

Gravitational Couplings of the Inflaton in Extended Inflation

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We discuss a new extended-inflationary scenario evading the difficulties of the original model. Our model can thermalize the energy in the bubble walls by the necessary epoch, and establish a Robertson-Walker frame in the bubble clusters. The essential new ingredient in our model is the observation that the coupling of inflaton to the Jordan-Brans-Dicke field is expected to be different from that of visible matter.

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Recently, La and Steinhardt¹ proposed a new inflationary-universe scenario, dubbed extended inflation, which allows the old inflation model of Guth² to succeed in percolating the true vacuum phase. It is based upon a Jordan-Brans-Dicke³ (JBD) theory coupled to an inflaton field, whose potential admits both a metastable state and a true ground state, separated by a potential barrier. Unfortunately, the original extended-inflation scenario suffered from some serious flaws, as pointed out by Weinberg⁴ and by La, Steinhardt, and Bertschinger.⁵ The problems found were twofold: While the true vacuum phase did indeed percolate, there were still problems of too many large bubbles of the new phase whose interiors could not be thermalized in time. There was also a problem with establishing a common Robertson-Walker frame in the various bubble clusters which would eventually coalesce to form our Universe.

In this Letter, we will show that these problems with extended inflation can be avoided in a new class of models promulgated at Damour, Gibbons, and Gundlach⁶ (DGG). They start with a generalized JBD model in which the JBD scalar field Φ couples with different strengths to "visible" matter and to "invisible" matter (thus leading to a violation of the weak equivalence principle). We will follow the line taken by DGG, and assume that the inflaton of extended inflation has an "invisible" coupling to the JBD scalar field. Since the iden-

tification of the inflaton is unknown, there is no reason to believe that it should couple to the JBD scalar field in the same way as does normal matter. Although this may seem *ad hoc*, it should be emphasized that such a situation in fact arises naturally in superstring theories,⁷ where the dilaton plays the role of the JBD scalar field. It is the existence of a new parameter, namely, the ratio of the couplings of visible matter and the inflation field to the JBD scalar field Φ , that allows us to evade the bounds on the ω parameter in JBD models.

We first express the action for our theory in the conformal frame in which the visible sector couples only to the metric $g_{\mu\nu}$, and not to the JBD field. This frame is referred to as the Jordan conformal frame. Following the metric and Riemann-tensor conventions in Ref. 8, the Brans-Dicke gravitational action $S_{\text{BD}}[g_{\mu\nu}, \Phi]$ is given by

$$S_{\text{BD}}[g_{\mu\nu}, \Phi] = \int d^4x (-g)^{1/2} \left[-\Phi R + \omega g^{\mu\nu} \frac{\partial_\mu \Phi \partial_\nu \Phi}{\Phi} \right], \quad (1)$$

where ω is the JBD parameter constrained by observations⁹ to be greater than 500 and Φ is the JBD scalar field.

Denoting the inflaton field by ψ_I , its action in the Jordan frame is

$$S_I[g_{\mu\nu}, \Phi, \psi_I] = \int d^4x (-g)^{1/2} \left[\frac{1}{2} (16\pi G_N \Phi)^{1-\beta} g^{\mu\nu} \partial_\mu \psi_I \partial_\nu \psi_I - (16\pi G_N \Phi)^{2(1-\beta)} V(\psi_I) \right]. \quad (2)$$

Denoting the field content of visible matter by ψ_V , in the Jordan frame the action for visible matter $S_V[g_{\mu\nu}, \psi_V]$ would simply be the action for a minimally coupled field, e.g., Eq. (2) with $\psi_I \rightarrow \psi_V$ and $\beta=1$. Expressing the action in the Jordan frame, the parameters of the model are β and ω . We further suppose that visible matter is described as usual via a perfect-fluid stress-energy tensor and will play no role in inflation.

