Cosmological Evolution of Global Monopoles and the Origin of Large-Scale Structure

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We study the evolution of global monopoles by means of numerical simulations, and find that the monopoles obey a scaling solution in which there are a fixed number of monopoles in every horizon volume. Monopoles which form at the grand unification scale can serve as seeds for galaxy and large-scale structure formation.

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The idea that a spontaneous symmetry-breaking phase transition in the early Universe can give rise to topological defects has played a very important role in the development of theories which attempt to explain the formation of galaxies and large-scale structure. Cosmic strings,[1,2] light domain walls,[3] and global textures[4] have all been proposed as possible seeds for large-scale structure formation, and in this Letter we show that global monopoles are also a viable and promising candidate. Of all these candidates, only cosmic strings are actually predicted by simple grand unified theories (GUTs), but since these GUTs now seem rather doubtful on both experimental and theoretical grounds, it is important to consider the alternatives.

Global monopoles[5] are the pointlike defects of a spontaneously broken non-Abelian global symmetry, but unlike gauge (magnetic) monopoles, they do not resemble particles. They carry an associated Goldstone-boson field whose energy density falls off as $1/r^2$, so the total energy of the Goldstone field diverges as $\sim r$ at large distances. In a realistic setting, this divergence will be cut off at the distance to the nearest antimonopole which is a large astrophysical scale. The evolution of a system of global monopoles will be dominated by the dynamics of this Goldstone-boson field which can become correlated at large distances and provide an interesting spectrum of density fluctuations.

In order to see if global monopoles could produce interesting density fluctuations, it is necessary to understand the dynamics of the Goldstone-boson field. In this paper, we study the evolution of this Goldstone-boson field with numerical simulations and show that global monopoles evolve according to a scaling solution with a fixed (small) number of monopoles per horizon volume at all times. The scaling-solution number density is many orders of magnitude smaller than the upper limits derived by Hiscock,[6] and we have explicitly demonstrated that the instability discussed by Goldhaber[7] is not an actual dynamical instability. (See Ref. 8 for a more complete discussion of this point.)

For definiteness, we will consider the simplest theory that can produce global monopoles: a scalar field with an O(3) symmetry which is spontaneously broken to O(2). The Lagrangian density is

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a-\tfrac{1}{4}\lambda(\phi^a\phi^a-\eta^2)^2\,, \tag{1}$$

where ϕ^a is the O(3)-symmetric Higgs field. The ground state of the theory has $|\langle\phi\rangle|=\eta$, but there are also states with a nontrivial topological charge which are local minima of the energy functional. The topological charge is given by

$$Q=\frac{1}{4\pi\eta^3}\int_{S^2}\epsilon_{\alpha\beta\gamma}\phi^\alpha\partial_\mu\phi^\beta\partial_\nu\phi^\gamma dx^\mu\wedge dx^\nu\,, \tag{2}$$

and the simplest $Q=1$ solution is the spherically symmetric configuration $\phi^a=\eta f(r)\hat{r}^a$, where $\hat{r}^a$ is a component of the unit vector in the radial direction, and the core function $f(r)\to 1$ as $r\to\infty$. Outside the core, the energy density of this global-monopole solution is dominated by the gradient term

$$\rho_{\text{grad}}=\tfrac{1}{2}\nabla\phi^a\cdot\nabla\phi^a=\eta^2/r^2\,. \tag{3}$$

Thus, the total energy of a monopole out to a radius R is $E_{\text{mon}}=4\pi\eta R$. In practice, the divergence for large R will cut off at roughly the distance to the nearest antimonopole. Clearly, any global SO(3) rotation of this solution is also a monopole solution of equal energy, but a reflection of the $Q=1$ solution changes it to an antimonopole ($Q=-1$) solution. For a single monopole, these rotations have no physical significance because of the global symmetry, but the relative orientation of multiple monopoles and/or antimonopoles can have physical consequences. For instance, a M-$\overline{M}$ pair released from rest will generally feel some transverse acceleration as they approach each other to annihilate. This implies that global monopoles should not be considered to be simple particles. They are topologically nontrivial field configurations whose dynamics is dominated by the behavior of the Goldstone-boson field at large distances.

