## Spacetime Singularities in String Theory and String Propagation through Gravitational Shock Waves

In Ref. 1 it is claimed that when a quantized string propagates through a gravitational shock wave, the expectation value of the square-string-mass operator $\langle M^2 \rangle \approx \sum_{n=1}^{\infty} n \langle N_n \rangle$ diverges. The expression used for $N_n$ in Ref. 1 corresponds to the linear approximation of the transformation between ⟨ and ⟩ operators. The exact transformation found in Refs. 2 and 3 can be linearized.[2]

The linear approximation holds only for large impact parameters $\rho_0$. One must use the exact transformation between $\alpha^i_{n<}$ and $\alpha^i_{n>}$ to include *all* impact parameters. We recently succeeded in computing $\langle N_n \rangle$ exactly[4] with the result

$$\langle M_n^2 \rangle_{\text{exact}} = A(-1)^{n+1} \times \int \frac{dp^{D-2}\,\Gamma(1-2\alpha' \mathbf{p}^2)}{\mathbf{p}^2 \Gamma(1+n-\alpha'\mathbf{p}^2)\Gamma(1-n-\alpha'\mathbf{p}^2)}, \quad (1)$$

$$A = \frac{(G\hbar p_U/\pi)^2}{(4\pi^2)^{D-4}}.$$

For large $n$, the asymptotic behavior results:

$$\langle M_n^2 \rangle_{\text{exact}} = \frac{A}{2\pi n}\left(\frac{\alpha'}{\pi}\ln n\right)^{1-D/2}. \quad (2)$$

Hence, the sum over $n$ in $\langle M^2 \rangle$ and $\langle N \rangle$ converges. As is clear by comparing Eq. (2) and $\langle M_n^2 \rangle_{\text{linear}} \approx 1/n$, the linear approximation does not hold for large $n$.

The integrand in Eq. (1) exhibits poles in the integration path. This is exactly what happens in the tree amplitudes of string models. These poles correspond to the tree-level string spectrum. As is usually expected, loop corrections provide a width to these resonances and will therefore shift the poles away from the integration path, leading to finite results.

In conclusion, test strings do propagate consistently in shock-wave space-times. We recall that the Klein-Gordon equation (for a point particle) is ill defined in this geometry, whereas the string equations are well behaved.[2,5] Analogous conclusions hold for quantum strings in the Schwarzschild geometry where a regular behavior was found at the horizon and at the $r=0$ singularity.[6] That is, strings feel the space-time singularities much less than point particles.

Furthermore, we would not be surprised by the presence of space-time singularities in string theory as long as one sticks to a geometry description using a metric tensor $G_{AB}(X)$ (in spite of the fact that it fulfills the string-corrected Einstein equations). We do not expect that a space-time description in terms of a Riemannian manifold with local coordinates $X^A$ will be meaningful at the Planck scale.

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