Although we will find it convenient to work in the Jordan frame, it is instructive also to express our results in the Einstein conformal frame where the coefficient of the Ricci scalar is constant. This frame is realized by use of a metric $\bar{g}_{\mu\nu}$, related to the metric in the Jordan frame by the conformal transformation $g_{\mu\nu} \rightarrow \Omega^{-2} \bar{g}_{\mu\nu}$, where $\Omega^2 = (16\pi G_N \Phi)^{-1}$. The action in the Einstein frame is

$$\bar{S} = \int d^4x (-\bar{g})^{1/2} \left[-\frac{1}{16\pi G_N} \bar{R} + \frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] + \bar{S}_I \left[\exp \left[-\frac{2\beta_I \phi}{\phi_0} \right] \bar{g}_{\mu\nu}, \psi_I \right] + \bar{S}_V \left[\exp \left[-\frac{2\beta_V \phi}{\phi_0} \right] \bar{g}_{\mu\nu}, \psi_V \right], \quad (3)$$

where ϕ is related to Φ by $16\pi G_N \Phi = \exp(2\beta_V \phi / \phi_0)$, with $\phi_0^2 = (3+2\omega)/16\pi G_N$. In Eq. (3), $\bar{S}_i[\exp(-2\beta_i \phi / \phi_0) \bar{g}_{\mu\nu}, \psi_i]$ ($i=I, V$) is the action for a minimally coupled field with metric $\exp(-2\beta_i \phi / \phi_0) \bar{g}_{\mu\nu}$. For example, for a scalar field,

$$\bar{S}_I[\exp(-\beta_I \phi / \phi_0) \bar{g}_{\mu\nu}, \psi_I] = \int d^4x (-\bar{g})^{1/2} \left[\frac{1}{2} \exp(-\beta_I \phi / \phi_0) \bar{g}^{\mu\nu} \partial_\mu \psi_I \partial_\nu \psi_I - \exp(-2\beta_I \phi / \phi_0) V(\psi_I) \right]. \quad (4)$$

In the Einstein frame the parameters of the model are β_I and β_V . They may be related to the model parameters of the Jordan frame by $\beta = \beta_I / \beta_V$ and $\omega = (\beta_V^{-2} - 6)/4$. The constraint $\omega > 500$ implies $\beta_V < 0.022$, while present observations are consistent with $|\beta_I| \lesssim 1$.⁶ It is not meaningful to mix the model parameters of the two frames. One should consider either (ω, β) or (β_I, β_V) as the parameter set.

Let us now write the metric in the standard Robertson-Walker form with scale factor $a(t)$. We will assume, as required for "extended" (as well as "old") inflation to occur, that during inflation $\psi_I = \psi_0$, and $V(\psi_0) = \rho_F$, where the energy density of the false vacuum, ρ_F , dominates the total-energy density. Setting $\Lambda = 8\pi G_N \rho_F$, we have the following equations of motion for $a(t)$ and $\Phi(t)$:

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{\Lambda}{3} (16\pi G_N \Phi)^{1-2\beta} + \frac{\omega}{6} \left(\frac{\dot{\Phi}}{\Phi} \right)^2 - \frac{\dot{a}}{a} \frac{\dot{\Phi}}{\Phi}, \quad (5)$$

$$\frac{\ddot{\Phi}}{\Phi} + 3 \frac{\dot{a}}{a} \frac{\dot{\Phi}}{\Phi} = \frac{4\beta\Lambda}{2\omega+3} (16\pi G_N \Phi)^{1-2\beta}.$$

These equations are most easily solved in terms of the dimensionless field $\chi \equiv 16\pi G_N \Phi$. This system of equations admits power-law solutions for $k=0$, just as in the original extended-inflation scenario:

$$a(t) = a(0)(1+Bt)^p, \quad p = (\omega - \beta + \frac{3}{2}) / (2\beta - 1)\beta, \quad (6)$$

$$\chi(t) = \chi(0)(1+Bt)^q, \quad q = 2 / (2\beta - 1).$$

Here $t=0$ signifies the onset of inflation, and B is given by

$$B^2 = \frac{4\Lambda\beta^2(2\beta-1)^2[\chi(0)]^{1-2\beta}}{(2\omega+3)(6\omega+9-4\beta^2)}. \quad (7)$$

It can easily be seen that the above results reduce to those found in Ref. 1 when $\beta=1$. This is as it should be, since in this case the conformal transformation that takes us from DGG's original action to ours will act on the visible and inflaton sectors identically.