Because a system of global monopoles is such a complex system, the only tractable way to study their dynamics is through numerical simulations of the SO(3)-symmetric Higgs field that admits the monopole solutions. The equation of motion for the ϕ^a field in a Friedmann-Robertson-Walker (FRW) spacetime is

$$\ddot{\phi}^a+2(\dot{a}/a)\dot{\phi}^a-\nabla^2\phi^a+a^2\lambda(\phi^2-\eta^2)\phi^a=0\,, \tag{4}$$

where a is the FRW scale factor, the dots denote derivatives with respect to time, and the spatial derivatives are with respect to comoving coordinates.

When one attempts to solve Eq. (4) numerically in an expanding universe on a fixed comoving grid, the a^2 coefficient of the potential term presents a bit of a problem because it implies that the comoving size of the monopole core must decrease as $1/a$ with time. In order to resolve the monopole core properly throughout an expanding-universe simulation, we must require that the monopole radius be at least a few times the grid spacing at the end of a run. The monopole radius will be a factor of a ($=\tau^2$ in the matter era) larger than this at the beginning of the run. This reduces the effective dynamical range of a simulation by a factor of a compared to the dynamic range that would be available with a monopole fixed size. More precisely, the useful length of a run will be the cube root (in the matter era) or the square root (in the radiation era) of the length of the run that can be done in flat space.

One way around this problem that has been tried in the case of domain walls[9] is to modify the equation of motion to keep the size of the topological defect fixed. This has some very serious side effects, however. In particular, it requires that the coefficient of the $\dot{\phi}^a$ term in Eq. (4) be modified. This will result in excess (unphysical) damping of the motion of the monopoles and excess redshifting of radiation with the expansion.

A much better way of dealing with this difficulty in the context of global monopoles (or global texture) is to allow the monopoles to have zero size and evolve the ϕ^a field according to the nonlinear-σ-model equation,

$$\left(\delta_{\alpha\beta}-\frac{\phi^\alpha\phi^\beta}{\eta^2}\right)\left(\ddot{\phi}^\beta+2\frac{\dot{a}}{a}\dot{\phi}^\beta-\nabla^2\phi^\beta\right)=0\,, \tag{5}$$

with the constraint $\phi^2=\eta^2$. This is just the limit of Eq. (4) when λ becomes large or alternatively when we are interested in excitations whose energies and inverse wavelengths are small compared to $\sqrt{\lambda}\eta$. This is *always* a good approximation in any astrophysical context. One drawback of this nonlinear-σ-model approach is that the core of the monopole is not particularly well modeled. The monopole energy density should scale as $1/r^2$ as we approach the core, but this power law gets truncated numerically at the scale of grid separation. Thus, our discrete monopoles are too light on the scale of the grid spacing, but since the energy of the monopole increases linearly with r, these difficulties at small r have little influence on the overall evolution. We have tested our numerics by comparing runs of varying resolution which started with an initial monopole-antimonopole pair, and found that our code usually does fairly well (errors in $\phi \lesssim 20\%$) at a distance of only two grid spacings from the monopole core. The errors get somewhat worse when the monopole-antimonopole pair undergoes rather violent acceleration just before they annihilate. But the errors are always small ($\lesssim 10\%$) at distances of more than three grid spacings from the monopole path, and the errors quickly dissipate after the annihilation.

Another potential difficulty in evolving Eq. (5) numerically is that ϕ^a will necessarily have large spatial variations in the vicinity of a monopole. The large gradients can cause large accelerations of ϕ^a which will make the field change by a large amount in a single time step. This can cause difficulties in applying the $\phi^2=\eta^2$ constraint and lead to large errors that show up as (among other things) large violations of energy conservation. Fortunately, this problem is easily resolved by taking small time steps. We have found that for a flat-space run for a time interval of $64\Delta x$, energy conservation is violated by 17% for $\Delta\tau/\Delta x=0.2$, 2.1% for $\Delta\tau/\Delta x=0.1$, and 0.24% for $\Delta\tau/\Delta x=0.05$. (For comparison, the 3D Courant-stability criterion requires that $\Delta\tau/\Delta x<1/\sqrt{3}$.) We have used $\Delta\tau/\Delta x=0.1$ or 0.08 for almost all of our production runs.

