that this inference remains true for the relativistic self-focusing of pulses with Gaussian initial intensity distributions.

The dynamic motion of the foci illustrated in Fig. 2 exhibits important characteristics. For example, the calculations show that the locus of the first focus in the (t,z) plane,[4] for the initial condition determined by Eq. (4) with $N_1=2$, $N_2=2$, $I_m=6I_r\cong 8.04\times 10^{20}$ W/cm^2, $\tau=500$ fs, and $r_0=3$ μm, reaches the minimal z for $I_0(t)=\frac{2}{3}I_r$. If the extreme intensity on the beam axis is $I_m\gtrsim I_r$, this locus has three reversal points. Two of them, corresponding to the same z, are due to $I_0(t)=\frac{2}{3}I_r$. The third and central one, corresponding to a greater z, occurs at $I_0(t)=I_m$. In principle, this feature establishes a clear diagnostic signature for relativistic self-focusing that enables it to be distinguished from the Kerr nonlinearity[4] which produces a locus with a single point of reversal.

The spatial character of propagation of sufficiently intense subpicosecond ultraviolet pulses in self-generated plasmas is significantly influenced by a relativistic-charge-displacement mechanism and two important classes of propagation have been distinguished. In a spatially homogeneous plasma, the interaction can produce self-focusing in the form of a pulsing waveguide involving a structure of alternating rings and focal zones. The location, formation, and nature of the focal regions, including the maximal intensities occurring in them, depend essentially on the properties of the initial transverse intensity distribution. The locus of the first focus in the (t,z) plane for pulses with Gaussian initial intensity distributions and maximum intensities $I_m \gtrsim I_r$ has three reversal points. For this class of pulses, the relativistic self-focusing length is minimal, if the intensity of the initial pulse on the axis equals $\frac{2}{3}I_r$.

A mechanism leading to a spatially stabilized mode of propagation is also revealed. In this case, the defocusing that occurs when the plasma column has a radius close to that of the beam is particularly significant. The effective action of the defocusing, which serves as a spatially distributed nonuniform energy loss, causes the rapid formation of intensity profiles that become quasistabilized along the beam axis. Therefore, it is accurate to state that the process of defocusing contributes directly to the dynamical development of stabilization. Further computational results using the procedures described above are available.[12]

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