Berry's Phase and Persistent Charge and Spin Currents in Textured Mesoscopic Rings

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We consider the motion of electrons through a mesoscopic ring in the presence of a classical, static, inhomogeneous, magnetic field. Zeeman interaction between the electron spin and this texture couples spin and orbital motion, and results in a Berry phase. As a consequence, the system supports persistent equilibrium spin and charge currents, even in the absence of conventional electromagnetic flux through the ring. We mention the possibility of analogous persistent mass and spin currents in normal ^{3}He.

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The quantum orbitial motion of electrons in mesoscopic normal-metal rings threaded by a magnetic flux produces striking interference phenomena such as persistent currents[1] and the Aharonov-Bohm effect.[2] Similarly, when a quantum spin adiabatically follows a magnetic field which rotates slowly in time, the phase of its state vector acquires an additional contribution known as the Berry phase.[3,4]

The purpose of this Letter is to explore the combination of these two quantum phenomena by examining the interplay between orbital and spin degrees of freedom for an electron moving in a mesoscopic ring.[5] To this end, we shall consider a ring which is placed in a classical, static, inhomogeneous, magnetic field, i.e., a texture,[6] as depicted in Fig. 1.

As a consequence of its orbital motion through the texture, the spin experiences a varying magnetic field, which results in a geometrical—or Berry—phase, besides the usual dynamical factor. This geometrical phase, which can be reformulated in terms of a spin-dependent gauge potential, leads to persistent equilibrium charge currents. It also leads to persistent spin currents and causes an Aharonov-Bohm interference effect. These phenomena should be distinguished from the related consequences of the conventional electromagnetic gauge potential.

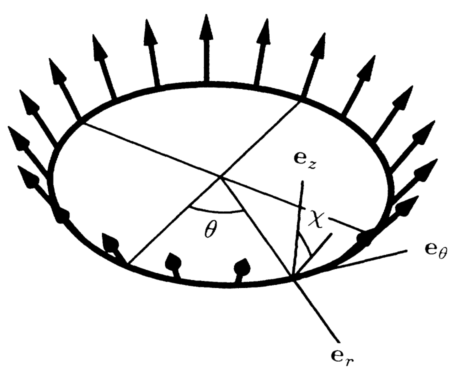

FIG. 1. Mesoscopic ring (thick circle) in an inhomogeneous magnetic field (arrows) with tilt angle χ. The local cylindrical coordinate system at the position θ is indicated by $\{\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z\}$.

Using a path-integral approach to decouple the orbital and spin motion, and an adiabatic approximation, we compute the equilibrium expectation values of the magnetization and persistent charge and spin currents. We derive explicit formulas for these quantities, and examine certain limits in detail. We comment on dynamical properties, and conclude with a brief discussion of experimental consequences for normal-metal and -^{3}He rings.

We consider noninteracting electrons of mass m, charge e, and spin $\frac{1}{2}$, confined to a ring of radius a, in the presence of a specified texture $B\mathbf{n}$. The ring is taken to be strictly one dimensional, lying in the x-y plane with its center at the origin, and the position of the electron is specified by the angular coordinate θ. The Hamiltonian for this system is taken to be

$$\hat{H} = \frac{1}{2ma^2}\left(\hat{p}_\theta - \frac{ea}{c}A_\theta\right)^2 - \gamma B\mathbf{n}(\hat{\theta})\cdot\hat{\boldsymbol{\sigma}}\,, \tag{1}$$

where $\hat{p}_\theta$ is the angular momentum operator conjugate to the coordinate operator $\hat{\theta}$, $\hbar\hat{\sigma}^i/2$ (with $i=1,2,3$) are the Cartesian components of the spin operator, and $\gamma = ge\hbar/4mc$ is the Bohr magneton.