We now turn to the questions of whether inflation occurs in this model and whether a "graceful exit" can be achieved. Clearly, a necessary condition for inflation to occur in our theory is that the exponent p be greater than 1 so that \ddot{a}/a is a positive definite during the vacuum-energy-dominated period. This first constraint can be written in terms of ω and β as

$$\omega + \frac{3}{2} > 2\beta^2. \quad (8)$$

In order that sufficient inflation occur, we require $a(t_{\text{end}})/a(0) > 10^{27}$, where t_{end} denotes the end of the

inflationary period. Following the analysis of Ref. 4, we relate $a(t_{\text{end}})/a(0)$ to $\Phi(t_{\text{end}})/\Phi(0)$ via $a(t_{\text{end}})/a(0) = [\Phi(t_{\text{end}})/\Phi(0)]^{p/q}$. Since $\Phi(t_{\text{end}}) \sim M_{\text{Pl}}^2$, and neglect of quantum-gravity effects requires $\Phi(0) > \rho_F^{1/2} \sim M^2$, we arrive at our second constraint,

$$\omega + \frac{3}{2} > \left[1 + \frac{54}{10 - 2 \log_{10}[M/(10^{14} \text{ GeV})]} \right] \beta. \quad (9)$$

Next, we turn to the constraints coming from percolation and thermalization of the phase transition. The basic techniques for calculating tunneling amplitudes in field theories were developed by Callan and Coleman,¹⁰ and require computation of the Euclidean action of the bounce configuration that interpolates between the true and false vacua. The tunneling rate per unit four-volume is then given by $\lambda = A \exp[-S_E(\psi_B)]$, where A is the prefactor containing information about fluctuations of the bounce configuration and $S_E(\psi_B)$ is the Euclidean action for this configuration (denoted by ψ_B).

This analysis was extended to include the effects of (classical) gravity by Coleman and De Luccia.¹¹ Unfortunately, their analysis is not directly applicable to our case due to the existence of the JBD field. In order to compute the tunneling rate in this case, we need to understand how to generalize the existing formalism to the case in which the false vacuum is "rolling" due to the evolution of the Φ field during the bounce. An attempt along these lines was pursued by Accetta and Romaneli¹² with some success. However, for our purposes, we follow previous work of ours on the subject⁸ and note that at late times (or equivalently, for large values of the JBD field) the variations of Φ can be neglected, and we can compute λ using the analysis of Ref. 8.

The Euclidean action for the ψ_I field in the truncated theory is given by

$$S_E(\psi_I) = \int d^4x \left[\frac{1}{2} \chi^{1-\beta} (\partial_\mu \psi)^2 + \chi^{2(1-\beta)} V(\psi_I) \right]. \quad (10)$$

As per our analysis in Ref. 8, the formalism of Ref. 10 can be applied to find a χ -independent bounce action. However, when proper care is taken in normalization and in projection of the functional determinant to the subspace orthogonal to the translational zero modes, a χ dependence appears in the prefactor:

$$\lambda(t) = \lambda_0 \chi^{2(1-\beta)} = \lambda_0 [\chi(0)]^{2(1-\beta)} (1+Bt)^{4(1-\beta)/(2\beta-1)}, \quad (11)$$

where λ_0 is the (constant) tunneling rate for $\chi=1$. Thus, the physical bubble-nucleation rate per unit four-volume is *time dependent* in this theory. This is quite unlike the

case in the original extended-inflation model at late times.^{8,12}

The parameter controlling the percolation properties of the phase transition in this theory is¹³ $\epsilon \equiv \lambda(t)/H^4(t)$, where $H(t)$ is the Hubble parameter $\dot{a}(t)/a(t)$. In our model, we have

$$\epsilon(t) = \frac{\lambda_0 \chi(0)^{2(1-\beta)}}{p^4 B^4} (1+Bt)^{4\beta/(2\beta-1)}. \quad (12)$$

