Experimental Observation of Spatiotemporal Wave Forms of all Possible Types of Soliton-Antisoliton Interactions in Josephson Transmission Lines

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We report the first observation of soliton-antisoliton (fluxon-antifluxon) collision processes of all types (annihilation, passing through, and pair creation) in a discrete Josephson transmission line of niobium. These collision processes were directly measured not only in time but also in space by a Josephson sampling system with precise control of the collision point. Negative phase shifts, contrary to the analytic solution for the sine-Gordon equation, were observed and attributed to the dissipation.

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Solitons are universally localized entities in various nonlinear physical systems.[1] They have been experimentally observed in macroscopic systems, first in water surfaces and then in optical systems, gas plasmas, acoustic systems, etc.[1] They also are believed to play an important role in physical properties of microscopic systems such as charge-density-wave states,[1] polymers such as polyacetylene,[1-3] magnetic systems,[1] and Josephson transmission lines.[1,4-6] Although the solitons in these systems are modeled by several equations (for example, the Korteweg–de Vries equation, the ϕ^4 equation, the sine-Gordon equation, etc.[7]), no direct observation has been made of the line shapes of microscopic solitons or the dynamics of interactions among them.

Recently, we have reported on the dynamics of the interaction between mesoscopic solitons in time and space coordinates in a Josephson transmission line.[8] In the previous work, however, the observation was limited to the annihilation process of a fluxon-antifluxon pair on collision. In this Letter we report the spatiotemporal observation of all possible types of fluxon-antifluxon collisions in a Josephson transmission line (JTL) obtained by control of the value of the line bias current. This is the first observation in solid-state systems.

The Josephson circuits were fabricated by use of a conventional niobium circuit technology.[9] A niobium junction is much more reliable compared to the Pb junction used in the previous report.[8] The superconducting circuit comprises four components,[8] a Josephson sampler, a discrete JTL, and two identical pulse generators (PGs). The JTL was discrete and was composed of 31 $Nb/AlO_x/Nb$ junctions of 4 μm$\times$4 μm with shunt resistance. The critical current of each junction was designed to be 0.32 mA. The measured total critical current I_0 of the JTL was 11.2 mA. The capacitance of the junctions was 9.18 $\mu F/cm^2$, as measured by the monitor SQUID (the two-junction interferometer) on the same chip. The spacing between the junctions was 60 μm. The loop inductance L of each section of the discrete JTL was 5.96 pH. Therefore the penetration length and the characteristic time of the JTL are $\lambda_J = 25.0$ μm and $\tau_J = 1.23$ psec. In an overlap structure, the bias current I_b flows uniformly and normal to the length of the JTL. $AuIn_2$ films were used as resistors. The shunt resistance for each junction was 1.33 Ω. The sampling gate was connected to the center of the JTL through a resistance of 2.48 Ω.

The key point for observing all types of collision processes is the feasibility to introduce a fluxon-antifluxon pair for any value of the line bias current. In previous work[8] the fluxon-antifluxon pair could not be introduced into the line when the bias current was high enough to achieve the passing-through process of the pair. In the present work, a fluxon and an antifluxon were initially set at each end of the JTL by using mutually coupled inductors, and then trigger pulses from the PGs released the fluxons to let them enter. The initial setting of fluxons made it easier to introduce fluxons into the JTL compared with our previous circuits.[8] The measuring system is essentially the same as the one described in the previous report,[8] except that the system was fabricated on Nb base. The system is basically an ordinary Josephson sampler with an extra electrical delay in addition to a mechanical delay line. The relative position of the pair in the JTL, or equivalently the sampling time after the introduction, was controlled by the mechanical delay. The absolute position of the center of gravity of the pair with respect to the measuring point was scanned by the electric delay which controls the time difference of introducing a fluxon and an antifluxon. Scanning the center of the pair for fixed relative position is equivalent to scanning the measuring position for fixed pair position.[10]

Figure 1 shows the spatiotemporal evolution of the wave forms of a fluxon-antifluxon pair passing through each other. In this case the two peaks of the fluxon and the antifluxon merge into a higher single peak at the col-