The texture $B\mathbf{n} = \nabla\times\mathbf{A}$ is a classical, inhomogeneous magnetic field which we shall take to be a cylindrically symmetric crown, with fixed magnitude B and spatially varying orientation $\mathbf{n}(\theta) = \mathbf{e}_r\sin\chi + \mathbf{e}_z\cos\chi$, as depicted in Fig. 1. Here $\mathbf{e}_r$ and $\mathbf{e}_z$ are radial and axial basis vectors for a cylindrical coordinate system located at the point θ on the ring. The parameter χ describes the deviation of the texture from the z axis, and will be referred to as the tilt angle.[7] Because of the cylindrical symmetry A_θ may be chosen to be independent of θ on the ring. Of course, the device causing the magnetic field on the ring will typically also cause a magnetic flux through the ring. The Zeeman term $\gamma B\mathbf{n}(\hat{\theta})\cdot\hat{\boldsymbol{\sigma}}$ couples the spin and orbital motion. However, for simplicity, we have omitted from the Hamiltonian any additional spin-orbit coupling caused, for example, by the potential which confines the electron to the ring. We point out that such conventional spin-orbit coupling[8] can also lead to a geometrical phase, as implied by Meir, Gefen, and Entin-Wohlman,[9] in the context of a tight-binding model with spin-dependent hopping. It should be emphasized, however, that while the texture on our ring does indeed couple the spin and orbital motion, it does so through the Zeeman term.

We note, in passing, that the Longuet-Higgins model of Jahn-Teller molecules bears a formal resemblance to the present model, Eq. (1), but, of course, describes a quite different physical system.[10] However, in neither the mesoscopic nor microscopic context has the issue of persistent equilibrium currents due to Berry's phase been previously addressed for systems described by Eq. (1).

The dimensionless equilibrium currents, which will be computed in the canonical ensemble, are the charge current $\langle \hat{J}^0\rangle = \langle \hat{p}_\theta - eaA_\theta/c\rangle/\hbar$ and the spin current $\langle \hat{J}^i\rangle = \langle (\hat{p}_\theta - eaA_\theta/c)\hat{\sigma}^i\rangle/\hbar$, where $\langle\cdot\rangle = Z^{-1}\mathrm{Tr}(\cdot)\times\exp(-\beta\hat{H})$, $Z=\mathrm{Tr}\exp(-\beta\hat{H})$ is the partition function at temperature $T=1/\beta k_B$, and Tr denotes the trace.[11] For compactness, we shall write $\hat{\sigma}^0$ for the identity operator in spin space so that we may assemble the charge and spin currents together as $\langle \hat{J}^\mu\rangle = \langle (\hat{p}_\theta - eaA_\theta/c)\hat{\sigma}^\mu\rangle/\hbar$, with $\mu = 0,\ldots,3$.

It will prove convenient to introduce the complete set of states $|\theta;\mathbf{n}(\theta),\alpha\rangle$, with $0\le\theta<2\pi$ and $\alpha=\pm1$, satisfying $\hat{\theta}|\theta;\mathbf{n}(\theta),\alpha\rangle = \theta|\theta;\mathbf{n}(\theta),\alpha\rangle$, and $\mathbf{n}(\hat{\theta})\cdot\hat{\sigma}|\theta;\mathbf{n}(\theta),\alpha\rangle = \alpha|\theta;\mathbf{n}(\theta,\alpha\rangle$, i.e., simultaneous eigenstates of $\hat{\theta}$ and $\mathbf{n}(\hat{\theta})\cdot\hat{\sigma}$. In terms of the more usual eigenstates $|\theta\rangle$ and $|\mathbf{e}_z,\alpha\rangle$, of $\hat{\theta}$ and $\hat{\sigma}^z$, respectively, these eigenstates are given by

$$|\theta;\mathbf{n}(\theta),\alpha\rangle = |\theta\rangle\otimes|\mathbf{n}(\theta),\alpha\rangle = |\theta\rangle\otimes e^{i\chi/2}e^{i\alpha\delta\theta}(\alpha\cos\tfrac{1}{2}\chi|\mathbf{e}_z,\alpha\rangle + e^{i\alpha\theta}\sin\tfrac{1}{2}\chi|\mathbf{e}_z,-\alpha\rangle)\,. \tag{2}$$

In order to have a single-valued basis under $\theta\to\theta+2\pi$, required in Berry's derivation, we impose periodic boundary conditions on our wave functions. As a particular consequence, the parameter δ is integral, although our final results will turn out to be independent of the choice of integer.