In order for the nucleation to be successful, clearly $\epsilon(t)$ must increase in time. This implies that $4\beta/(2\beta-1) > 0$, which in turn implies that either $\beta < 0$ or $\beta > \frac{1}{2}$. This will be subsumed by other constraints. The time t_{end} for which $\epsilon(t)$ is larger than some critical value of order unity corresponds to the end of the inflationary period.^{1,4,13}

We now turn to the constraint coming from the requirement that the bubble clusters that will comprise the observable universe have enough time to thermalize their energy. The point is that the typical bubble cluster consists of a large bubble, together with much smaller ones. Most of the energy of the bubble is tied up in the bubble

walls, and collisions with other bubbles are required in order to allow this energy to spread through the bubble interior. The question is how long does it take for this energy to become thermalized, so as to lead to a homogeneous and isotropic universe.

If too many large bubbles are still completing the thermalization process at cosmologically sensitive times, severe conflicts with big-bang predictions will clearly ensue. Thus, we must demand that the fraction of space in such bubbles be less than some predetermined value when the temperature is T . The volume fraction $\mathcal{V}_>(r, t_{\text{end}})$, the fraction of the volume contained in bubbles greater than a given (comoving) size r at the end of inflation, can be calculated exactly as in Ref. 4: $\mathcal{V}_>(r, t_{\text{end}}) \approx \ln[p^{-1}(t_{\text{end}})](r_0/r)^\delta$. Here r_0 is the asymptotic comoving size of a bubble nucleated at t_{end} , $\delta \equiv 4\beta^2/(\omega + \frac{3}{2} - 2\beta^2)$, and $p(t)$ is the probability of finding a point in space in the false vacuum at time t : $p(t) \sim \exp[-c(t/t_{\text{end}})^{(\rho-1)\delta}] \sim \exp[-c(r_0/r)^\delta]$, where c is a constant of order unity.

Imposing the condition that $\mathcal{V}_>(r, t_{\text{end}})$ to be less than 10^{-n} when the temperature is T , we arrive at the constraint

$$\omega + \frac{3}{2} < \left[2 + \frac{4\{23 + \log_{10}[M/(10^{14} \text{ GeV})] + \log_{10}[(1 \text{ eV})/T]\}}{n + \log_{10}\{\ln[p^{-1}(t_{\text{end}})]\}} \right] \beta^2. \quad (13)$$

It is not unreasonable to suppose that $p(t_{\text{end}}) < 1/e$ as in Ref. 4, so that for $M \sim 10^{14}$ GeV and $n \approx 5$ at recombination ($T \approx \frac{1}{3}$ eV), we have the constraint $\omega + \frac{3}{2} < 20.7\beta^2$. Note that setting $\beta=1$ we recover the results (and constraints) of Ref. 4. The result is that whereas before the limit was $\omega \lesssim 20$, the limit now is $\omega/\beta^2 \lesssim 20$, which can easily be satisfied for $\omega > 500$.

Finally, we turn to the question of reestablishing a common Robertson-Walker frame in all the bubble clusters that will coalesce to form our Universe. We must

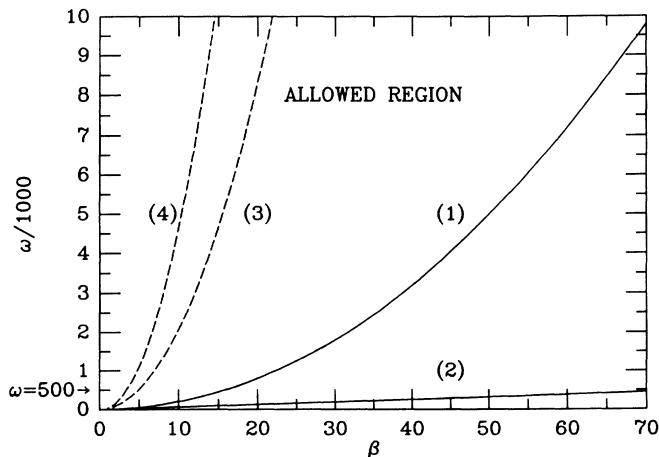


FIG. 1. The allowed region in ω - β space is indicated. Curves 1, 2, 3, and 4 correspond to the constraints in Eqs. (8), (9), (13), and (14), respectively. The observational limit $\omega = 500$ is indicated.