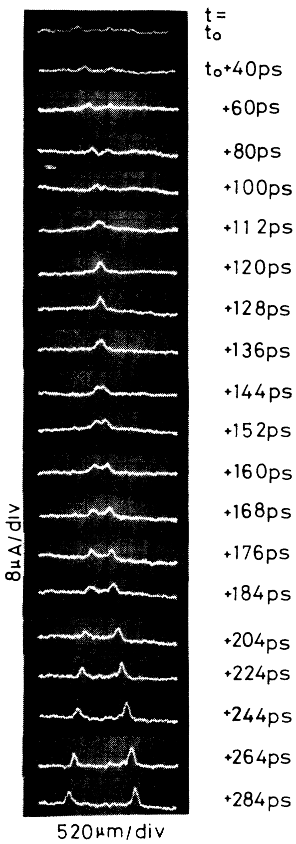

FIG. 1. Fluxon-antifluxon passing-through wave forms in time and space. The unit of vertical axis is 8 μA/div. The horizontal axis denotes the distance in the JTL converted from the electrical delay time. The time for each wave form is directly obtained from the mechanical delay time indicated at the right-hand side of them. The line bias current I_b is $0.87 I_0$. A gradual increase of the sensitivity of the sampler with delay time is perhaps an artifact of the sampling system, which is not understood at the present moment.

lision, and then they recover the original forms decreasing the peak height. The positions of the fluxon and the antifluxon are plotted as a function of time for two values of the line bias current in Fig. 2. It can be seen from Fig. 2 that the fluxons have achieved the steady propagation velocities as is expected. It should be noted that the velocities increase just before interacting with the antifluxon. Hence, it is considered that the interaction between a fluxon and an antifluxon is attractive. After the interaction the propagating fluxons recover the velocity equal to the one before interaction. The unit of the horizontal axis in Fig. 1 was roughly estimated to be 520 μm/div. The interaction extends over about 60 ps in time and about 200 μm in space for $I_b = 0.87 I_0$. The interaction length is comparable to the result for lead alloy junctions, but the interaction time is longer compared with the lead alloy JTL.[8] The propagating velocity is estimated to be $\sim 5\times10^6$ m/s for $I_b = 0.87 I_0$ which is slow compared with the JTL composed of lead alloy junc-

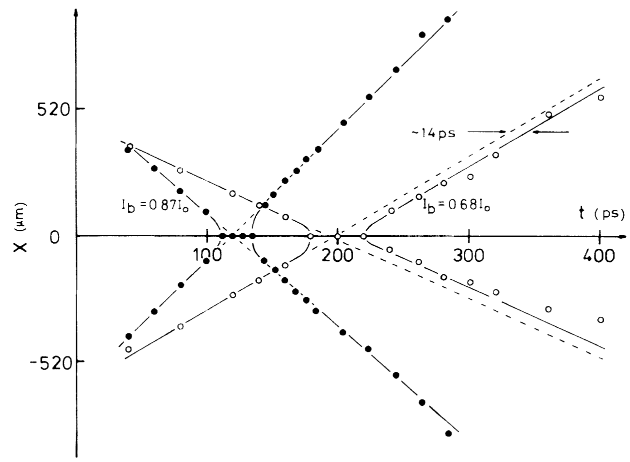

FIG. 2. Experimentally obtained fluxon-antifluxon collision trajectories in time and space for two values of the line bias current.

tions.[8] The relatively long interaction time and low velocity are attributed to the greater value of surface impedance of niobium[9] as compared with lead.

Figure 2 shows the negligibly small phase shift of soliton interaction[11] for $I_b = 0.87 I_0$. But the phase shift observed for $I_b = 0.68 I_0$ was about $+14$ ps time delay, or equivalently $11\tau_J$, as shown in Fig. 2. The sign of the observed phase shift was always negative (positive time delay) and opposite to that calculated for the lossless sine-Gordon equation. The dissipation increases proportionally to the square root of the pulse voltage height, while the acceleration is proportional to the pulse voltage height. Because the pulse height increases at the collision,[12] the dissipation overcomes the acceleration at the collision, resulting in the negative phase shifts.[13] The numerical calculation[14] shows that the phase shift is always negative independent of the bias current for the experimental loss parameter, and that the magnitude of the phase shift decreases with increasing line bias current. The numerical result agrees with the experimental result.