Inserting complete sets of states, one sees that the equilibrium expectation value of the current operator $\hat{J}^\mu$ can be written in terms of the thermal propagator $G(\theta_f,\alpha_f;\theta_i,\alpha_i)$ as

$$\langle \hat{J}^\mu\rangle = \frac{1}{Z}\sum_{\alpha_i,\alpha_f}\int_0^{2\pi}d\theta_i\int_0^{2\pi}d\theta_f\langle\theta_i;\mathbf{n}(\theta_i),\alpha_i|\hat{J}^\mu|\theta_f;\mathbf{n}(\theta_f),\alpha_f\rangle G(\theta_f,\alpha_f;\theta_i,\alpha_i)\,, \tag{3}$$

where $G(\theta_f,\alpha_f;\theta_i,\alpha_i)$ is given by

$$G(\theta_f,\alpha_f;\theta_i,\alpha_i) = \langle\theta_f;\mathbf{n}(\theta_f),\alpha_f|e^{-\beta\hat{H}}|\theta_i;\mathbf{n}(\theta_i),\alpha_i\rangle\,. \tag{4}$$

To evaluate $G(\theta_f,\alpha_f;\theta_i,\alpha_i)$ we construct a path-integral representation in which the orbital and spin degrees of freedom are decoupled, in the sense that the spin evolves (in imaginary time) in the presence of an external magnetic field which depends parametrically on the path of the orbital motion.[12] This is achieved through a simple extension of the standard technique[13,14] of inserting complete sets of states at infinitesimally separated imaginary-time slices. One then recognizes that the matrix elements regroup to give a weighted average over Feynman paths $\theta(\tau)$ of a spin propagator in the presence of a $\theta(\tau)$-dependent magnetic field. Thus, after a little algebra, one obtains the representation

$$G(\theta_f,\alpha_f;\theta_i,\alpha_i) = \int_{\theta(0)=\theta_i}^{\theta(\beta)=\theta_f}\mathcal{D}_c\theta\langle\mathbf{n}(\theta_f),\alpha_f|\hat{U}_\theta(\beta)|\mathbf{n}(\theta_i),\alpha_i\rangle e^{-S_0[\theta]}\,, \tag{5}$$

where the Euclidian orbital action $S_0[\theta]$ is given by

$$S_0[\theta] = \frac{ma^2}{2\hbar^2}\int_0^\beta d\tau\,\dot{\theta}(\tau)^2 - i\frac{ea}{\hbar c}A_\theta\int_0^\beta d\tau\,\dot{\theta}(\tau)\,, \tag{6}$$

and the subscript c indicates that the measure $\mathcal{D}_c\theta$ is compact.[15] The spin propagator $\hat{\mathbf{U}}_\theta(\beta)$ describes the evolution of the spin in the imaginary-time–dependent magnetic field $B\mathbf{n}(\theta(\tau))$ and satisfies the Schrödinger-Bloch equation $\partial_\tau\hat{U}_\theta(\tau) = \gamma B\mathbf{n}(\theta(\tau))\cdot\hat{\sigma}\hat{U}_\theta(\tau)$, with initial condition $\hat{U}_\theta(0)=\hat{I}$.

To solve the problem of the motion of a quantum-mechanical spin in the presence of the magnetic field $B\mathbf{n}(\theta(\tau))$ we use the adiabatic approximation,[8,16] valid for large β, in the form studied by Berry[3] and, subsequently, many others.[4] In this approximation, a system prepared in a nondegenerate eigenstate of $\gamma B\mathbf{n}(\theta(0))\cdot\hat{\sigma}$ evolves into the eigenstate of $\gamma B\mathbf{n}(\theta(\beta))\cdot\hat{\sigma}$ with the same quantum numbers, thereby acquiring two factors, a dynamical one due to Zeeman splitting, $\exp(\alpha\beta\gamma B)$, and a geometrical one, $\exp[i\alpha\Gamma(\beta)]$. The latter factor occurs because the usual adiabatic theorem does not determine the phase of the final state. Thus, the spin evolution is approximated by

$$\hat{U}_\theta(\beta)|\mathbf{n}(\theta_i),\alpha\rangle = e^{\alpha\beta\gamma B + i\alpha\Gamma(\beta)}|\mathbf{n}(\theta_f),\alpha\rangle\,, \tag{7}$$