require that there be some way for the system to remember the original (pre-bubble-nucleation) coordinates. Weinberg⁴ argues that such a record can be found in the time evolution of $a(t)$, or equivalently $\Phi(t)$. Since a constant Hubble parameter and $\Phi(t)$ correspond to the de Sitter situation with *no* distinguished frame, we must require that there be sufficient variations of these quantities. The relevant time interval over which these variations should take place is between t_{end} and $t(r_{\text{univ}})$, the time when bubbles with asymptotic sizes equal to that of the observed universe were nucleated. The actual interval may, in fact, be shorter since we expect that homogeneity and isotropy must hold by the time of recombination or perhaps even nucleosynthesis. Using the fact that

$$H(t)/H(t_{\text{end}}) = [r_{\text{as}}(t)/r_0]^{1/(\rho-1)} \approx (M/T)^{\rho-1},$$

if we require a variation of m orders of magnitude in the Hubble parameter $H(t)$, we arrive at our final constraint

$$\omega + \frac{3}{2} < \beta(2\beta-1)(1+m^{-1}\{23 + \log_{10}[M/(10^{14} \text{ GeV})] + \log_{10}[(1 \text{ eV})/T]\}) + \beta. \quad (14)$$

Taking $M \sim 10^{14}$ GeV, $T \sim 10^2$ keV, and $m=1$ as in Ref. 4 leads to the constraint $\omega + \frac{3}{2} < 19\beta(2\beta-1) + \beta$.

We plot the constraints given by Eqs. (8), (9), (13), and (14) together with the constraint that $\omega > 500$ in Fig. 1. It should be clear from this plot that there is am-

ple room for all these constraints to be satisfied. If we make the constraints tighter [i.e., demanding that a smaller fraction of space still be thermalizing at recombination, or more orders of magnitude variation in $H(t)$], larger values of β will be required.

The constraints on (ω, β) can be easily translated to constraints on the parameters in the Einstein frame, (β_I, β_V) . Notice from Fig. 1 that for large ω , the most restrictive limits come from Eqs. (8) and (13). These two constraints can be combined into a β_I -independent constraint on β_I of

$$0.11 \leq |\beta_I| \leq 0.35. \quad (15)$$

Although a factor of 3.2 between the upper and lower bounds for β_I may not seem large, it should be remembered that β_I appears in the exponent. To illustrate this point, consider simply for the purpose of comparison, the implications of a similar range in β_V . The observational upper bound on β_V is $\beta_V \leq 0.022$. A hypothetical range in β_V of a factor of 3.2 would correspond to $0.0069 \leq \beta_V \leq 0.022$. Again the factor of 3 between the upper and lower bounds seems unduly restrictive. However, this range of β_V would correspond to an allowed range in ω of $5288 \geq \omega \geq 500$, which would not be considered overly restrictive. Likewise, we could define a parameter $\omega_I = (\beta_I^{-2} - 6)/4$ analogous to ω for visible matter. The allowed range of ω_I corresponding to the limits of Eq. (15) would be $20 \geq \omega_I \geq 0.54$.

The moral is that it is not productive to argue about whether the range of allowed values of the model parameters is sufficiently large, so long as no fine tuning (in the technical sense) is required. The real question is whether the ultimate theory of gravity will give our model. This question also arises in all models of extended inflation.

We now consider the question of reheating in our model. The energy of the bubble walls is in the form of inflaton matter. In order to convert this energy to visible (i.e., standard) matter, we may add couplings between the inflaton and visible sectors. One example might be $\lambda \psi_V^2 \psi_I^2$. The only constraints on such couplings are that

a reasonable reheating temperature be obtainable (in order that nucleosynthesis and/or baryogenesis be able to occur).

To conclude then, we have constructed a model of extended inflation in which the inflaton couples to the JBD field differently than standard visible matter. This yields a theory in which the JBD field is massless (i.e., no potential for Φ is required) and which meets all the requirements for an acceptable inflationary model with *no* fine tuning. If this avenue is to be pursued further, a realistic particle-physics model with the required couplings must be constructed.

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