Figure 3 shows the observed wave forms in time and space of a fluxon-antifluxon collision for the higher line bias current $I_b = 0.92 I_0$. One can see the spread of a steplike wave form after the collision between the fluxon pair. The steplike wave form is considered to be a propagation of fluxon array.[15] The amplitude of the steplike wave form increases with increasing line bias current.[14] It seems that many pairs of fluxon and antifluxon are generated at the collision point of JTL in Fig. 3. The pair creation may be explained by the following mechanism.[16] The potential energy (magnetic energy) of a fluxon and an antifluxon is converted into the kinetic energy (electric energy) at their collision.[12] Hence, the two peaks of the fluxon and the antifluxon merge into a higher-voltage single peak at their collision. The higher-voltage part of the JTL receives more input power from the bias current, because the input power is propor-

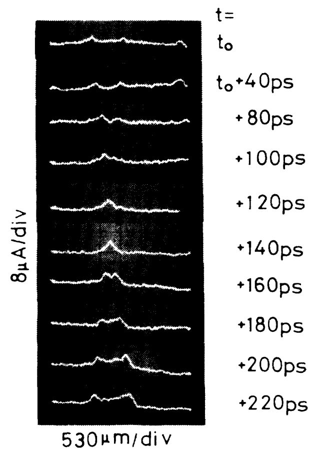

FIG. 3. Fluxon-antifluxon pair-creation wave forms in time and space. The line bias current I_b is $0.92I_0$.

tional to the voltage and the bias current. If the input power increases over the threshold energy, an additional fluxon-antifluxon pair will be created, satisfying a quantum condition inside the JTL. Once a new fluxon-antifluxon pair is created, the creation of infinite pairs is induced because of inertia (capacitance). And then they spread from the initial colliding position to the ends of the JTL.

For the lower line bias current, the fluxon-antifluxon annihilation is observed similarly to the case of the JTL composed of lead alloy junctions.[8] It was estimated that the interaction area for the annihilation of a fluxon pair extended over about 120 ps in time and about 200 μm in space at $I_b = 0.65 I_0$.[14]

Figure 4 shows the numerical results for the fluxon-antifluxon interaction. The parameters used in the simulation for the JTL were the values close to the ones fabricated. The normalized shunt conductance[17] Γ of the junction and the normalized damping resistance[17] κ of each section of the JTL were 0.6 and 0.3 (0.59 and 0.28 in the experiment), respectively. Figures 4(a), 4(b), and 4(c) show the annihilation, the passing through of a fluxon pair, and the pair creation of fluxons, which were, respectively, obtained for $I_b = 0.65 I_0$, $I_b = 0.86 I_0$, and $I_b = 0.95 I_0$. It can be seen from the numerical result that the two peaks of a fluxon and an antifluxon merge into a higher single peak at the colliding point as seen in the experimental results.

Figure 5 shows the phase diagram numerically calculated for the interaction types. The horizontal axis and the vertical axis represent, respectively, the bias current and the dissipation. The boundary of the phase diagram was previously studied numerically[17] and analytically.[18] The observed interaction types are marked in the figure by triangles, open circles, and solid circles. The annihilation of fluxon pairs occurs in the lower bias region, the passing through of them in the higher bias region, and the pair creation in the highest line bias current. The