where the phase of the geometrical factor, i.e., Berry phase,[3] is given by

$$\alpha\Gamma(\beta) = -\mathrm{Im}\int_0^\beta d\tau\langle\mathbf{n}(\tau),\alpha|\frac{\partial}{\partial\tau}|\mathbf{n}(\tau),\alpha\rangle = \alpha\phi^g(\delta)\int_0^\beta d\tau\,\dot{\theta}(\tau)\,, \tag{8}$$

and the geometric flux $\phi^g(\delta)$ is determined in terms of the tilt angle χ as

$$\phi^g(\delta) = \tfrac{1}{2}(\cos\chi - 1 - 2\delta)\,. \tag{9}$$

Equation (8) may be obtained by inserting the adiabatic form, Eq. (7), into the Schrödinger-Bloch equation and then using the explicit form of the spin eigenstates, Eq. (2). Note that $\Gamma(\beta)$ depends on the Feynman path[17] $\theta(\tau)$ which is not, in general, closed.[18] As stated above, A_θ is independent of θ; we may therefore exchange it for the (dimensionless) electromagnetic flux through the ring, $\phi^{\rm em} = (ea/\hbar c)A_\theta$.

At this stage we convert the path integral for G, Eq. (5), into one over extended paths,[13,15] thereby introducing the winding number ν for each path. Following the method described in Chap. 23 of Ref. 12, we conclude that the thermal propagator is given by

$$G(\theta_f,a_f;\theta_i,a_i) = \delta_{a_f,a_i}(\kappa/\sqrt{2\pi})\exp[a_i\beta\gamma B + i(\theta_f-\theta_i)\Phi(a_i;\delta) - \kappa^2(\theta_f-\theta_i)^2/2] \times\Theta_3(\pi\Phi(a_i;\delta)+\pi i\kappa^2(\theta_f-\theta_i);2\pi i\kappa^2)\,, \tag{10}$$

where $\kappa^2 = ma^2/\beta\hbar^2$ is a dimensionless parameter which measures the ratio of the thermal energy and the spacing between energy levels for a free particle on the ring. For a ring of radius 3000 Å and a temperature of 10 mK, κ is of order unity. We have introduced (i) the spin-dependent (dimensionless) flux $\Phi(\alpha;\delta) = \phi^{\rm em} + \alpha\phi^{g}(\delta)$, which combines the usual electromagnetic contribution and the purely geometrical Berry phase; and (ii) $\Theta_3(z;t) = \sum_{\nu=-\infty}^{\infty}\exp(i\pi t\nu^2 + 2i\nu z)$, the Jacobi theta function.[19]

We now use the thermal propagator to compute the partition function, currents, and magnetization. The partition function is given by

$$\begin{aligned} Z &= \sum_{\alpha=\pm1}\int_0^{2\pi} d\theta\, G(\theta,\alpha;\theta,\alpha) \\ &= \sqrt{2\pi}\kappa \sum_{\alpha=\pm1} e^{\alpha\beta\gamma B}\Theta_3(\pi\Phi(\alpha;0);2\pi i\kappa^2) \\ &= \sum_{\alpha=\pm1}\sum_{n=-\infty}^{\infty} e^{-\beta\mathcal{E}_{n,\alpha}}\,, \end{aligned} \tag{11}$$

with the effective energy spectrum $\mathcal{E}_{n,\alpha} = (\hbar^2/2ma^2)[n - \Phi(\alpha;0)]^2 - \alpha\gamma B$. This spectrum makes explicit the interpretation of the geometrical phase as a spin-dependent gauge potential.[20] In the last step we have used the identity[13]

$$\Theta_3(z;t) = (-it)^{-1/2}\exp(z^2/i\pi t)\Theta_3(z/t;-1/t)\,.$$

Note that Eq. (11) reduces to the exact partition function when the tilt angle χ vanishes.