We used two different types of initial conditions in our study of monopole evolution: randomly oriented fields and M-$\bar{M}$ pairs initially at rest. We have also numerically solved for the ground-state configuration of a M-$\bar{M}$ pair when the positions of the poles were held fixed with a 2D code. We found that the gradient energy of the field between the poles confined itself to a narrow "string" with a width proportional to the grid spacing. This seems to confirm the suggestions by Barriola and Vilenkin[5] and Turok[4] that the field between a monopole and a antimonopole might collapse to a very thin "string." However, when we ran our dynamical simulations without artificially fixing the positions of the monopole and antimonopole, we found that they would accelerate and annihilate before the field between them could collapse to a string. As the M-$\bar{M}$ pair was accelerating toward each other, the fields were significantly distorted from the stationary (or Lorentz-boosted) monopole configurations, but the fields more closely resembled the stationary monopole or antimonopole solutions than a M-$\bar{M}$ pair separated by a U(1) string. Thus, it is reasonable to conclude that these stringlike configurations do not occur in realistic situations.

The initial conditions for the "randomly oriented fields" configuration were chosen by randomly assigning the field direction at every fourth grid point and then using a four-point cubic interpolation scheme for the intermediate points (our boundary conditions were periodic). After the fields at the intermediate points are assigned, their amplitudes are normalized to meet the condition $\phi^2=\eta^2$.

Figure 1 shows the configuration of the field in a partial slice through one of our matter-era simulations at expansion factors of (a) $a=2.25$, (b) $a=9$, and (c) $a=49$ from the start of the run. The horizon grew from $0.125L_{\rm box}$ to $0.875L_{\rm box}$ from the start of the run until the time when the final picture is shown. ($L_{\rm box}$ is the size of the smallest dimension of the computational box.) Fig-