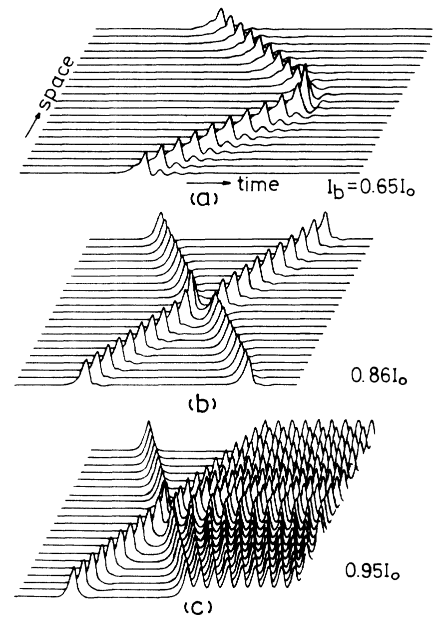

FIG. 4. Numerical simulations for fluxon-antifluxon interactions. (a)–(c) represent a fluxon-pair annihilation, a fluxon-pair passing through, and a fluxon-pair creation, respectively. The bias currents for (a), (b), and (c) are $0.65I_0$, $0.86I_0$, and $0.95I_0$, respectively.

bias-current dependence of the boundary between the annihilation and the passing through was analyzed by McLaughlin and Scott.[18] The bias condition for the pair creation shown in Fig. 5 may also be explained in terms of a power balance equation. The results of numerical calculations were found qualitatively to agree with the

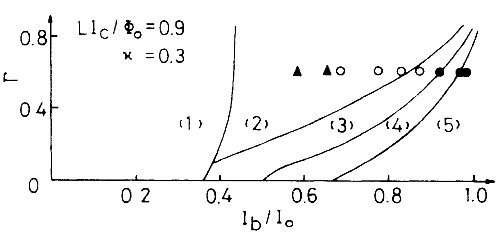

FIG. 5. Bias-current dependence of the interaction types of fluxon-antifluxon collisions. ▲, ○, and ● denote annihilation, passing through, and pair creation, respectively, which have been experimentally observed. (2), (3), and (4) denote the numerically obtained regions corresponding to annihilation, passing through, and pair creation, respectively. The bias current in the region (1) is too small for single fluxons to propagate down on the discrete JTL. In the region (5) the propagation of a fluxon is followed by the creation of many pairs of fluxons, resulting in a voltage transition of the whole JTL. In the figure I_c, Φ_0, Γ, and κ are the single-junction critical current of the JTL, the flux quantum, the normalized shunt conductance, and the normalized damping resistance, respectively. See also Ref. 17.

experimental result as shown in Fig. 5. But external noise, on-chip cross talks, and the nonuniformity of the bias current and the critical currents of junctions should be taken into account for the quantitative agreement. According to numerical simulation,[14] the continuous pair creation occurs at a lower bias current in a system with increasing value of inductance of a discrete JTL. This suggests that the continuous pair creation is explained in terms of the overshoot motion of "the pendulum" at the collision point which leads to the divergence of the quantum-mechanical phase difference.[16]

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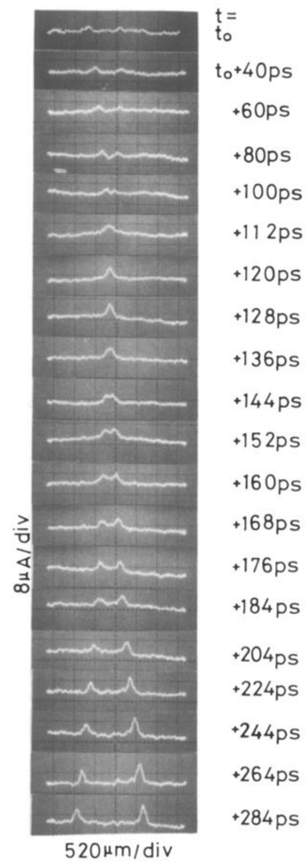

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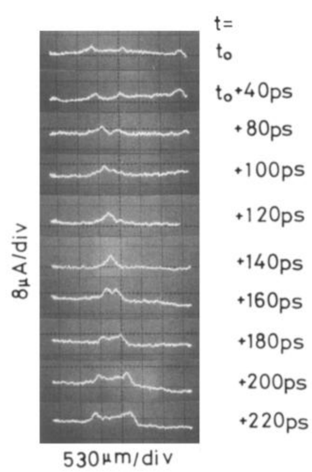

FIG. 3. Fluxon-antifluxon pair-creation wave forms in time and space. The line bias current I_b is $0.92I_0$.