The charge current $\langle\hat J^0\rangle$ can be expressed as $\kappa^2 \times\partial\ln Z/\partial\phi^{\rm em}$, and hence

$$\langle\hat J^0\rangle = \frac{1}{Z}\sum_{\alpha,n}[n-\Phi(\alpha;0)]e^{-\beta\mathcal{E}_{n,\alpha}}\,. \tag{12}$$

From Eqs. (3) and (10) we see that the spin current is given by

$$\begin{aligned} \langle\hat J^3\rangle + \tfrac{1}{2}\sin^2\chi &= \kappa^2\cos\chi\frac{\partial}{\partial\phi^g}\ln Z \\ &= \cos\chi\frac{1}{Z}\sum_{\alpha,n}\alpha[n-\Phi(\alpha;0)]e^{-\beta\mathcal{E}_{n,\alpha}}\,. \end{aligned} \tag{13}$$

The other two components of the spin current $\langle\hat J^{1,2}\rangle$ vanish. Note that physically observable quantities are independent of the integer δ, and that the charge current vanishes whenever $\Phi(\alpha;0)$ is integral or half integral. This is the case, e.g., when $\phi^{\rm em}=0$ and $\chi=\pi/2$. Also note that under the latter conditions the spin current reduces to $-\frac{1}{2}$. The dependence of the spin current on χ and $\phi^{\rm em}$ is depicted in Fig. 2. The magnetization is given by $\langle\hat\sigma^3\rangle = Z^{-1}\cos\chi\sum_{\alpha,n}\alpha e^{-\beta\mathcal{E}_{n,\alpha}}$, while the other two components $\langle\hat\sigma^{1,2}\rangle$ vanish. Finally, the extension to many noninteracting electrons amounts to replacing $Z^{-1}\exp(-\beta\mathcal{E}_{n,\alpha})$ by the normalized Fermi distribution in the above formulas.

We now discuss the charge and spin currents and magnetization in the low-temperature limit, which we indicate by the subscript 0. The charge current reduces to the sawtooth function $\langle\hat J^0\rangle_0 = -\{\Phi(1;0)\}$, where $\{\Phi\} = \Phi - [\Phi + \frac{1}{2}]$, and $[\Phi]$ is defined to be the largest integer less than or equal to Φ. Similarly, for the spin current we find $\langle\hat J^3\rangle_0 = -\frac{1}{2}\sin^2\chi - \{\Phi(1;0)\}\cos\chi$, and for the magnetization $\langle\hat\sigma^3\rangle_0 = \cos\chi$. Recall that $\Phi(1;0)$ depends on χ, as shown by Eq. (9). It is interesting to note that the low-temperature limit of the spin current can also be written in the following form $\langle\hat J^0\hat\sigma^3\rangle_0 = \langle\hat J^0\rangle_0\langle\hat\sigma^3\rangle_0 - \frac{1}{2}\sin^2\chi$. The last term in this equation, which is due to the geometrical phase, makes explicit the significance of the correlation in the quantum-mechanical fluctuations of $\hat\sigma^3$ and $\hat p_\theta - eaA_\theta/c$ induced by the effective spin-orbit coupling. In particular, note that when $\chi=\pi/2$ the magnetization component $\langle\hat\sigma^3\rangle = 0$. Despite this, as a consequence of the effective spin-orbit coupling generated by the geometrical phase, the spin current does not vanish: fluctuations which produce a positive z component of spin and a negative z component of orbital angular momentum (or *vice versa*) give a larger contribution to the partition function than those with parallel z components.

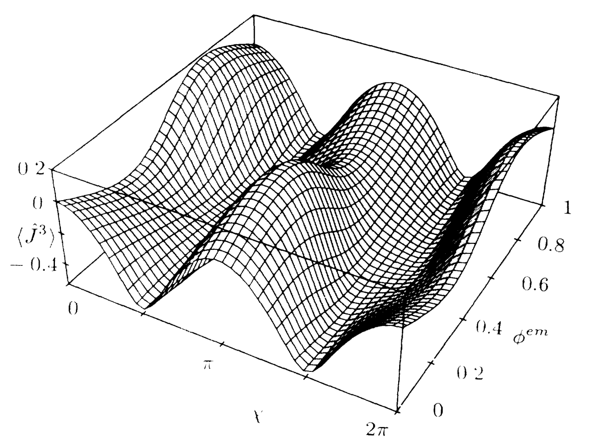

FIG. 2. Spin current $\langle\hat J^3\rangle$ as a function of tilt angle χ and electromagnetic flux $\phi^{\rm em}$ for $B=50$ G, $T=1$ mK, and $a=3000$ Å.