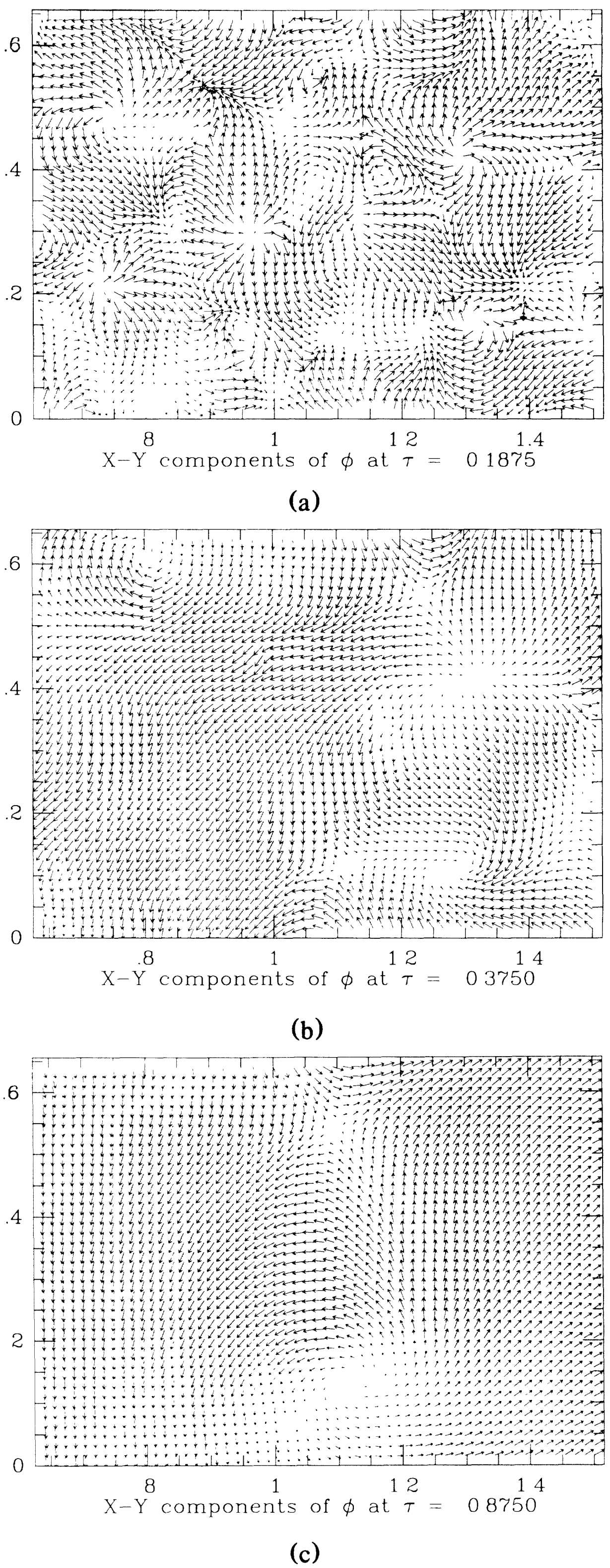

FIG. 1. Part of a slice through a 96×64^2 matter-era simulation at (a) $\tau=0.1875L_{\text{box}}=1.5\tau_0$, (b) $\tau=0.375L_{\text{box}}=3\tau_0$, and (c) $\tau=0.875L_{\text{box}}=7\tau_0$, where τ_0 is the initial conformal time and L_{box} is the comoving box size. The slice shown contains 65×41 grid points.

ure 1(c) is somewhat atypical in that the slice was chosen so that a few monopoles would be apparent. About half of the similarly sized slices at $\tau=0.875L_{\text{box}}=7\tau_0$ do not show any evidence of monopoles. Only the x and y components of the field are shown, so the amplitude of the field appears small when the z component is large (recall that $\phi^2\equiv\eta^2$).

The evolution of the global monopoles shown in Fig. 1 can be explained quite simply. On scales larger than the horizon the field is completely uncorrelated, so if we examine the field on the surface of a sphere with a radius larger than the horizon, we will find that there is a high probability that the sphere will contain a monopole or an antimonopole. As the Universe expands, we expect that many M-$\bar{M}$ pairs will annihilate and the field will become correlated on scales close to the horizon size. Thus, we expect of order 1 monopole or antimonopole per horizon volume at all times. Figure 2 shows the evolution of the comoving monopole (plus antimonopole) density N_{mon} multiplied by the horizon volume τ^3 for

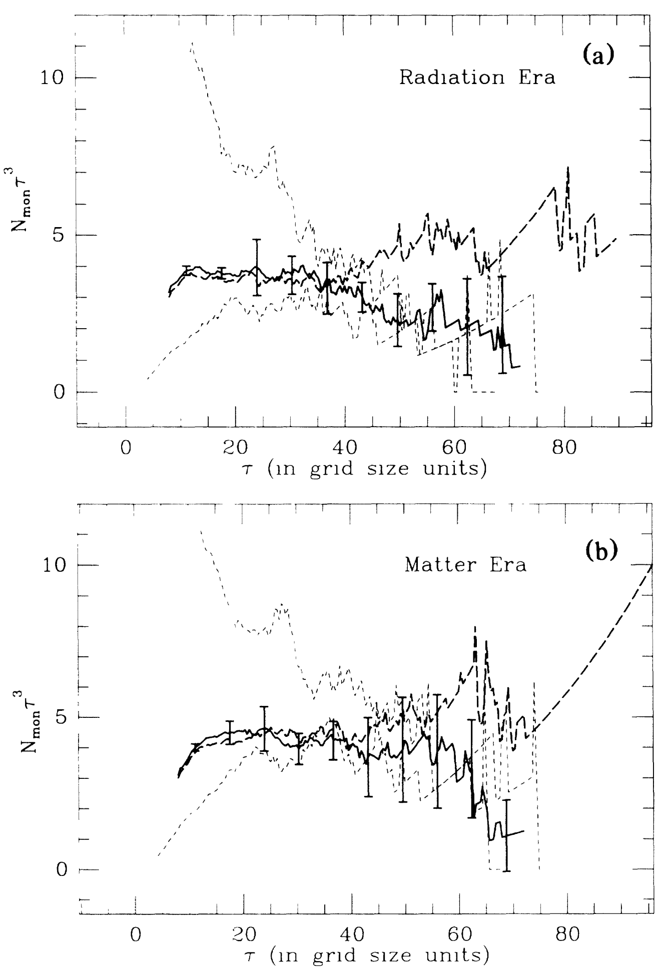

FIG. 2. The number of global monopoles plus antipoles per horizon volume, $N_{\text{mon}}\tau^3$, is plotted vs time for (a) several radiation-era runs and (b) several matter-era runs.

several different runs in (a) the radiation era and (b) the matter era. The heavy solid curves with error bars are the average of three 64^3 simulations and one 96×64^2 simulation (with statistical error bars). The heavy dashed curves represent 96^3 simulations, and the light dashed curves are runs that started with different initial monopole densities. These latter curves demonstrate that the system quickly relaxes toward the scaling density if the initial state has a higher or lower density. Unphysical effects of the periodic boundary conditions can enter at times as early as $\tau=40\Delta x=\tau_0+32\Delta x$ for the 64^3 runs and $\tau=56\Delta x=\tau_0+48\Delta x$ for the 96^3 runs because this is the first time in which waves emitted from the same point traveling in opposite directions can meet. (We expect these effects to become important somewhat later than this, however.) Our values for the comoving monopole density at scaling are $N_{\rm mon}=(3.5\pm1.5)/\tau^3$ in the radiation era and $N_{\rm mon}=(4.0\pm1.5)/\tau^3$ in the matter era.

A proper treatment of the density fluctuations from global monopoles will require adding gravity to our simulations, and so our present understanding of the density fluctuations from monopoles is rather speculative. A rough estimate of the amplitude of the fluctuations can be obtained by multiplying the typical energy of a monopole (evaluated at $R\approx1/N_{\rm mon}^{1/3}$) by $N_{\rm mon}$ and dividing by the total matter density. This yields $\delta\rho/\rho\approx100G\eta^2$ for the matter era which suggests that a GUT-scale value $\eta\sim10^{16}$ GeV would provide fluctuations of the right amplitude. Since the gravitational field of a static monopole[5] exerts no force on nonrelativistic matter, it seems reasonable that the dominant fluctuations will come from M-$\bar{M}$ pairs accelerating toward each other in order to annihilate. Since the monopoles will generally move at relativistic (even highly relativistic) speeds, the gravitational acceleration on the surrounding matter will be large. If the accelerating monopoles closely resemble Lorentz-boosted versions of the stationary solution, then we would expect that the density perturbations would be linear structures in analogy to the sheetlike "wakes" predicted by the cosmic-string model.[10] Our numerical simulations show, however, that the monopoles become significantly distorted before they annihilate,[11] but it might still be reasonable to expect filamentlike perturbations anyway. The spectrum of fluctuations should be broadly of the Harrison-Zeldovich form (like cosmic strings or global textures), but with significant non-Gaussian features on large scales. The apparent filamentlike nature of the perturbations provides an advantage over global texture because it may provide a way to generate correlations on the scale required to fit Abell-cluster observations. It might seem that cosmic strings would be a more attractive scenario for structure formation because they naturally provide the sheetlike perturbations that seem to be observed.[12] However, global monopoles have the advantage that their coherence length is larger than the coherence length of strings so that the dominant fluctuations which occur near equal matter and radiation density will be at a somewhat larger scale. We will address these issues more fully in a subsequent work which will contain calculations of the gravitational field produced by evolving global monopoles.

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