We now make the additional restriction to small tilt angles, $\chi \ll 1$ and, for simplicity, we shall also assume that ϕ^{em} is integral. Then, to quadratic order in χ, we find that for the spin magnetization $\langle \sigma^3 \rangle_0 \simeq 1 - \chi^2/2$, and for the charge and spin currents $\langle \hat{J}^0 \rangle_0 \simeq -\langle \hat{J}^3 \rangle_0 \simeq \chi^2/4$.

We have concentrated on the equilibrium properties of textured rings in order to display persistent currents. For integral ϕ^{em}, these currents can be thought of in the following way: (i) The field causes paramagnetic alignment of the spin; (ii) the geometrical phase induces an effective coupling between the spin and orbital angular momentum such that nonzero orbital angular momentum is preferred, causing a charge current; and (iii) the spin current is dominated by quantum fluctuations, as described earlier. There is also an intriguing Aharonov-Bohm–type interference phenomenon associated with real-time propagation which can be simply derived following the scheme presented here; we shall discuss this and related issues in a forthcoming publication. Besides making an experimental search for the effects described here in mesoscopic normal-metal rings, e.g., using NMR spin-echo experiments, it would also be interesting to look for analogous mass and spin currents in the Fermi-liquid regime of normal ^{3}He in a mesoscopic ring. As the quasiparticle excitations in this regime carry spin but no charge, persistent currents should arise purely from the geometrical phase.

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[4]For a collection of relevant reprinted articles and commentary, see A. Shapere and F. Wilczek, *Geometric Phases in Physics* (World Scientific, Singapore, 1989).

[5]By mesoscopic we mean sufficiently small that quantum-mechanical phase coherence is preserved around the ring. As a rough criterion the ring radius should be less than 3000 Å if the temperature is 10 mK.

[6]One might, for example, attempt to produce a texture by locating the mesoscopic ring between a pair of Helmholtz coils, with the current in one coil reversed. Alternatively, the ring could be fabricated from an intrinsically ferromagnetic material, or supported on an inhomogeneous, insulating ferromagnetic substrate.

[7]The path-integral description presented here has two useful virtues. First, it is readily extendible to arbitrary textures, although for the sake of clarity we shall only give results for the cylindrically symmetric case. This particular case happens also to have an exact solution which we will publish elsewhere. Second, in contrast with the exact solution of the cylindrically symmetric case, the present description clearly exhibits the significance of the Berry phase with regard to persistent currents, and is applicable to a wide class systems which are not amenable to exact solution.

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[9]Y. Meir, Y. Gefen, and O. Entin-Wohlman, Phys. Rev. Lett. **63**, 798 (1989). Anticipating the experiments of Lévy *et al.* (Ref. 1), these authors show that spin-dependent hopping provides a mechanism for *period halving* of persistent-current oscillations.

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[11]The physical currents are obtained by multiplying $\langle \hat{J}^0 \rangle$ by $e\hbar/ma$, and $\langle \hat{J}^i \rangle$ by $\hbar/ma$.

[12]For a related approach to the dynamical propagator, see H. Kuratsuji and S. Iida, Prog. Theor. Phys. **74**, 439 (1985).

[13]See, for example, L. S. Schulman, *Techniques and Applications of Path Integration* (Wiley, New York, 1981).

[14]See also R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[15]By compact we mean that the paths $\theta(\tau)$ lie on the cylinder segment $S^1 \times [0,\beta]$. We shall also use the extended measure $\mathcal{D}_e\theta$ which, by contrast, indicates integration over paths on the strip $R^1 \times [0,\beta]$.

[16]M. Born and V. F. Fock, Z. Phys. **51**, 165 (1928).

[17]Typically $\theta(\tau)$ is continuous but nondifferentiable, and this roughness induces rapid variation in $B\mathbf{n}$ which could violate the assumption of adiabaticity. However, such an objection could be raised for any quantum-mechanical application of Berry's phase based on the adiabatic approximation, since there are, strictly speaking, no classical dynamical variables. Nevertheless, we shall assume that, as with many path integrals (see, e.g., Ref. 13, p. 39), the dominant contribution to the path integral comes from near-classical, smooth paths and, hence, that roughness does not invalidate our final results.

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